Lecture # 27

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Another Example

1	2	3

$0.5\pm$ $0.5\pm$ $0.5\pm$

1.5 ± 0.050
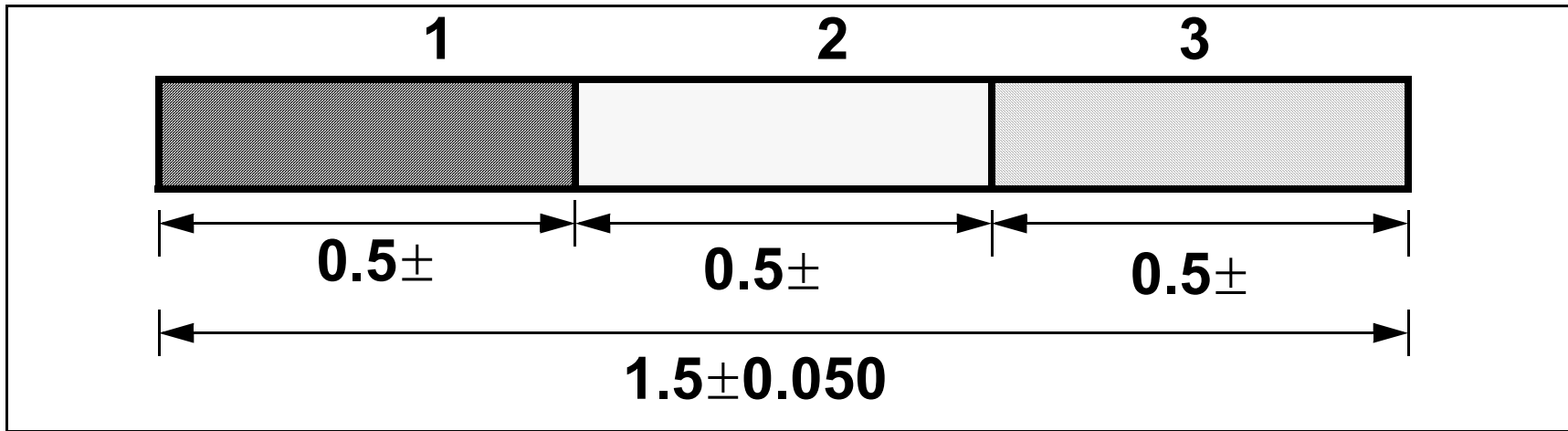
Three identical parts

Manufacturing process -- $C_p = 1.5$ (at least)

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What will the individual process σ_X be, if tolerances are obtained by simple division?

Find **(i) standard deviation for the processes**
(ii) tolerances for individual parts

Another Example

Hole	Pin
1.00 ± 0.015	0.965 ± 0.015
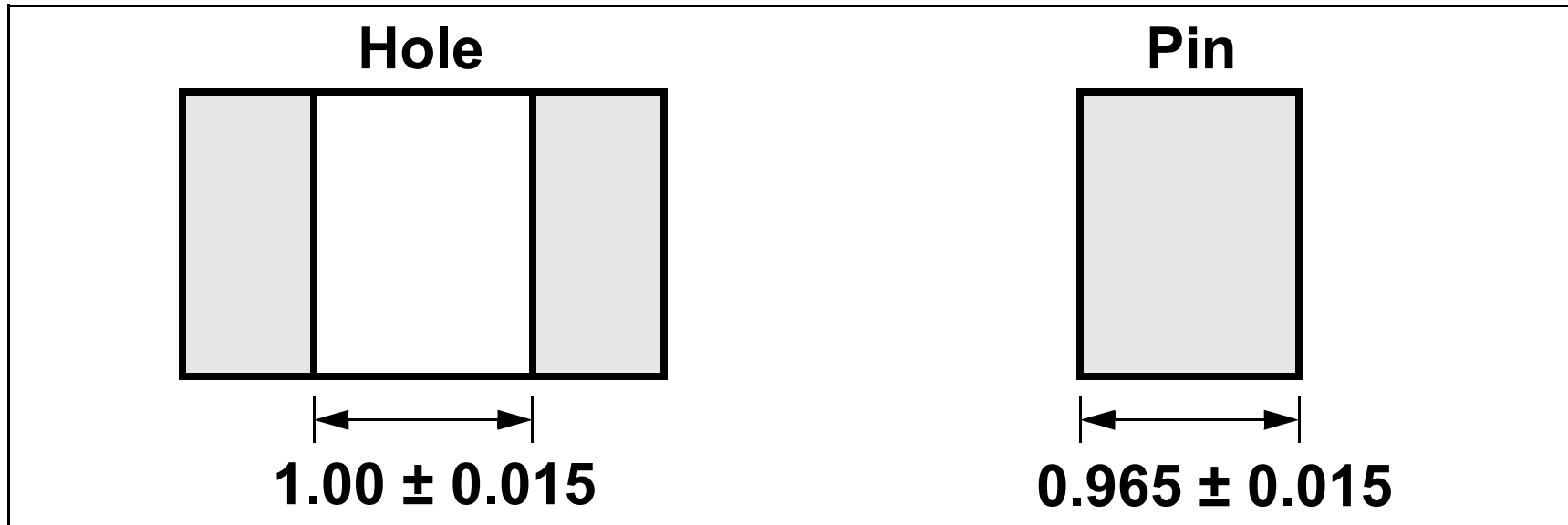
For now, let's assume hole & pin producing processes are centered at the nominal values and that processes have Cp values of 1.0.
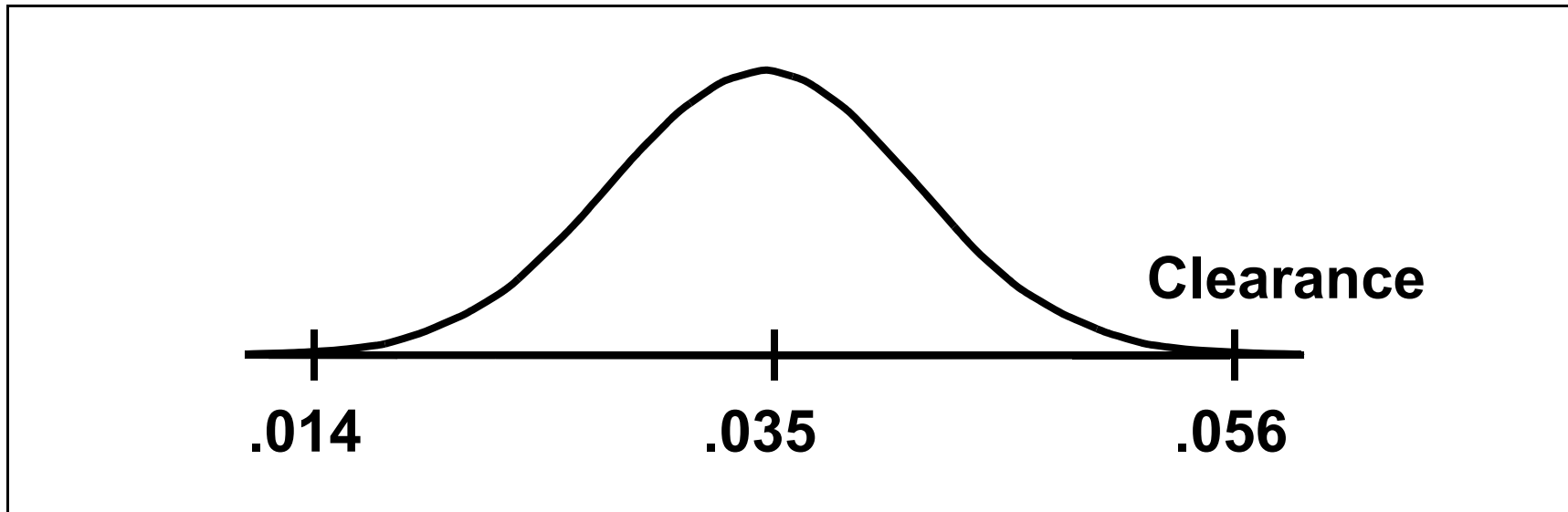
What does clearance dist. look like??
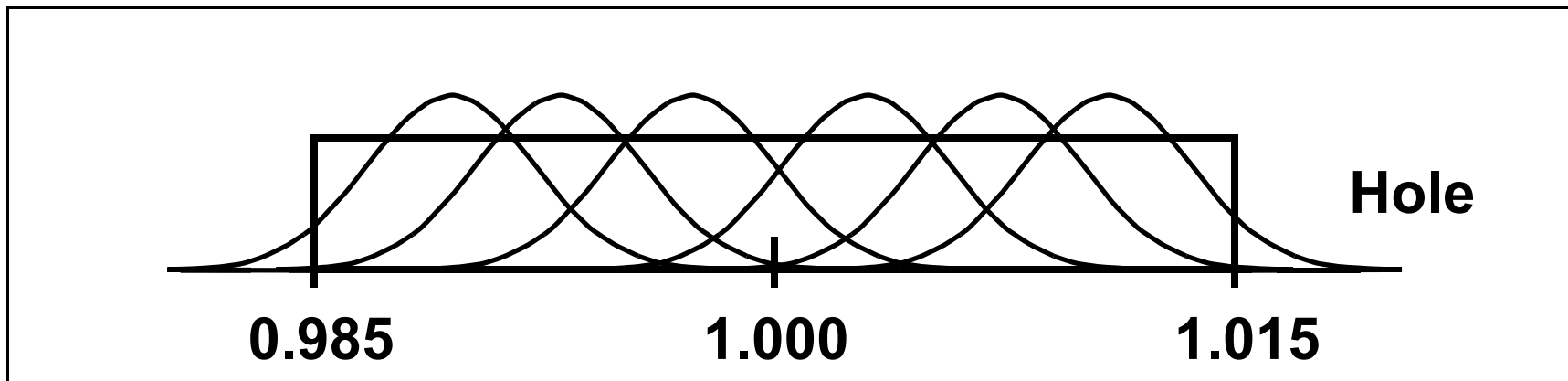
Clearance Distribution

$$C = H - P$$

$$\mu_C = \mu_H - \mu_P = 0.035$$

$$\sigma_C^2 = \sigma_H^2 + \sigma_P^2 \qquad ---- \qquad \sigma_c = 0.007$$

Clearance

.014　　　　.035　　　　.056

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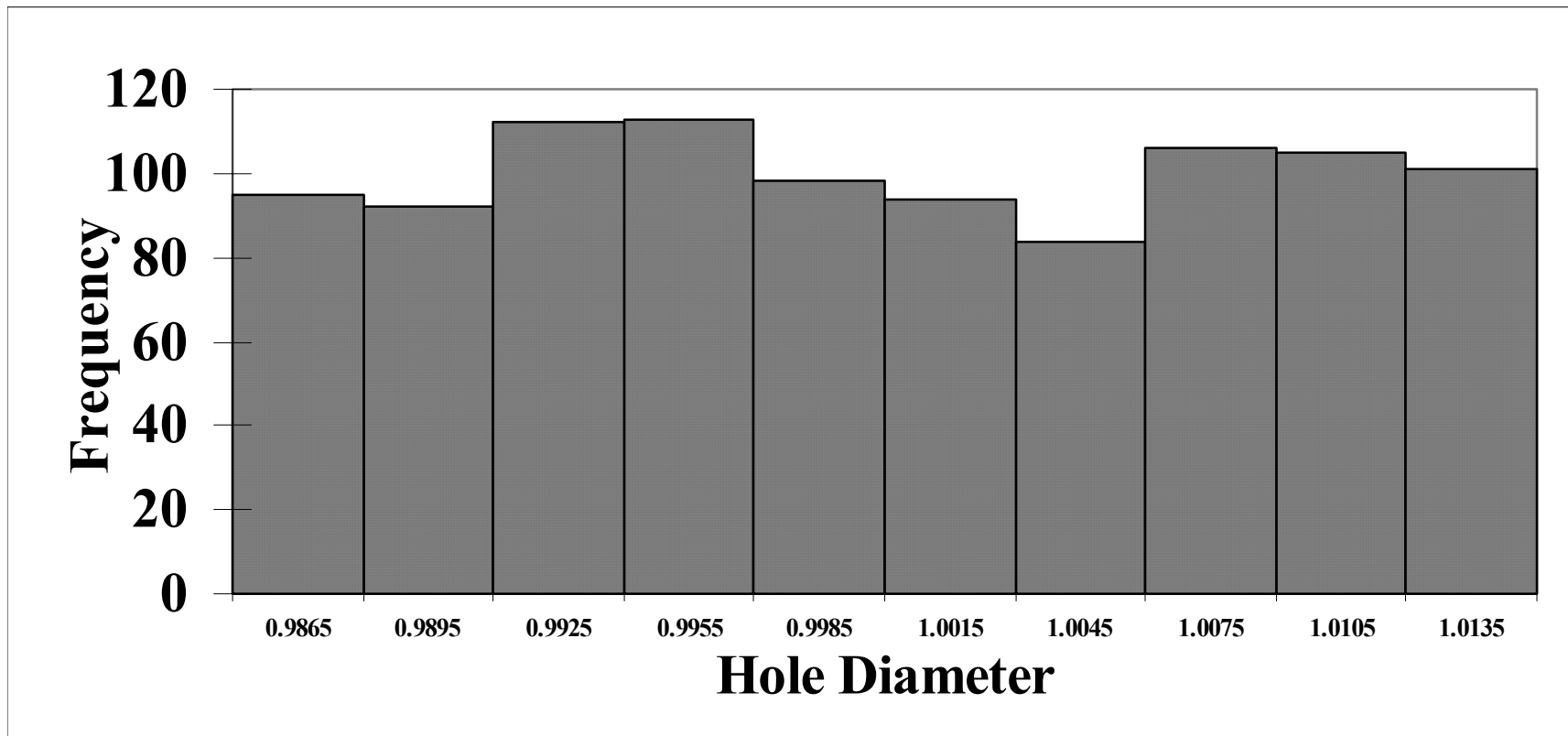**Michigan Technological University**
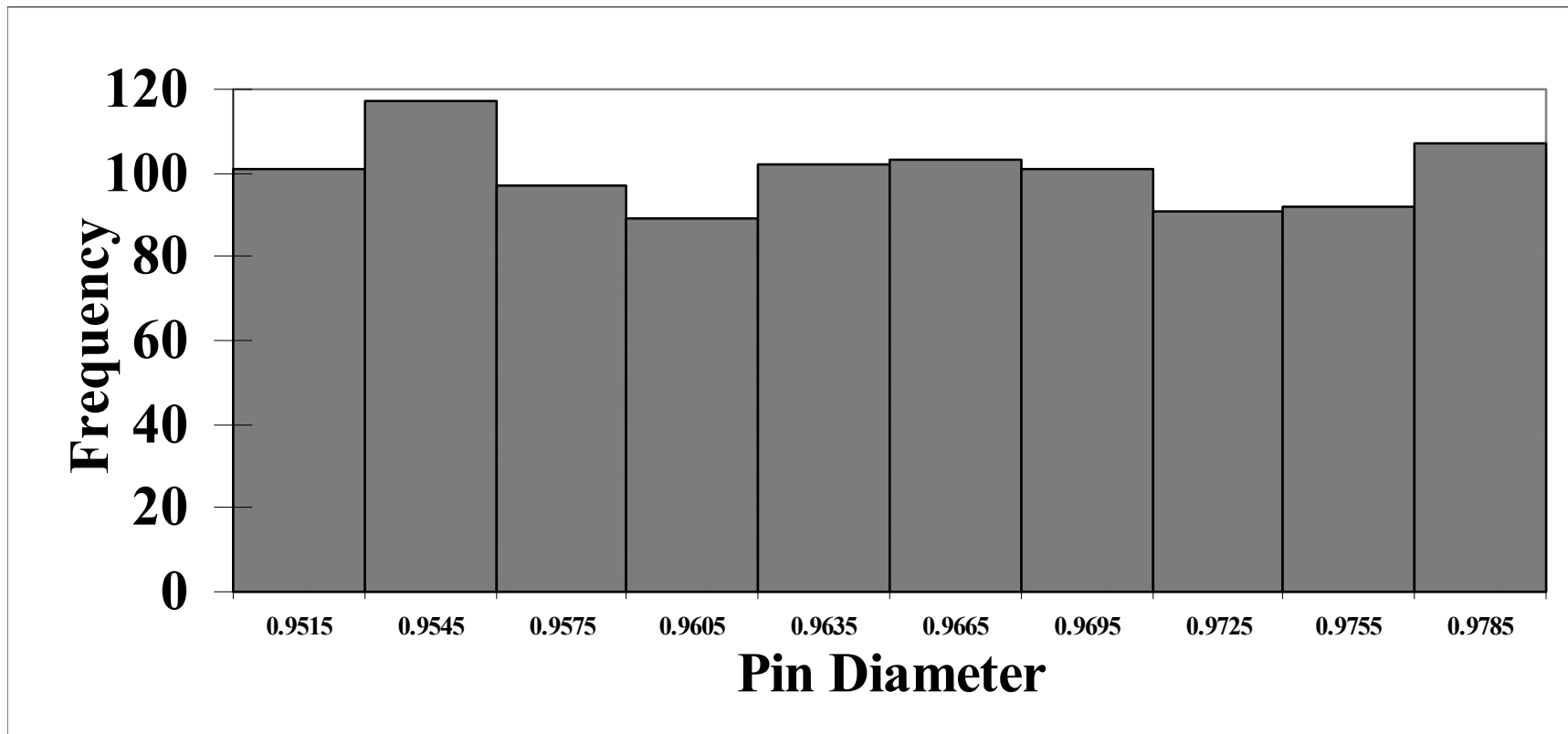
What can go Wrong?

- We have already seen that if our individual processes (in this case pin and hole) do not remain centered - the results can be disastrous.

- What if our processes are not maintained in a state of statistical control?

Hole

0.985   1.000   1.015

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Histogram - Hole Dimension

Histogram - Pin Dimension

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Histogram - Clearance Dimension

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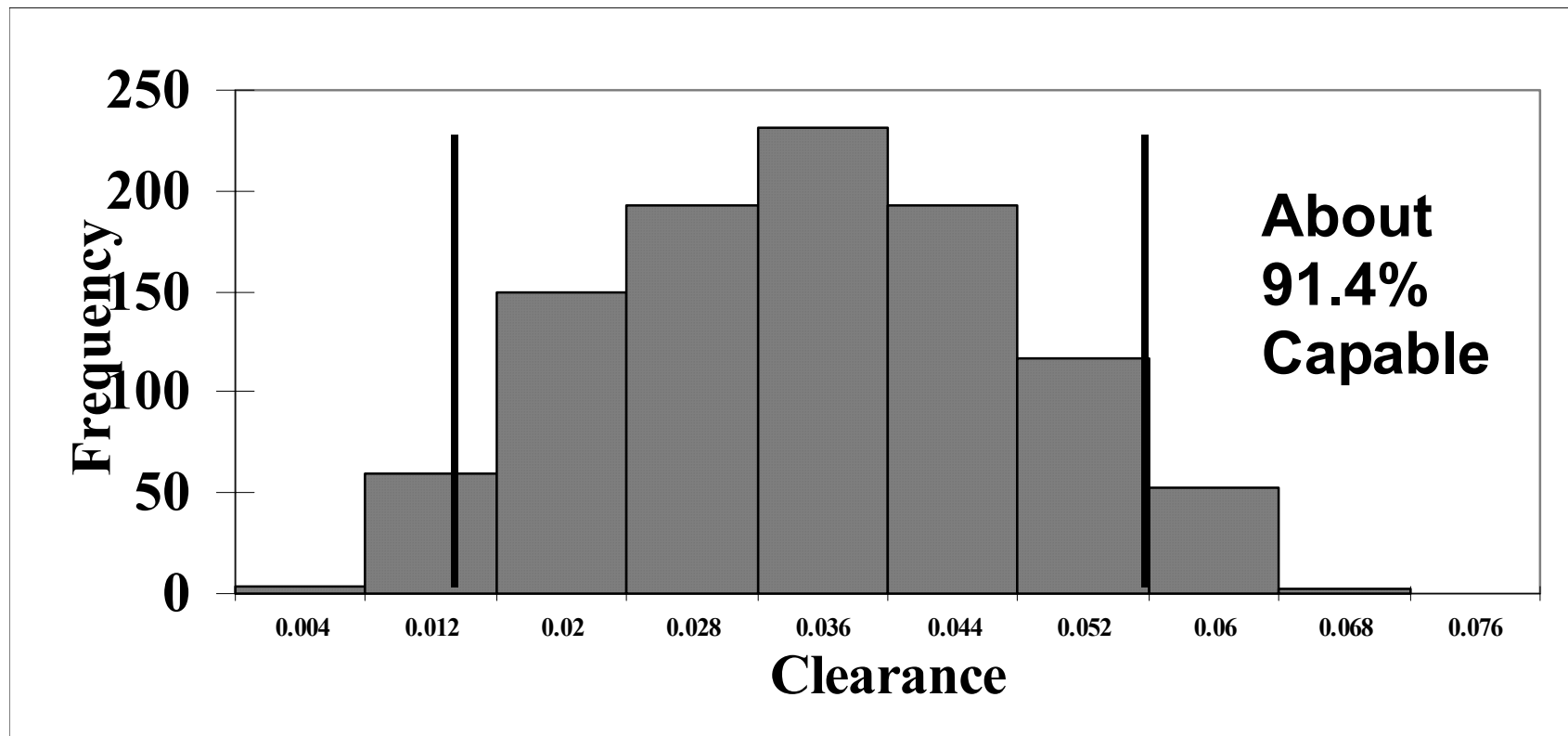
Capability Assessment
via a Loss Function

Quality loss (vertical axis)

Quadratic Loss Function

x

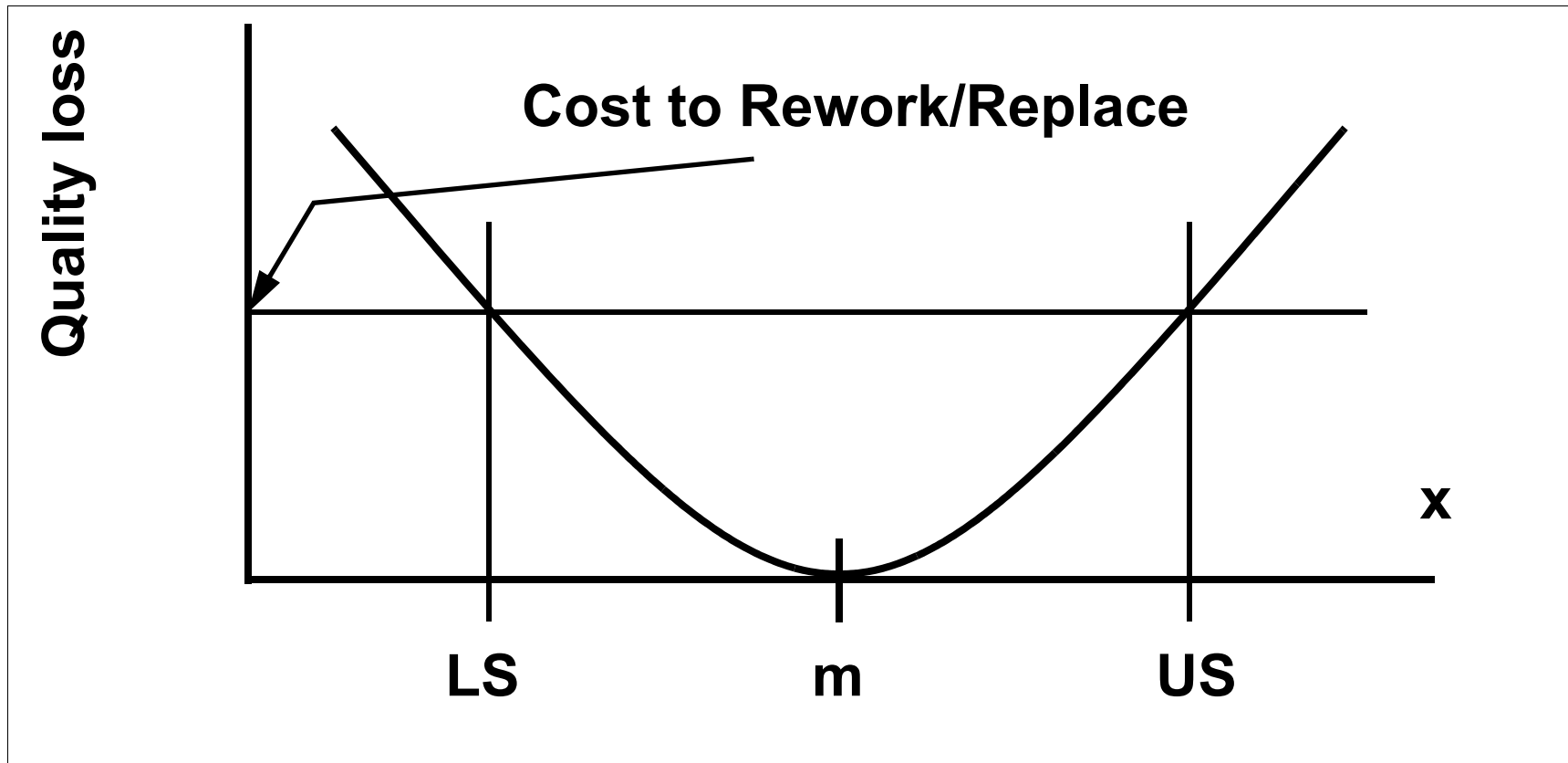**Nominal
Quality characteristic**
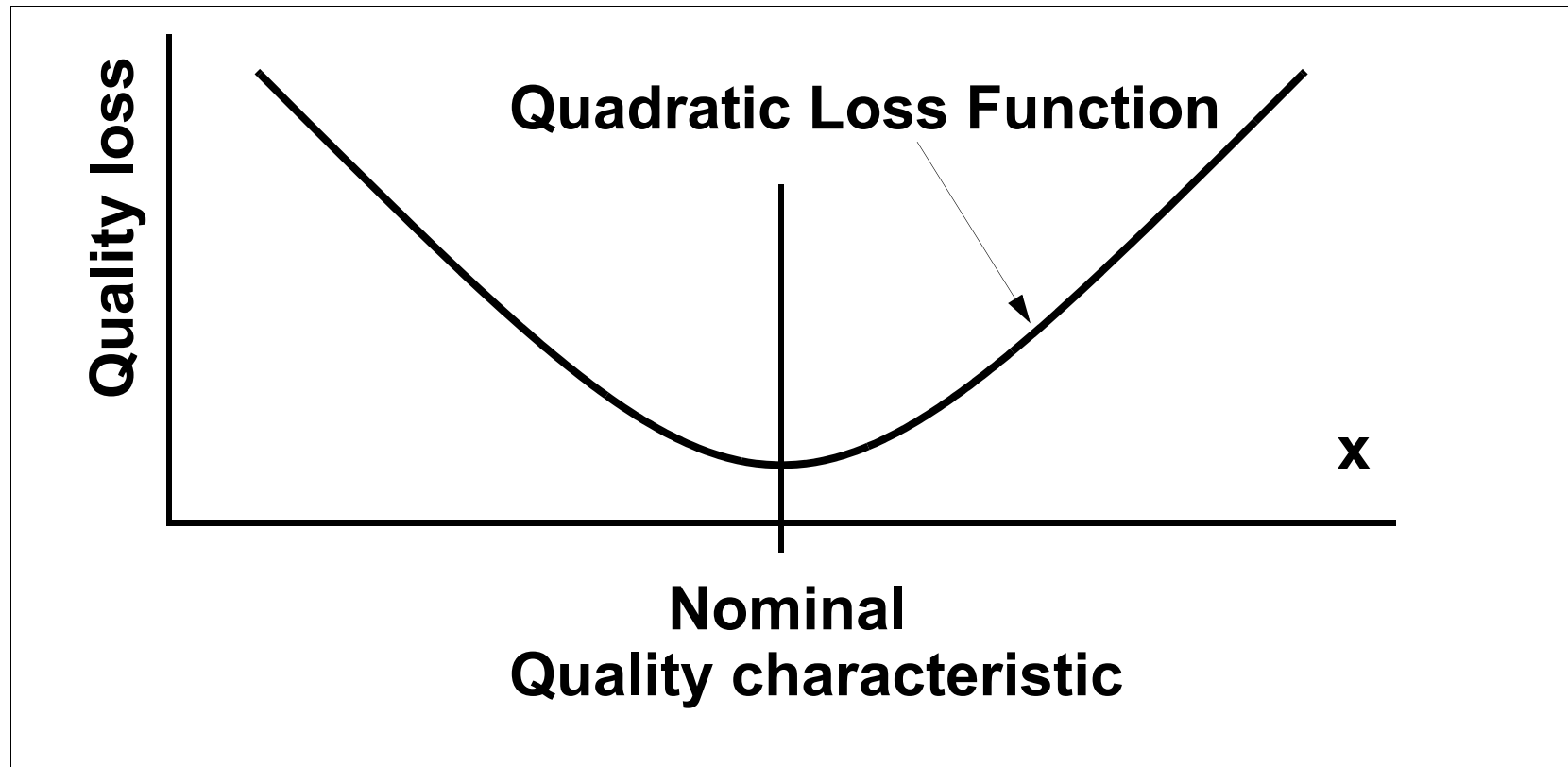
Defining the Loss Function

$$L = L_0 + k(x - m)^2$$

- for an x value of 0, the loss, L, is $0.
  This means that m = 0 and $L_0 = 0$

- when x= 1.5, the loss, L, is $2.25
  $L/(x-m)^2 = k$ $k = \$2.25/(1.5 - 0)^2 = 1$

  The $2.25 loss is obtained from the fact that when x=1.5, the probability of a $45 complaint is 5%, and $45 * (0.05) = $2.25

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Loss Function Interpretation
of Engineering Specifications

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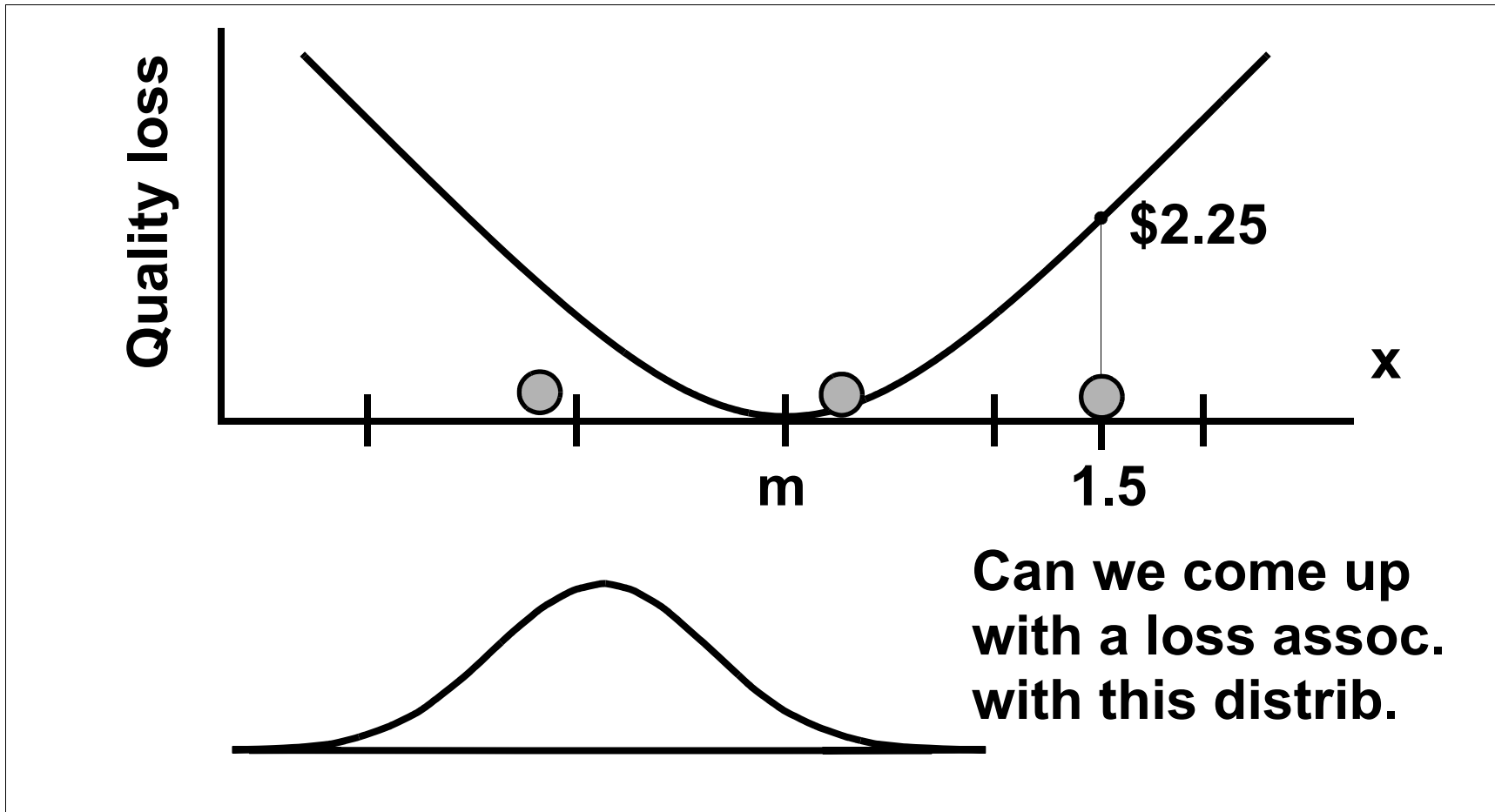
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More on Loss / Specifications

So, we want to set our specifications at the point of indifference: where the loss = cost of replacement

If cost of replacement = $2 for our previous example,

$$L = 1(x - 0)^2 = 2 \qquad \text{-----} \qquad x = \pm 1.414$$

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More on the Loss Function

Quality loss

$2.25

m   1.5   x

Can we come up with a loss assoc. with this distrib.

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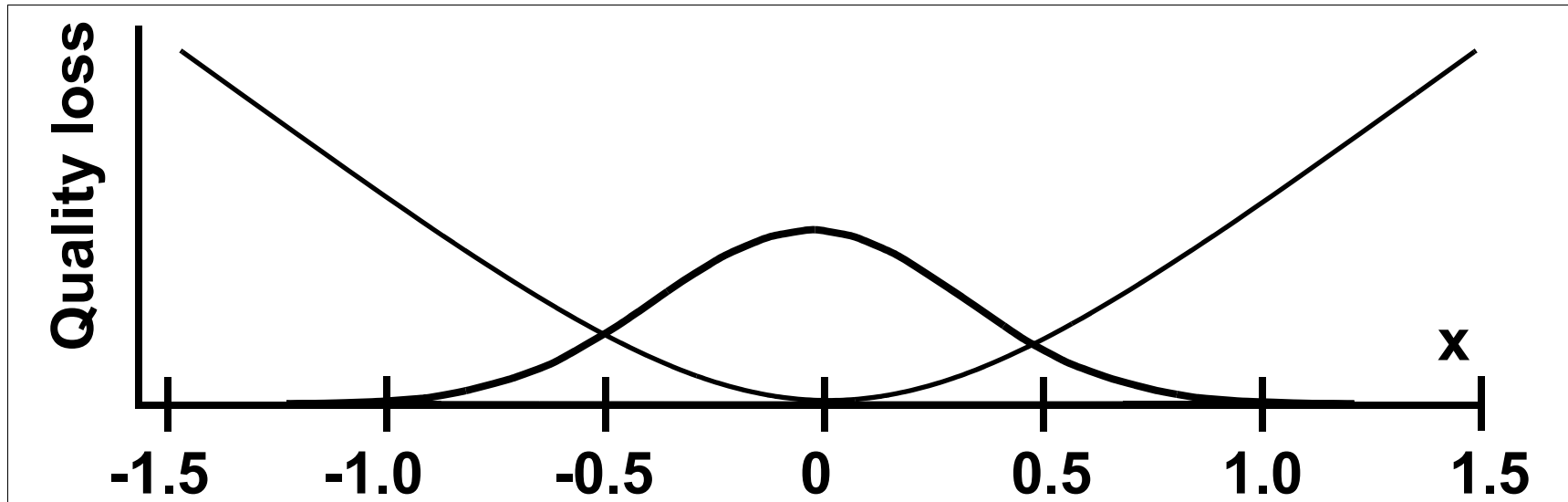
Expected Process Loss

$$E[L(X)] = \int_{\text{all } X} L(t)f(t)\,dt \qquad \text{where,}$$

$$L(x) = k(x - m)^2 \qquad \text{and} \qquad \text{f(t) is the pdf}$$

after simplification,

$$E[L(X)] = k\left\{(\mu_X - m)^2 + \sigma_X^2\right\}$$

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Loss Function: Example #1

m=0, k=$1.0, LSL=-1, USL=1,
X normal w/ mean 0 and σ_x = 0.33

E[L] = 1 [0.33^2 + $(0-0)^2$] = $0.11   **How to interpret??**

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Loss Function: Example #2

$\sigma_x = 0.58$

$E[L] = 1 [0.58^2 + (0-0)^2] = \$0.33$

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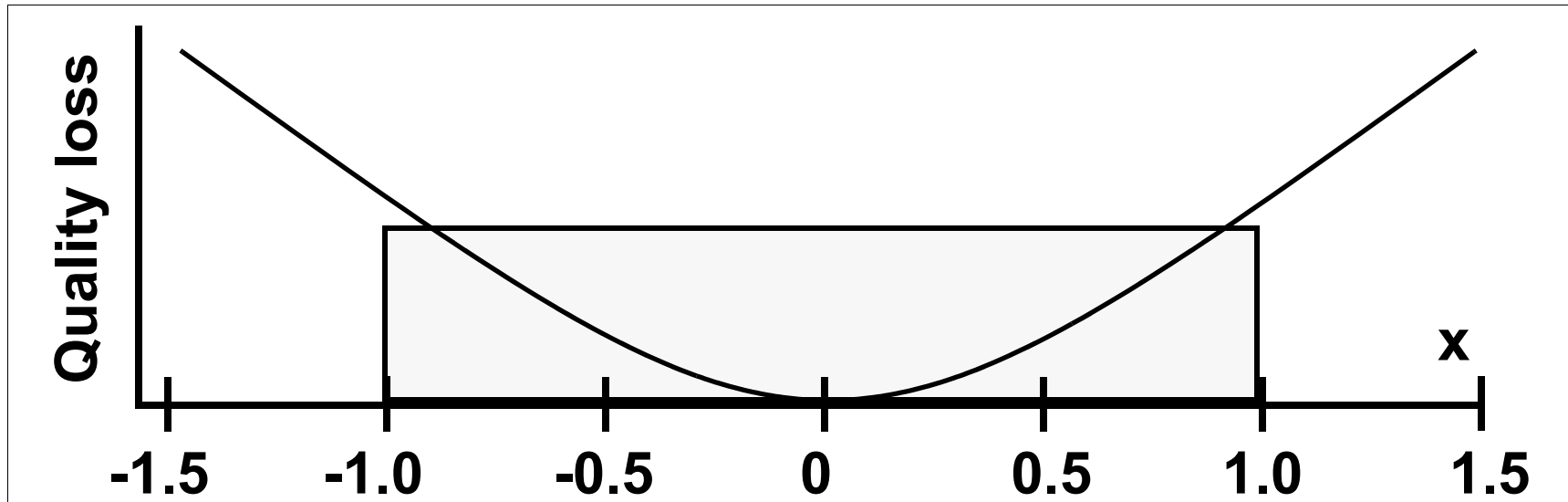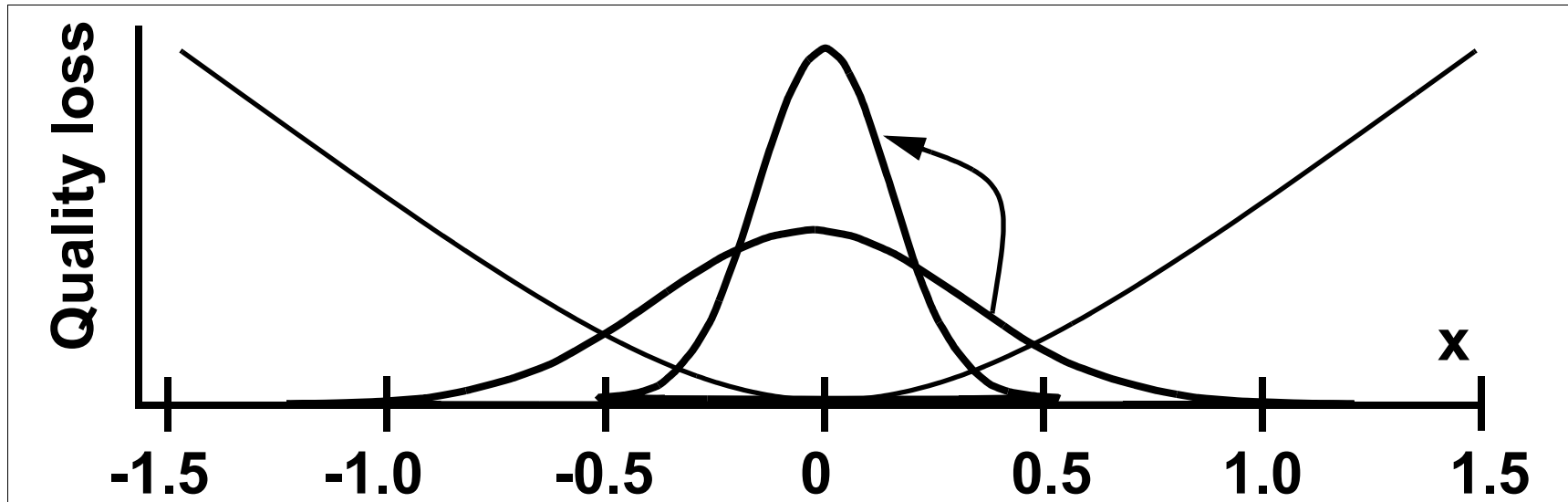
Filling in a Past Blank

Should know from prior Statistics Class
For a Uniform Distribution between a & b

$$f(x) = 1/(b-a)$$

$$\mu_X = \int_a^b xf(x)dx = \int_a^b \frac{x}{(b-a)}dx = \left.\frac{x^2}{2(b-a)}\right|_a^b = \frac{(b+a)}{2}$$

$$\sigma_X^2 = \int_a^b (x-\mu_X)^2 f(x)dx = \frac{(b-a)^2}{12}$$

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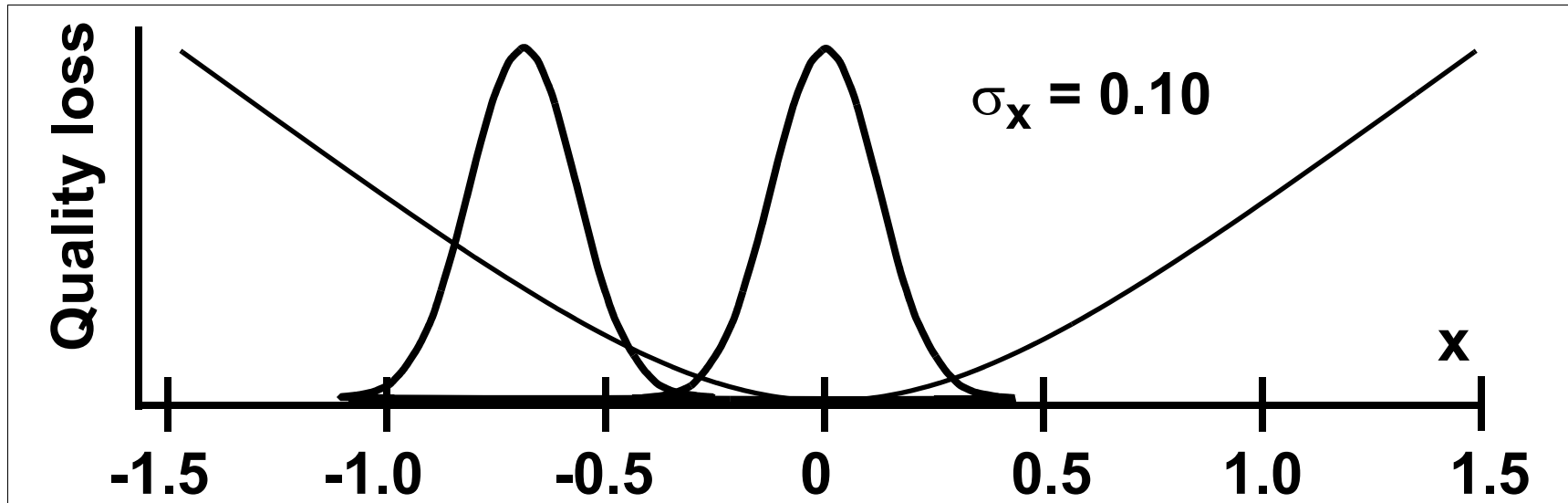**Michigan Technological University**

Loss Function: Example #3

Say we reduce $\sigma_X = 0.33$ to $\sigma_X = 0.10$

$E[L] = 1 [0.10^2 + (0-0)^2] = \$0.01$

Loss Function: Example #4

$\sigma_X = 0.10$

Can save $0.10 by reducing process mean from 0 to -0.7

$$E[L] = 1\,[0.10^2 + (-0.7 - 0)^2] = \$0.50 \quad WOW!!$$

Taguchi's comment:

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