Lecture # 24

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Process Capability

The extent to which a process produces parts that meet design intent.

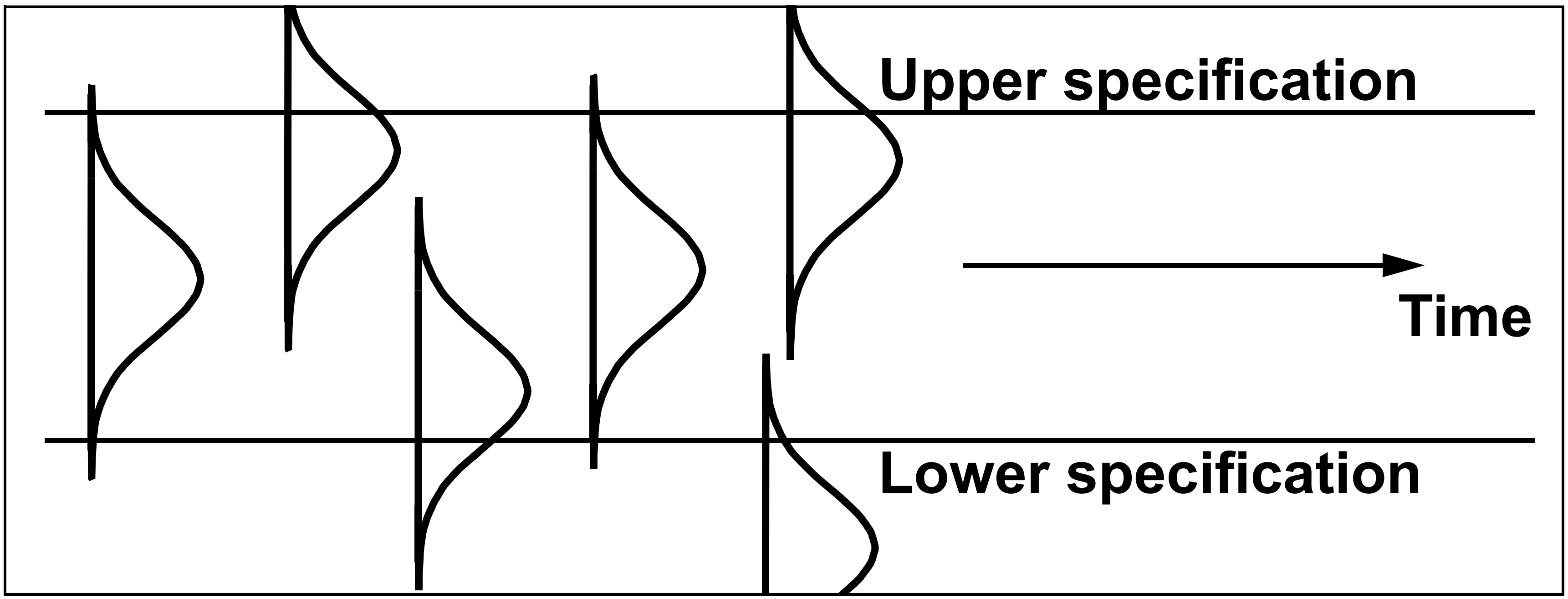
Most often, how well the process meets the engineering specifications.

Process capability -- when we quote a number for this we do not want it dependent on time.

Rule: Never assess process capability until the process is "in-control"

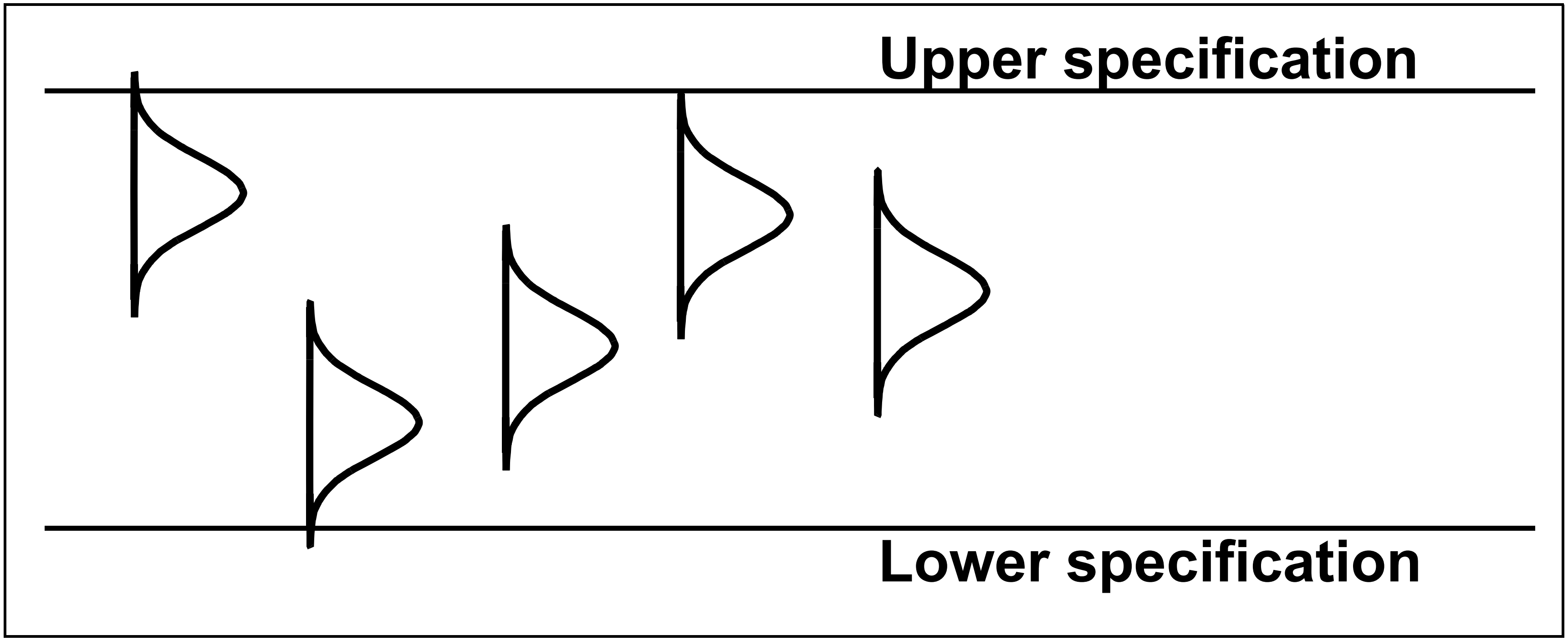
Why Process Stability

Let's say process is unstable -- mean changes vs. time

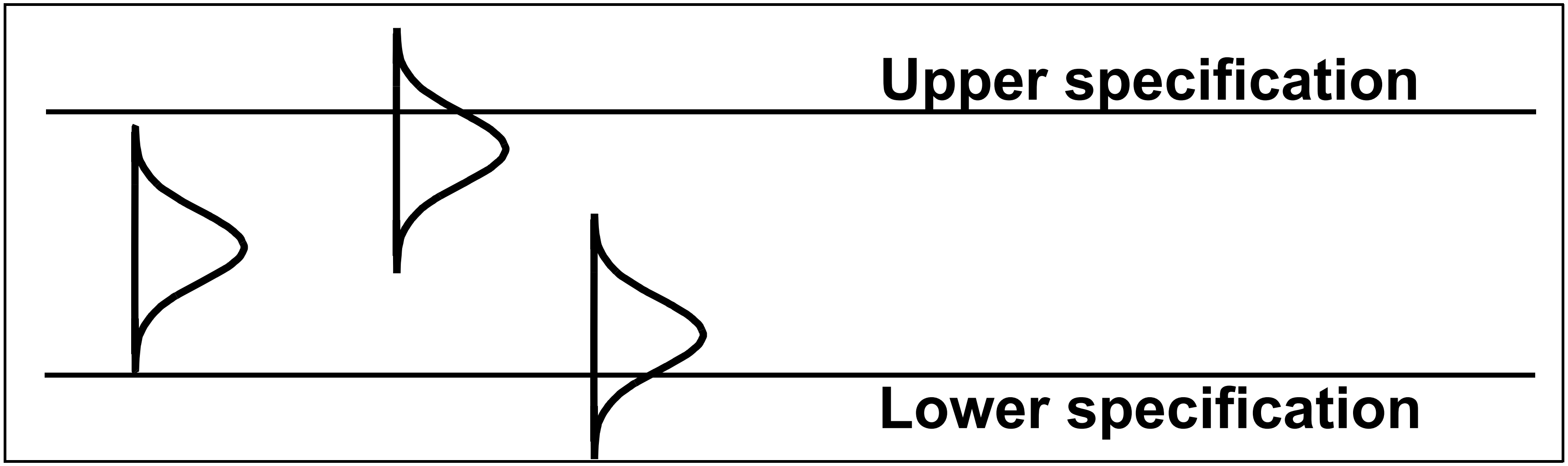


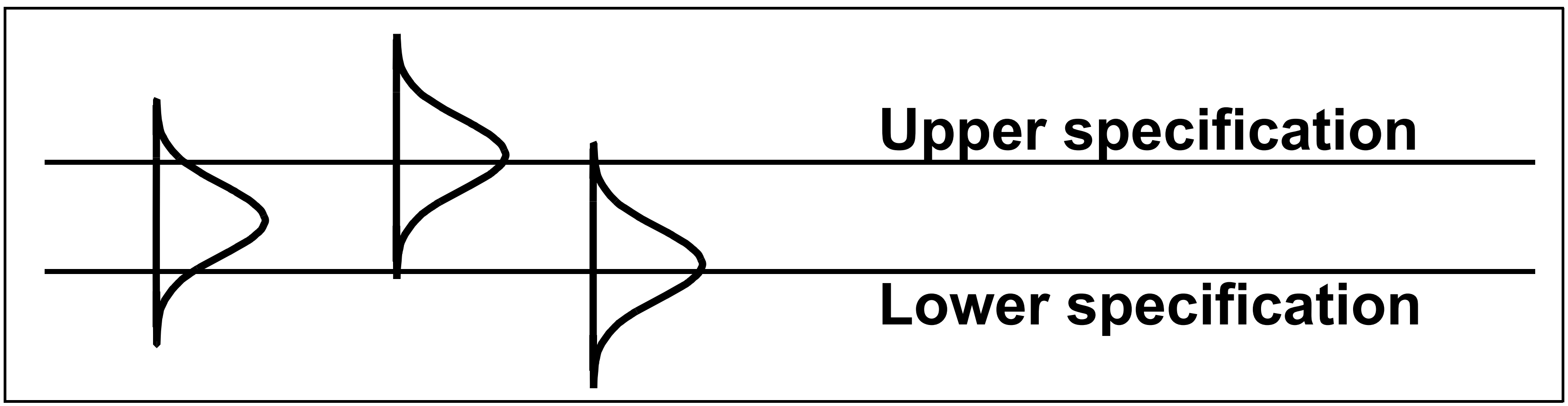
What % of the process output meets the specifications??

Process Variability & Specifications



Process variation is small relative to the width of the engineering specifications


Upper specification
Lower specification


Upper specification
Lower specification

Cylinder Boring - Case Study

Sample	1	2	3	4	5	$\overline{X}$	R
1	205	202	204	207	205	204.6	5
2	202	196	201	198	202	199.8	6
3	201	202	199	197	196	199.0	6
4	205	203	196	201	197	200.4	9
5	199	196	201	200	195	198.2	6
6	203	198	192	217	196	201.2	25
7	202	202	198	203	202	201.4	5
8	197	196	196	200	204	198.6	8
9	199	200	204	196	202	200.2	8
10	202	196	204	195	197	198.8	9
11	205	204	202	208	205	204.6	6
12	200	201	199	200	201	200.2	2
13	205	196	201	197	198	199.4	9
14	202	199	200	198	200	199.8	4
15	200	200	201	205	201	201.4	5
16	201	187	209	202	200	199.8	22
17	202	202	204	198	203	201.8	6
18	201	198	204	201	201	201.0	6
19	207	206	194	197	201	201.0	13
20	200	204	198	199	199	200.0	6
21	203	200	204	199	200	201.2	5
22	196	203	197	201	194	198.2	7
23	197	199	203	200	196	199.0	7
24	201	197	196	199	207	200.0	10
25	204	196	201	199	197	199.4	5
26	206	206	199	200	203	202.8	7
27	204	203	199	199	197	200.4	7
28	199	201	201	194	200	199.0	6
29	201	196	197	204	200	199.6	8
30	203	206	201	196	201	201.4	10
31	203	197	199	197	201	199.4	6
32	197	194	199	200	199	197.8	6
33	200	201	200	197	200	199.6	4
34	199	199	201	201	201	200.2	2
35	200	204	197	197	199	199.4	7

X-double-bar = 199.95

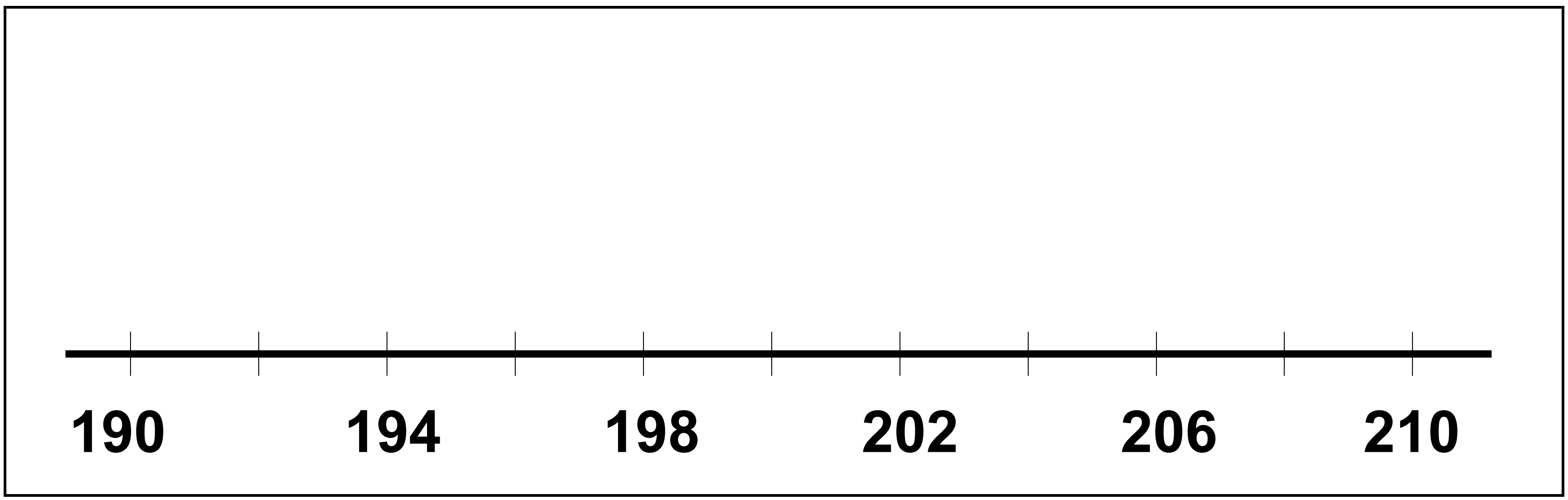
R-bar = 6.61

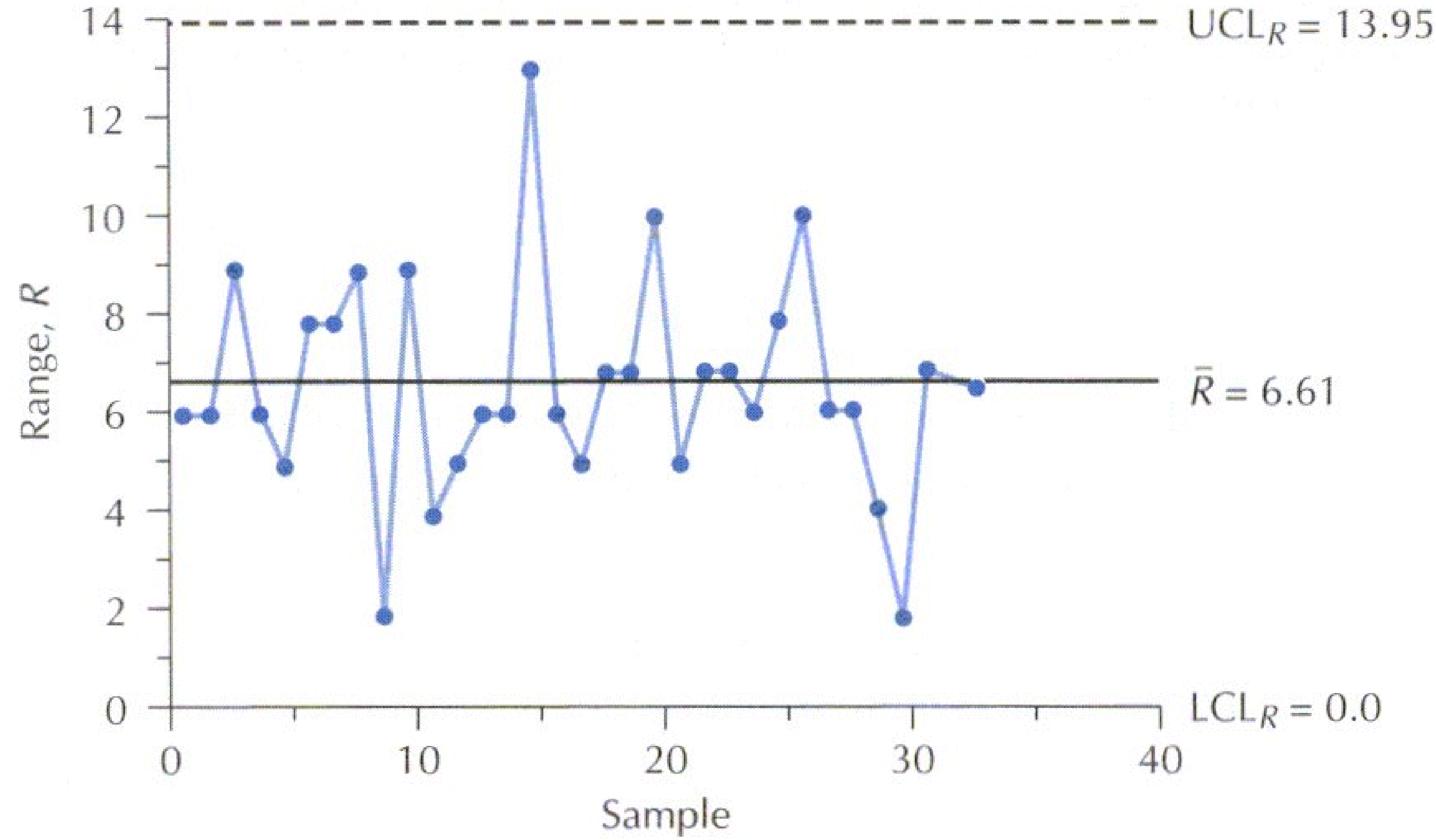
<u>Process is "in control"</u>

$$\hat{\sigma}_X = \bar{R} / d_2 = 6.61 / 2.326 = 2.8418$$

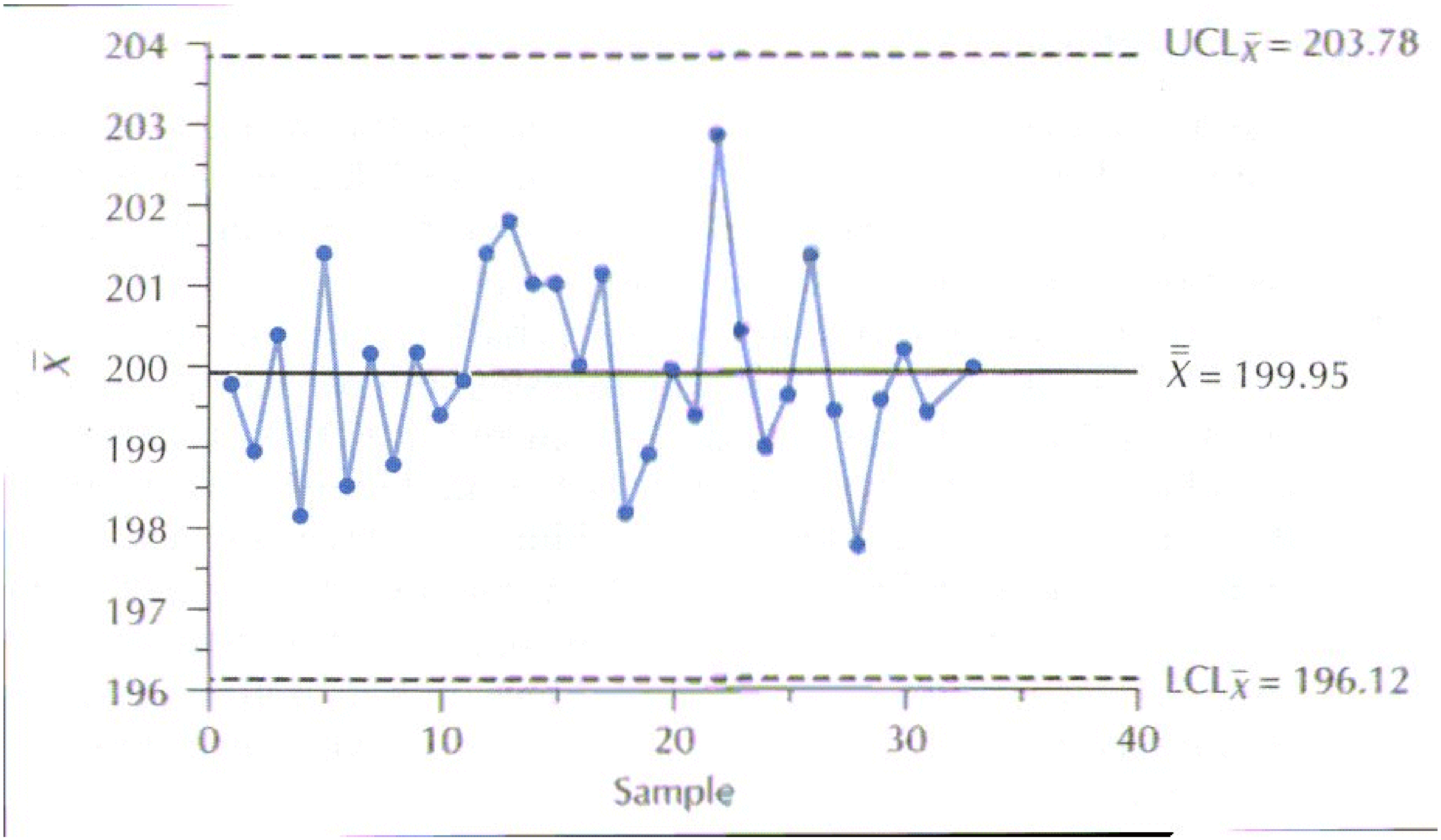
Histogram shows individuals normally distributed

Specifications are 199 +/- 4 : 195 - 203





14
12
10
8
6
4
2
0
Range, R
UCL$_R$ = 13.95
$\bar{R}$ = 6.61
LCL$_R$ = 0.0
0
10
20
30
40
Sample



204
203
202
201
200
199
198
197
196
$\bar{X}$
$UCL_{\bar{X}}$ = 203.78
$\bar{\bar{X}}$ = 199.95
$LCL_{\bar{X}}$ = 196.12
0
10
20
30
40
Sample

Z(lo) = (195-199.95)/2.8418 = -1.742
prob = .0407

Z(hi) = (203-199.95)/2.8418 = 1.073
prob = .8586

Capability = 85.86% - 4.07% = 81.8%
Process is not capable (want % > 99.73% as a min.)

Would centering the process at the nominal value help??

Could calculate probability for this case as well.

What action should we take??

Specifications & Control Limits

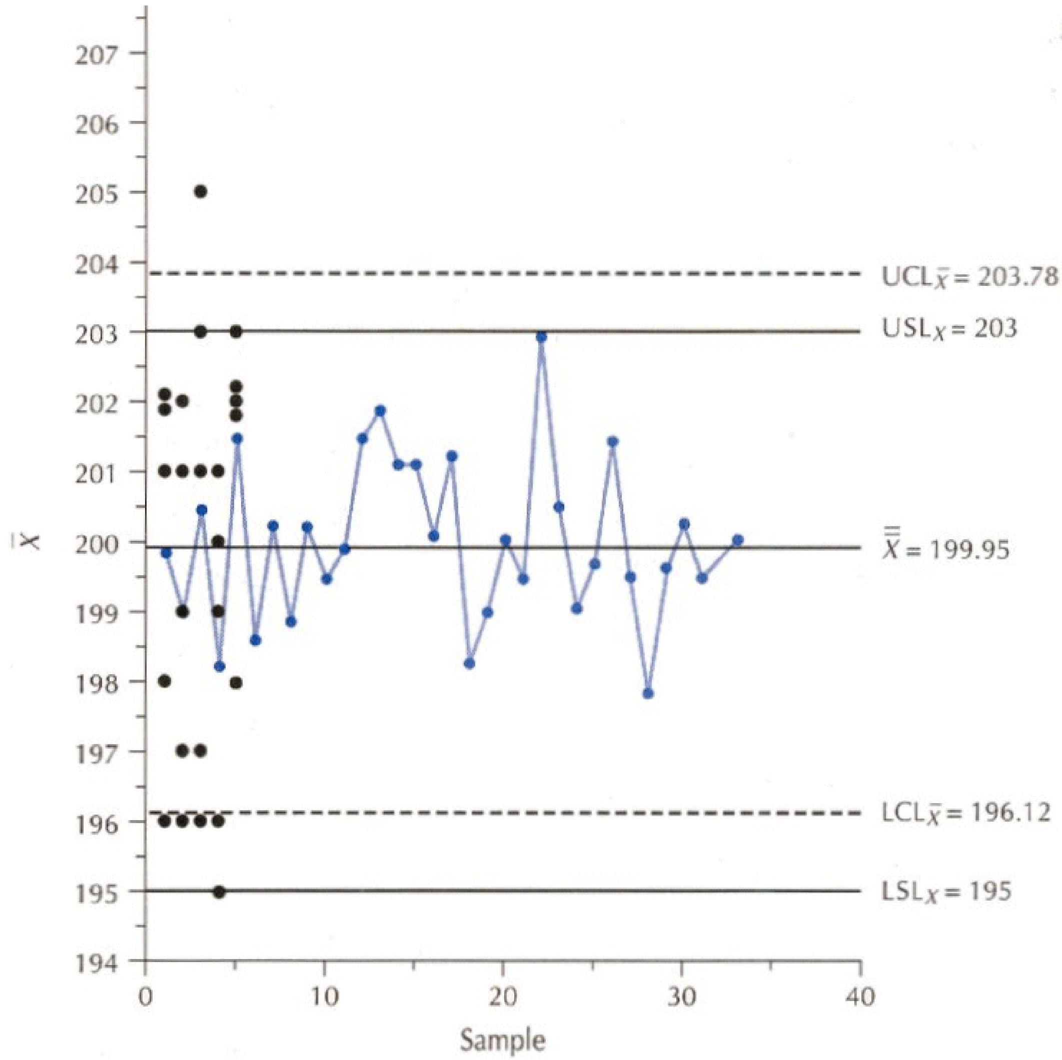
Specification Limits

- Characteristic of the part in question
- Based on functional considerations
- Compare to individual part measurements
- Establish part's conformability to design intent

Control Limits

- Characteristic of the process in question
- Based on process mean and variability
- Dependent on sample size, n, and α risk
- Establish presence/absence or special causes (local faults) in the process

Specifications on Control Charts



Process Capability Indices

- $C_p = \dfrac{(USL - LSL)}{6\sigma_X}$ Want $\geq$ 1

- Capability Index C_{pk}

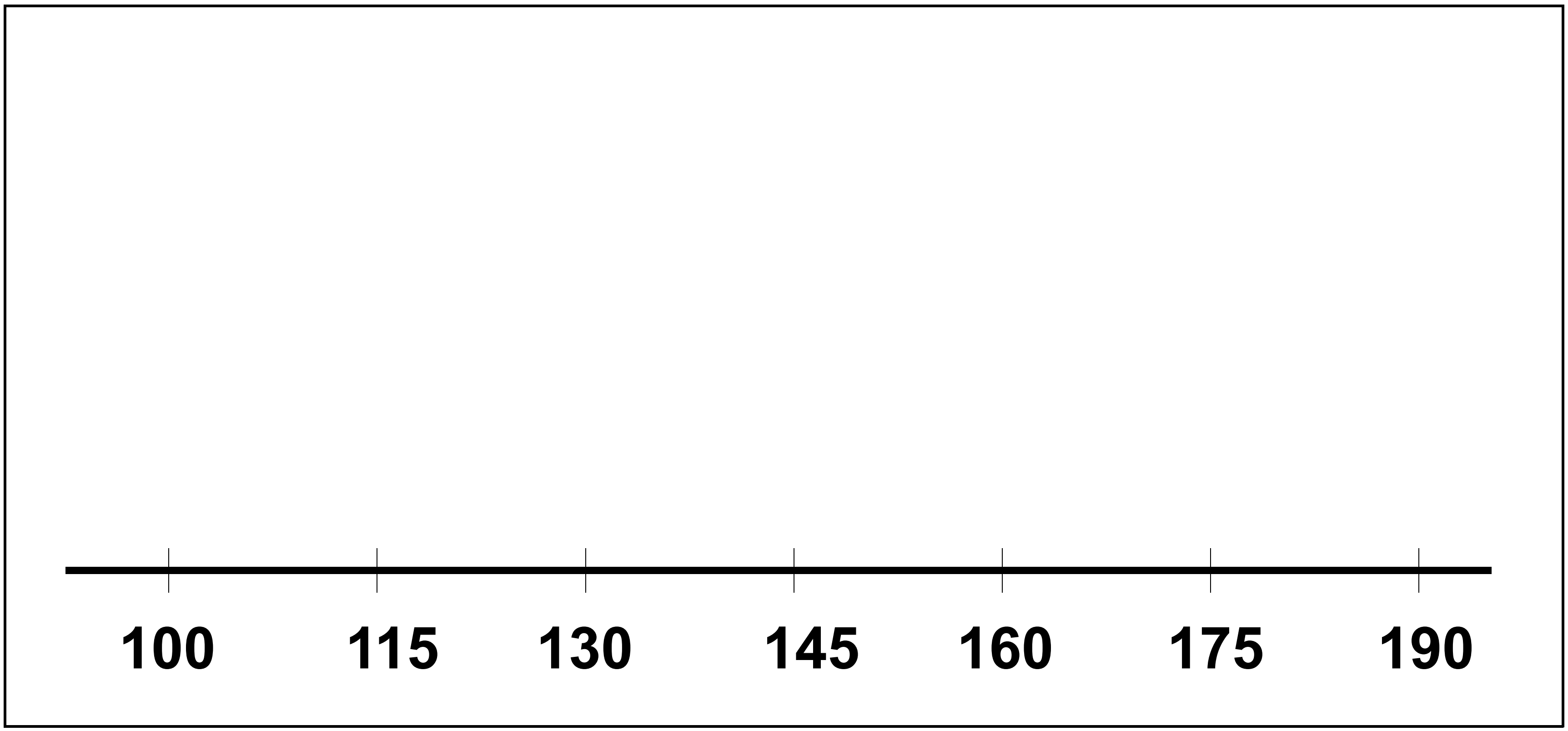
$$Z_{USL} = \frac{(USL - \mu_X)}{\sigma_X} \qquad Z_{LSL} = \frac{(LSL - \mu_X)}{\sigma_X}$$

$$Z_{min} = \min[Z_{USL}, -Z_{LSL}]$$

$$C_{pk} = Z_{min}/3$$ Want $\geq$ 1

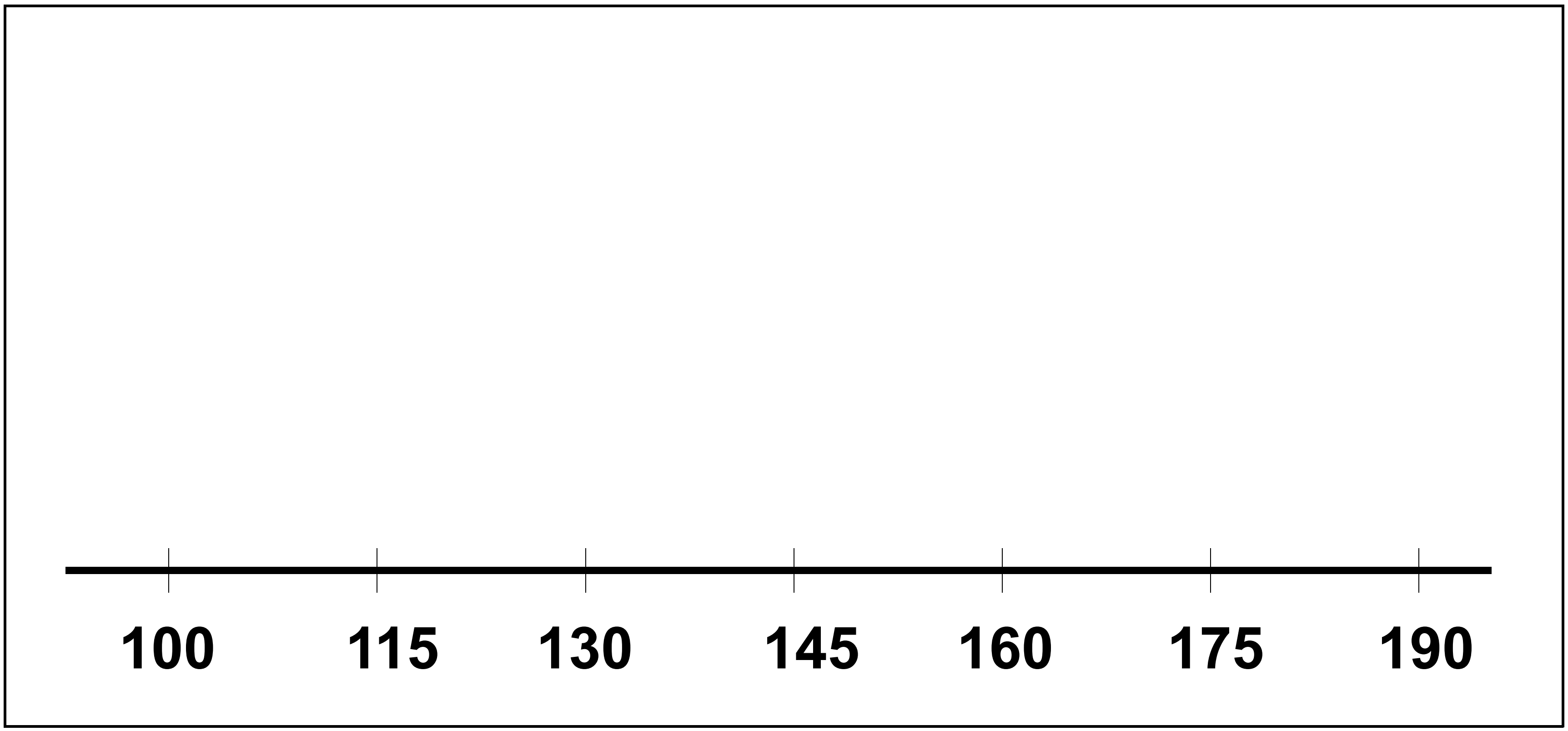
Example #1

Mean = 130, Sigma-X = 10, Nominal = 145, Specs: 100-190



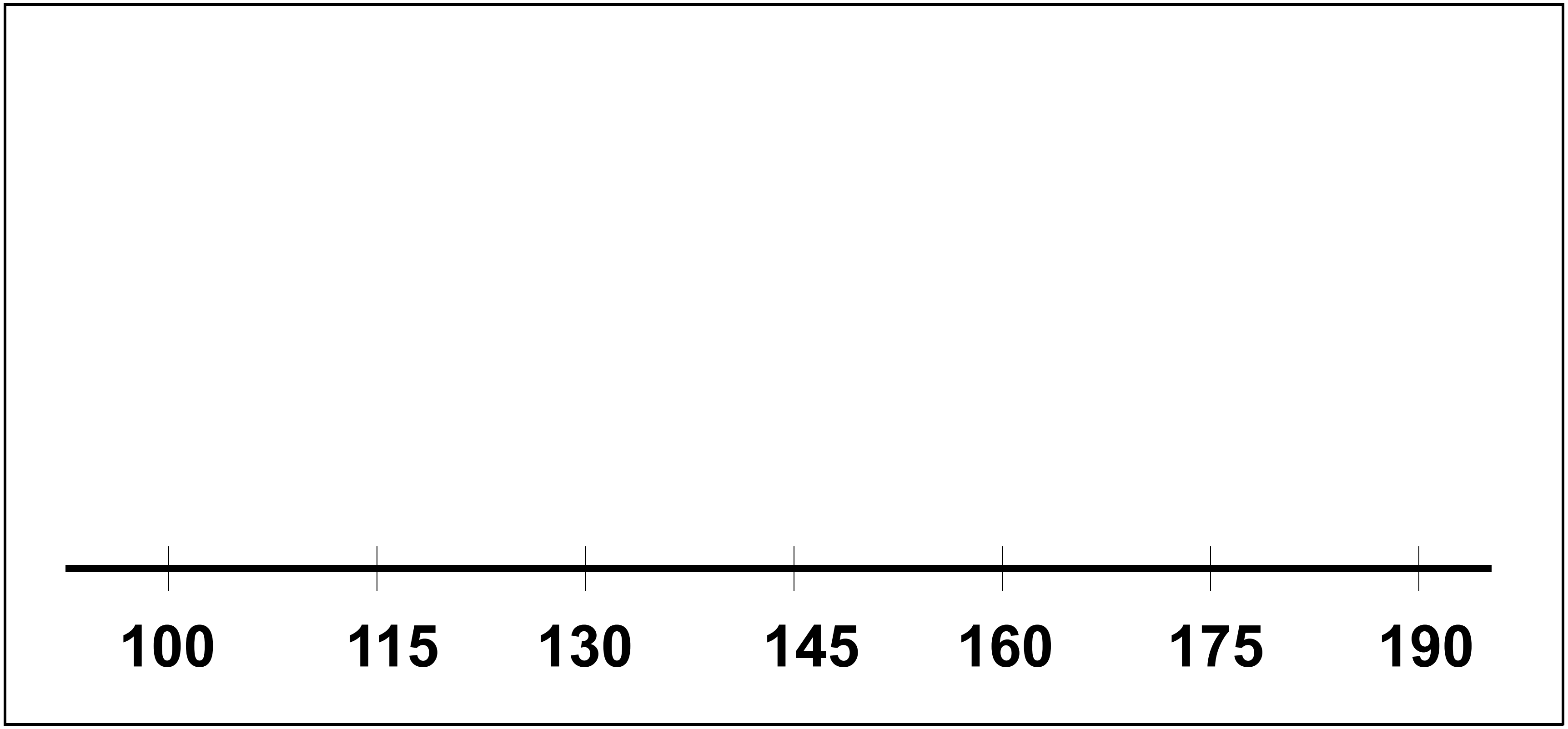
Example #2

Shift mean to 145

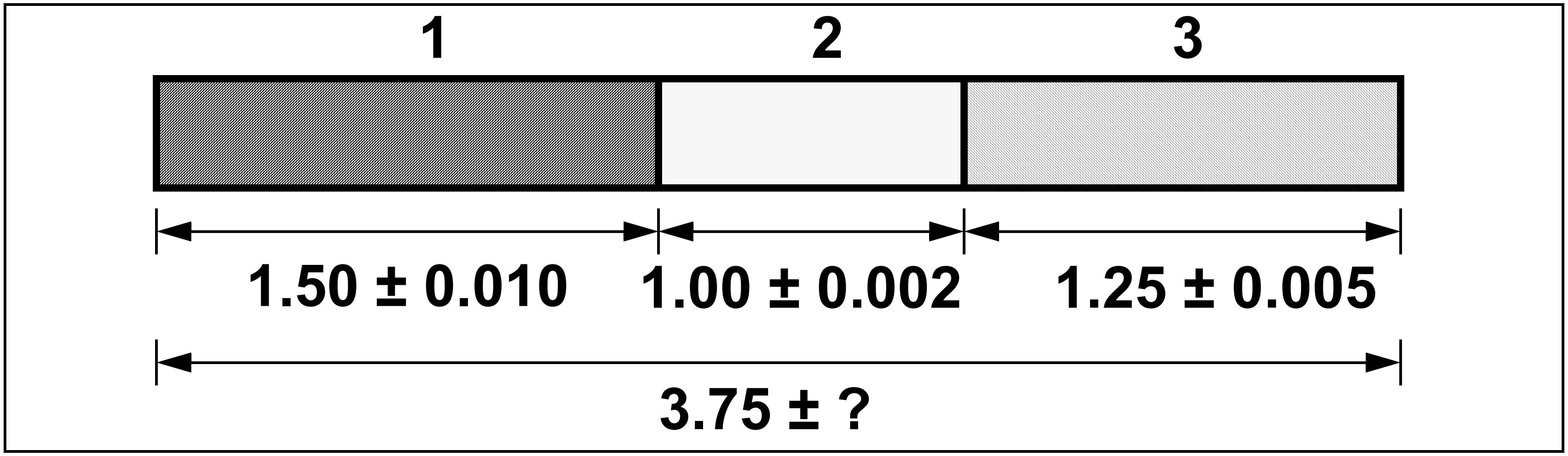


Example #3

A: Mean = 145, Sigma-X = 15
B: Mean = 130, Sigma-X = 10



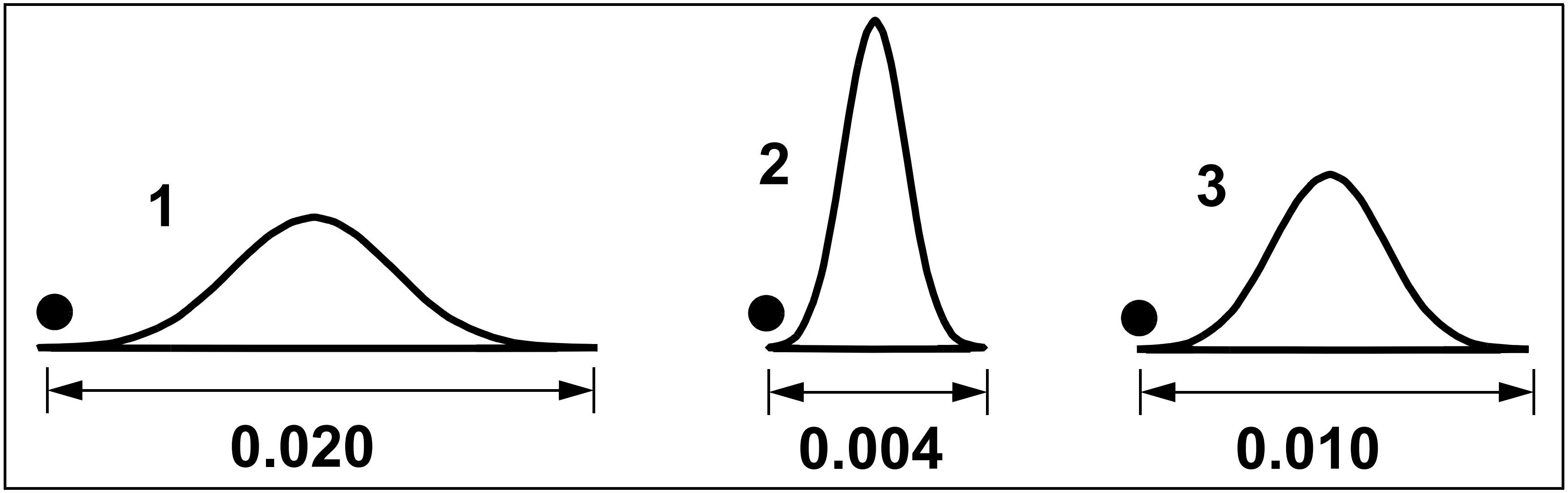
Assembly Tolerances



We might naively set the assembly tolerance by simply adding tolerances: ± 0.017

Concerned that parts 1, 2, & 3 might be selected right at the tolerances - want assembly to be ok

Individuals & Assemblies

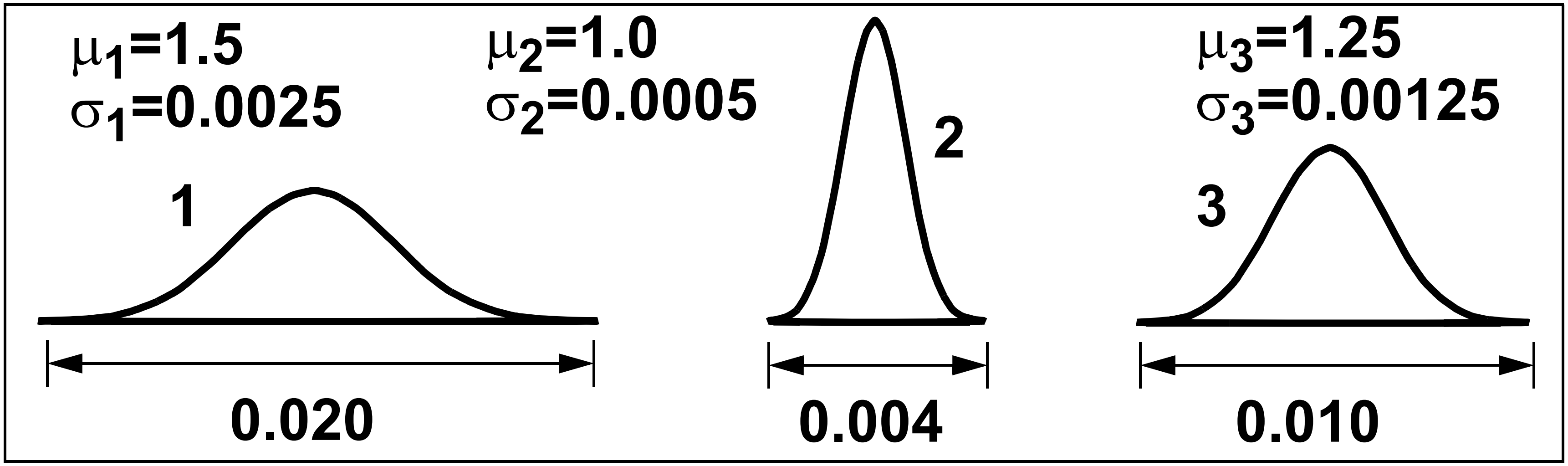


Let's assume specs are ±4σ from the mean/nominal

Probability of a point at or below -4σ = 0.00003

Probability of simultaneously obtaining 3 such points: $(0.00003)^3$ = 2.7 E-14 (1 in 37 trillion!!)

Describing the Assembly

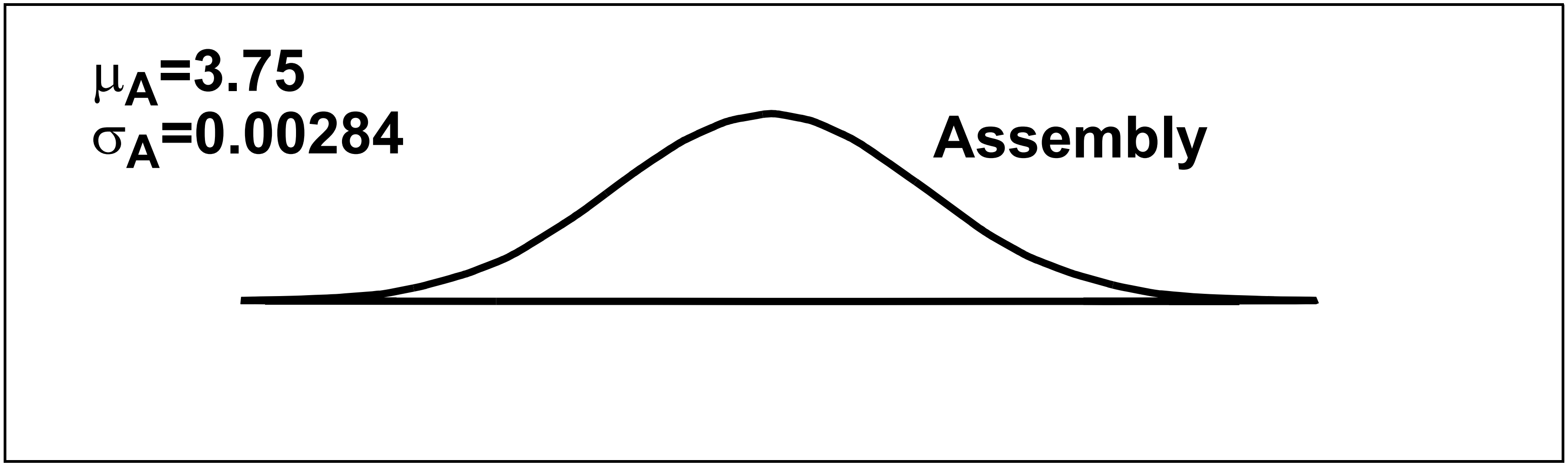


$$X_A = X_1 + X_2 + X_3$$

$$\mu_A = \mu_1 + \mu_2 + \mu_3 = 3.75$$

$$\sigma_A^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2 = 8.0625 \times 10^{-6} \qquad \sigma_A = 0.00284$$

Assembly Distribution



If we again assume that the specs are $\pm 4\sigma_A$ from the mean/nominal, then the tolerance is ± 0.01136

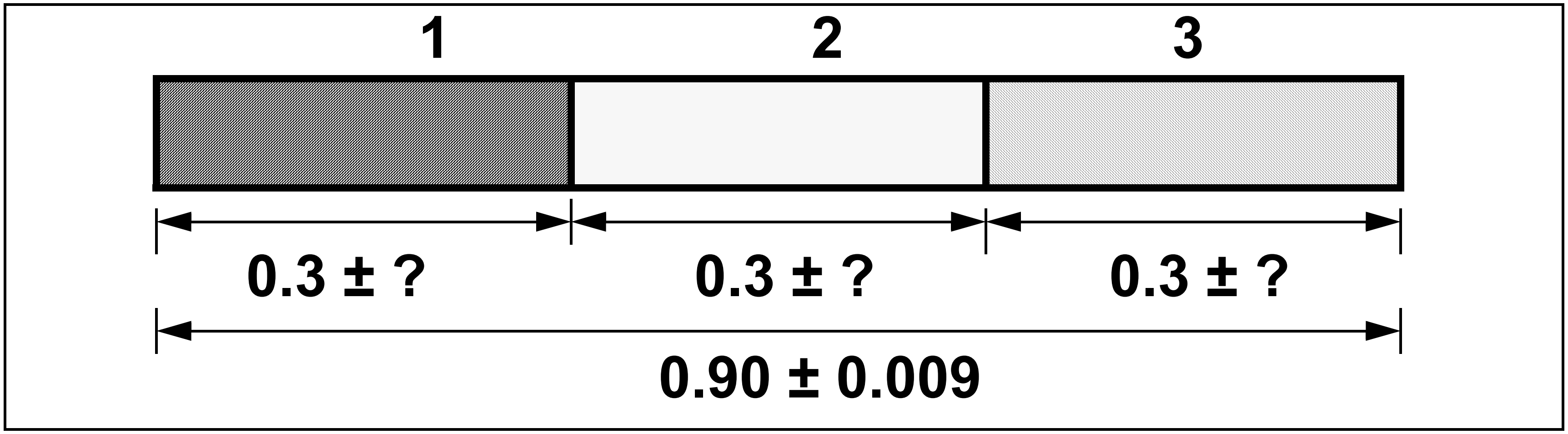
This differs significantly from that obtained by adding!!!

Forces at Work

- **Random assembly**
- **Statistical (so far normal) distribution of part dimensions**
- **Additive Law of Variances**

- **In our example we also assumed that the process was centered at the nominal value.**
- **Tolerances at ±4σ**

Another Assembly Example



How do we obtain the tolerances on the individual parts? Divide 0.009 by 3 = 0.003 ??

Let's use the relations that we have developed to obtain the unknown tolerance. Assume 1=2=3

Remember that $X_A = X_1 + X_2 + X_3$

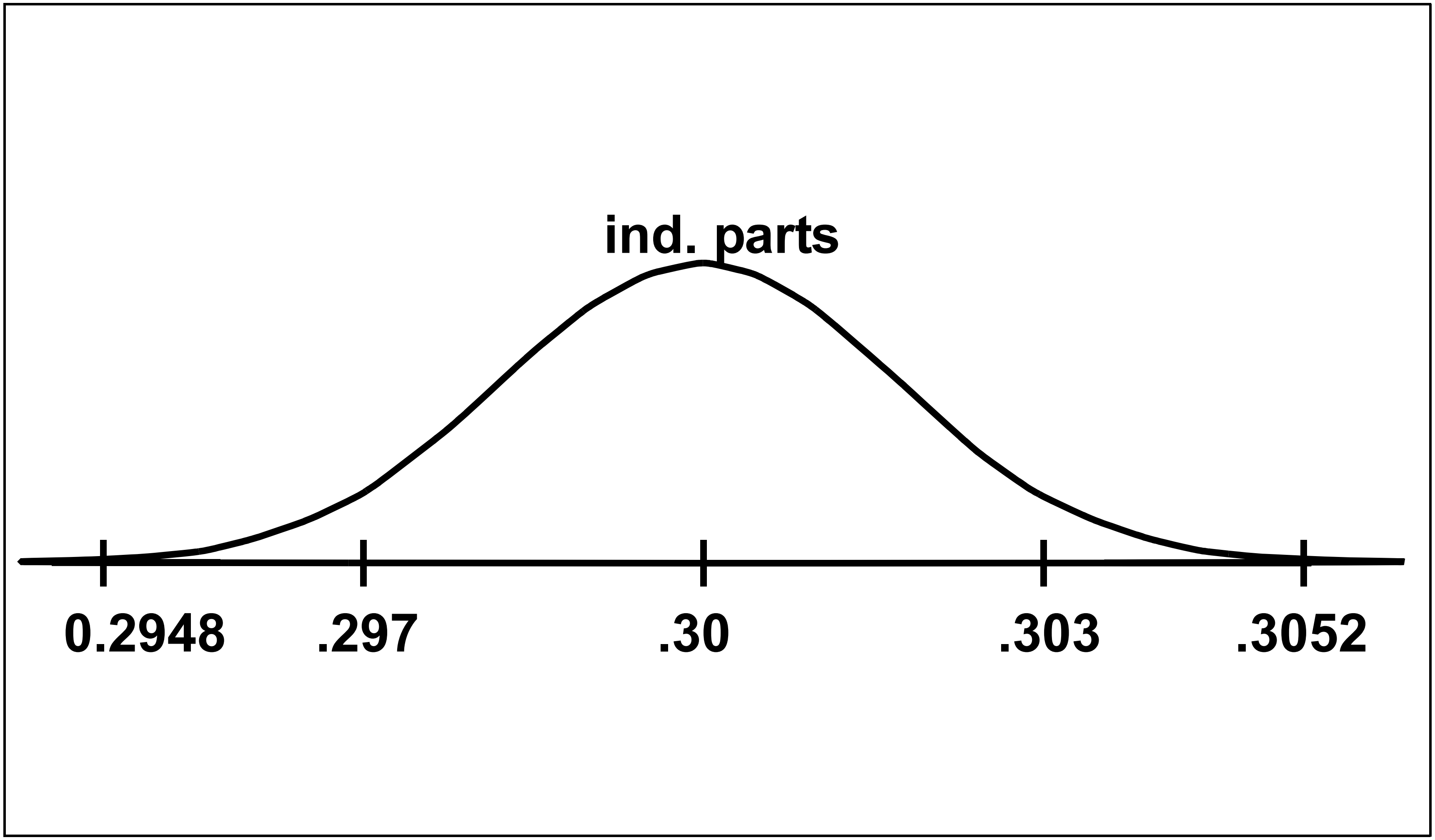
Mean of individual distributions at 0.30

Assembly has tolerance of ± 0.009

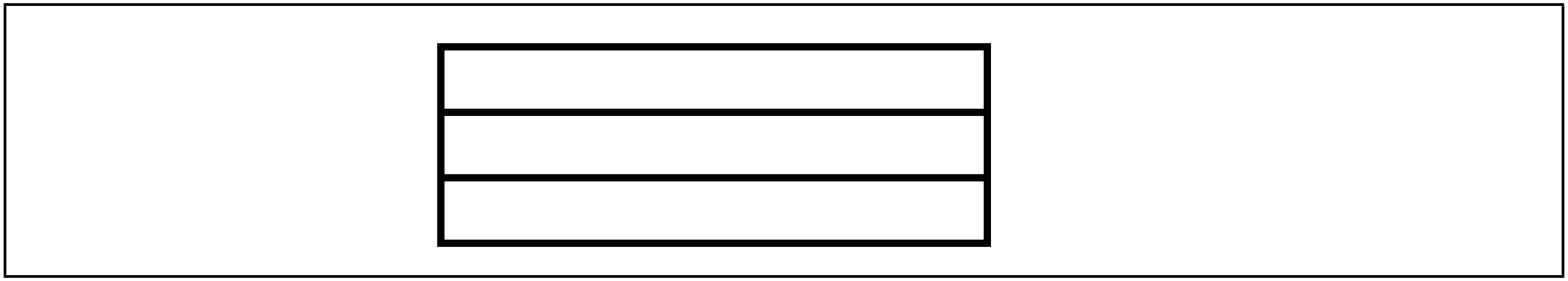
If tolerances are at $\pm 4\sigma_A$ **, then** σ_A **= ??**

Since $\sigma_A = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2} = \sqrt{3\sigma_p^2}$ **,** σ_p **= ??**

If we again put the specs for the individual parts at $\pm 4\sigma_p$ **, this turns out to be ± 0.0052**


ind. parts
0.2948
.297
.30
.303
.3052

Effect of Process Centering

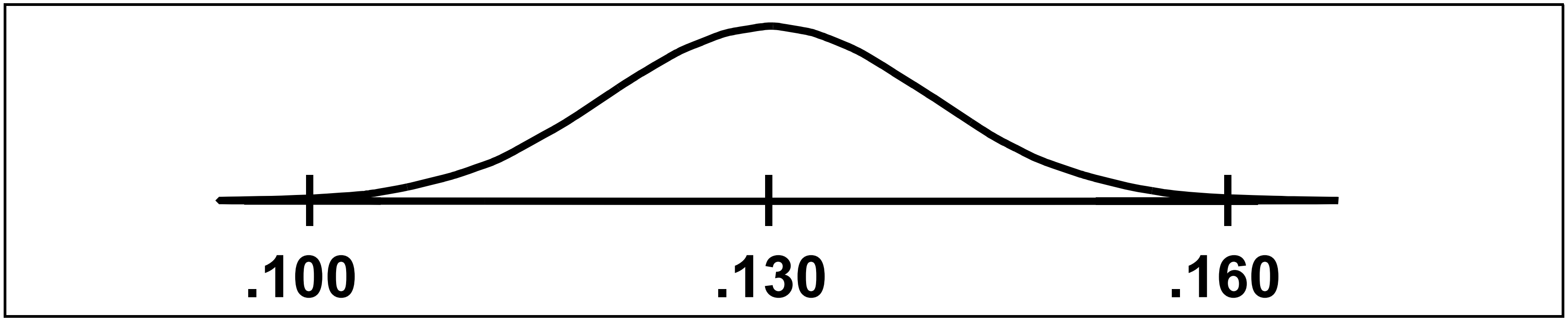


Assembly: 0.390 ± 0.050 inch (0.34 -- 0.44)
If tolerances are at ±3σ_A , σ_A = 0.017

Since $\sigma_A = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2} = \sqrt{3\sigma_p^2}$, σ_p **= 0.010**

So, for individ. parts: 0.130 ± 0.030 inch

What if we reduce σ_p from 0.010 to 0.005??



What if we shift process mean to 0.115??