Lecture #16

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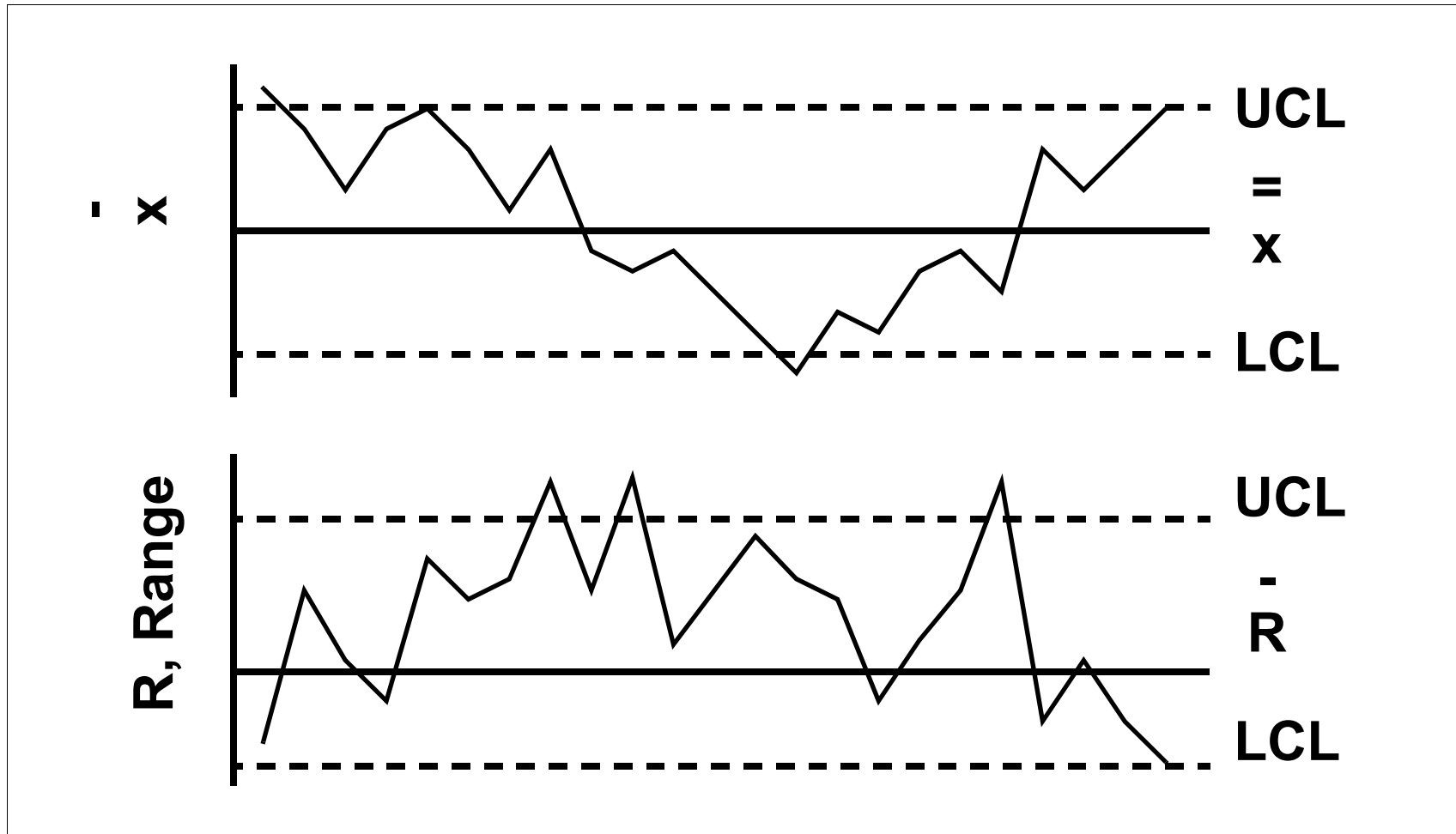**Quality Engineering (MEEM 4650 / 5650)**
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Chapter 6 - Making Control Charts

Sample			X_{ij}				
1	2	7	6	4	4		
2	8	6	4	7	5		
3	7	1	6	5	3		
k	5	6	3	4	5		

$$\overline{\overline{X}} = \sum_{i=1}^{k} \overline{X}_i / k \qquad\qquad \overline{R} = \sum_{i=1}^{k} R_i / k$$

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Chart Guidelines

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Xbar Chart Details

$$\mu_{\overline{X}} \pm 3\sigma_{\overline{X}} \qquad \overline{\overline{X}} \pm 3\sigma_{\overline{X}} \qquad \sigma_{\overline{X}} = \text{??}$$

Control Limits

$$\overline{\overline{X}} \pm A_2 \overline{R}$$

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R Chart Details

$$\mu_R \pm 3\sigma_R \qquad\qquad \overline{R} \pm 3\sigma_R$$

Control Limits $\qquad$ $\text{UCL} : D_4\overline{R}$ $\qquad\qquad$ $\text{LCL} : D_3\overline{R}$

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Control Charts (cont.)

Now we know how to construct control charts!!

We know to look for points beyond the limits -- is this enough?

How do we interpret the control charts??

Remember, our goal is to learn about the process -- we want to extract as much information as possible from the charts.

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Interpreting Control Charts

- We will look for more than just points beyond the limits, i.e., extreme points.

- Behavior that suggests presence of special causes.

- Trends / cyclic behavior

- High proportion of points near / beyond the limits

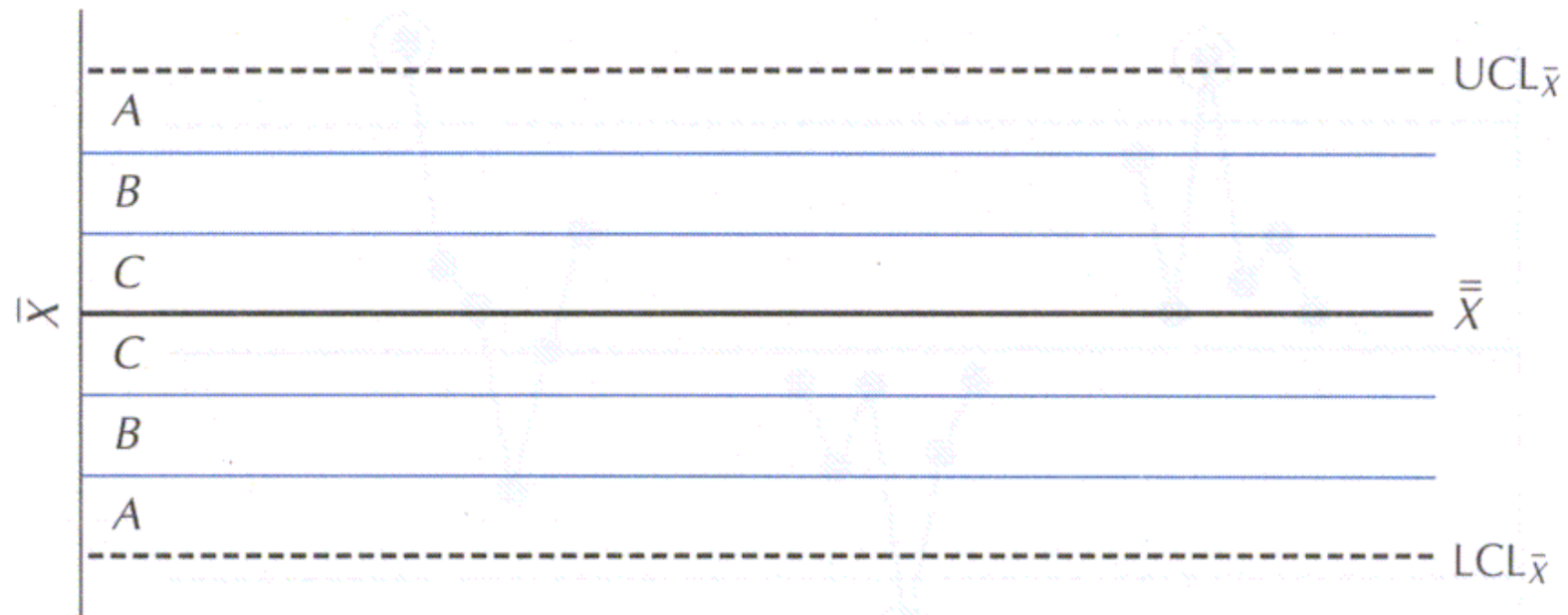- Sudden shifts in level

- In general, any non-random behavior

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8 Rules for Chart Interpretation

- **Test 1: Extreme points**

- **Test 2: 2 out of 3 points in zone A or beyond**

- **Test 3: 4 out of 5 points in zone B or beyond**

- **Test 4: Runs above / below the centerline**

- **Test 5: Linear trend**

- **Test 6: Oscillatory trend**

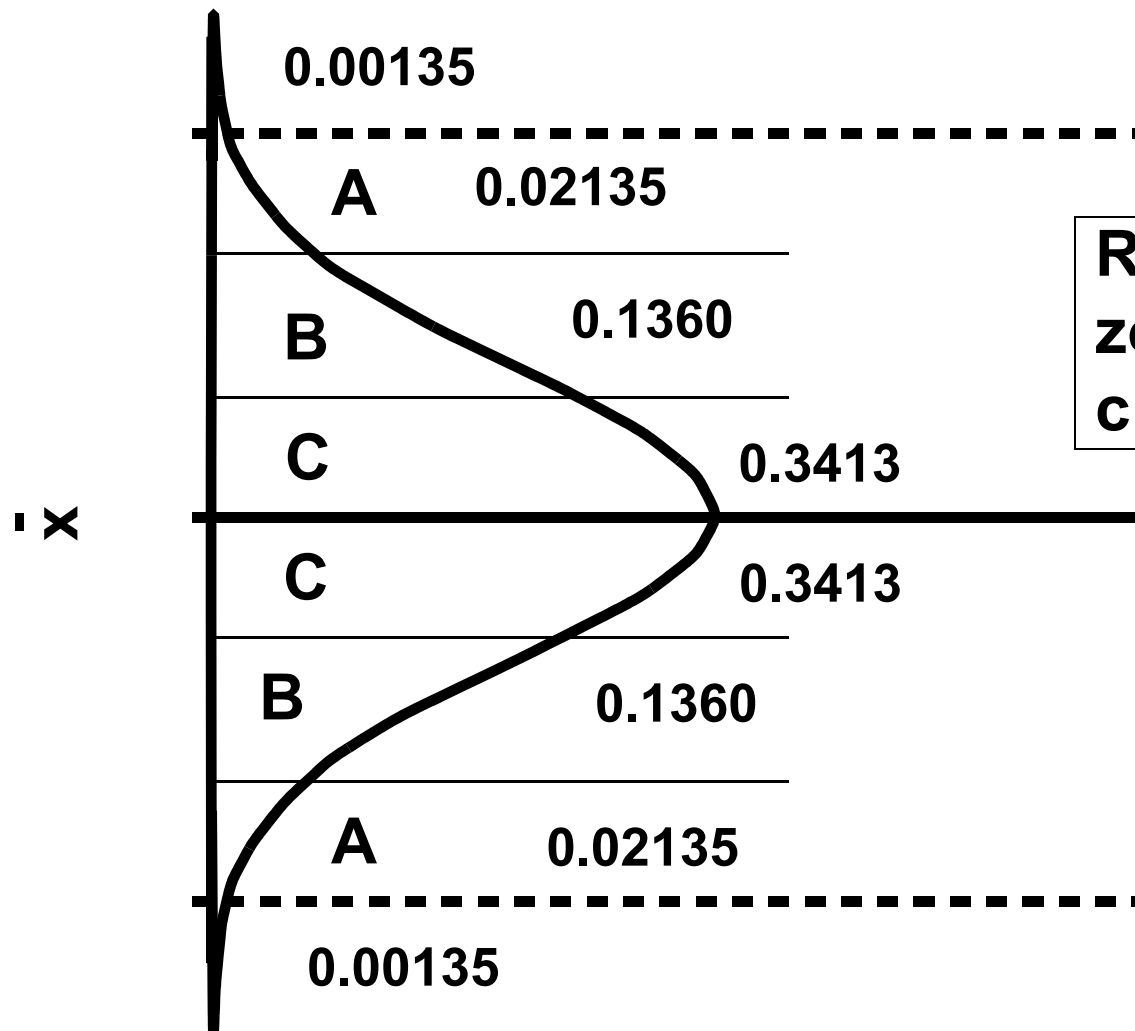- **Test 7: Avoidance of zone C**

- **Test 8: Run in zone C**
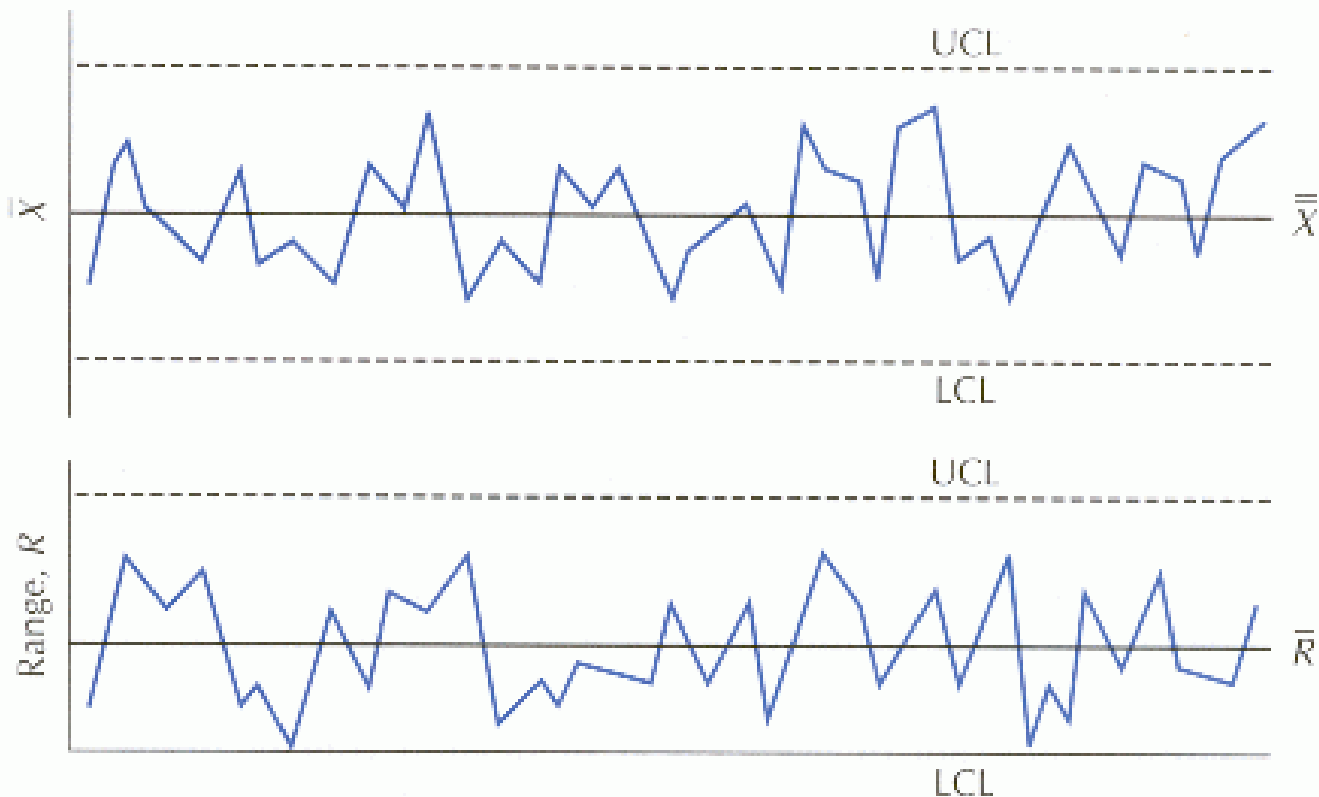
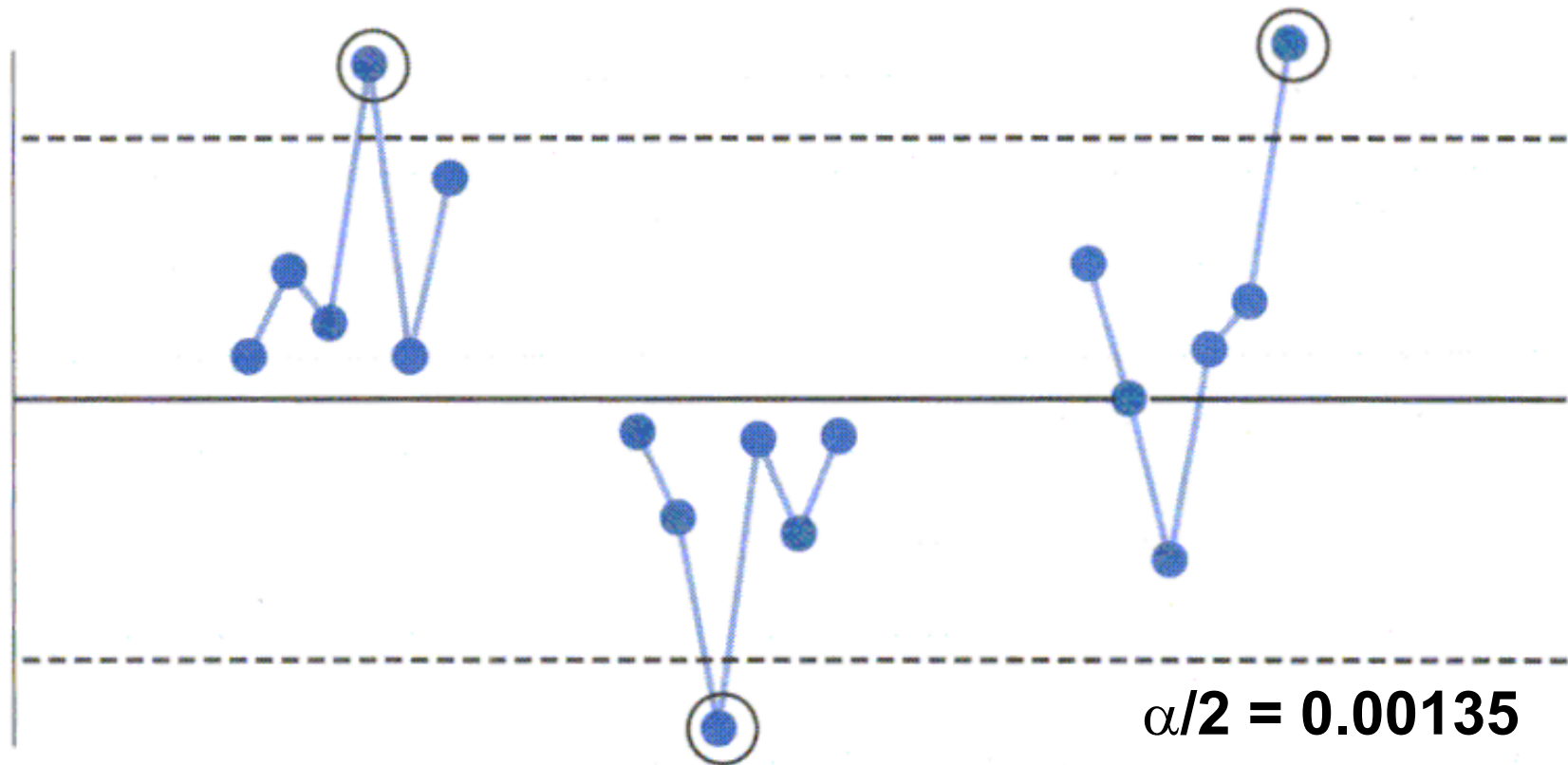**Figure 6.2** Appearance of a Process in Good Statistical Control

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Figure 6.3 Control Chart Zones to Aid Chart Interpretation

0.00135

A   0.02135

B   0.1360

C   0.3413

$\bar{x}$

C   0.3413

B   0.1360

A   0.02135

0.00135

Remember: No zones on the R chart

Test 1 examples - Extreme Points

$\alpha/2 = 0.00135$

Test 2: 2 out of 3 Points in Zone A or Beyond

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Basis for Test 2

Prob (A Point in Zone A or Beyond) = .0227

Prob (Two Points in a Row in Zone A or Beyond)
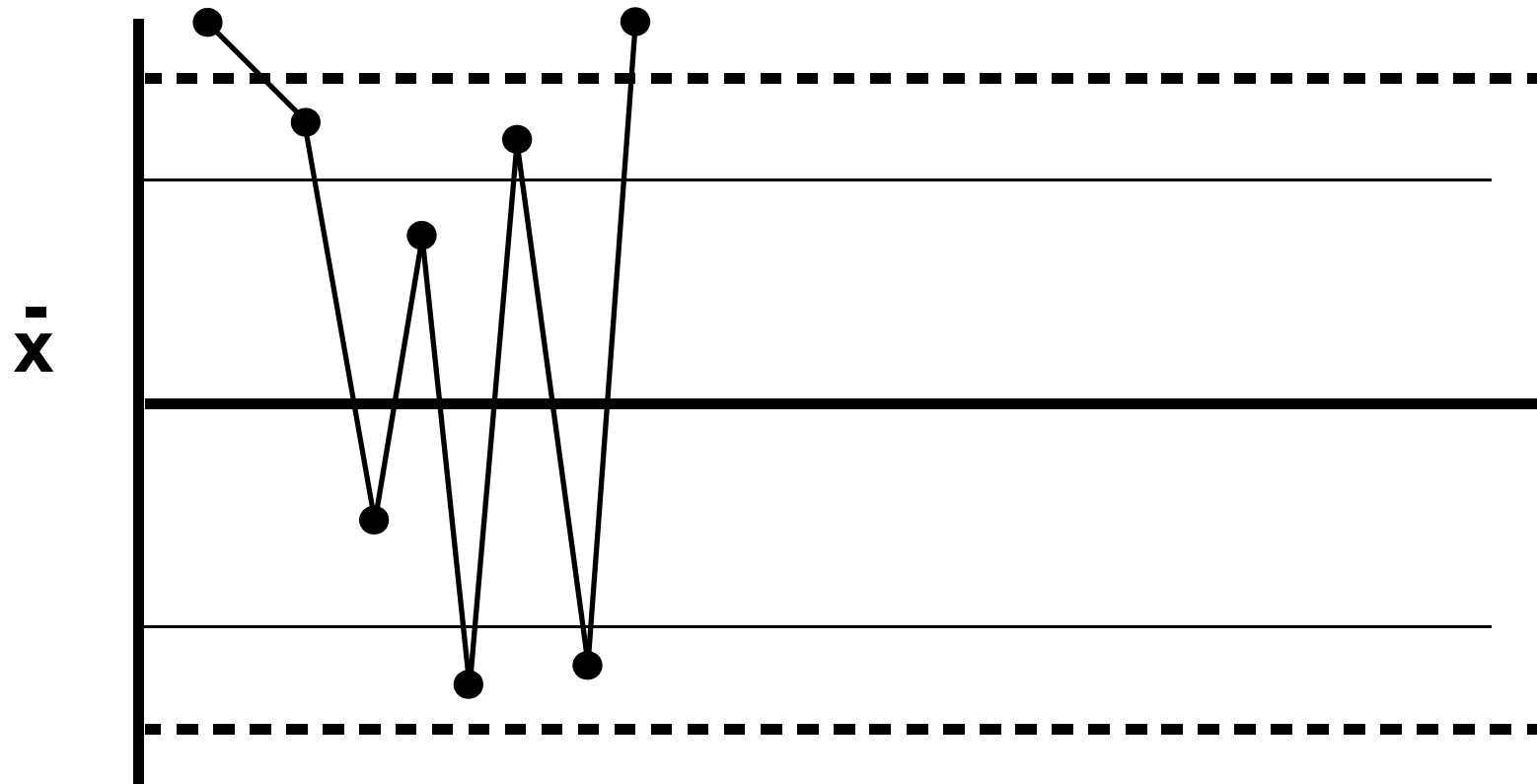$$= (0.0227) * (0.0227) = 0.00052$$

Very small relative to $\alpha/2 = 0.00135$
-- 2 points in a row in zone A is too restrictive

What about 2 out 3 in zone A or beyond? Two ways for this to occur: (A -- not A -- A)   OR   (not A -- A -- A)

$$= 2 * (0.0227)^2 * (0.9773) = 0.0010 \text{ -- close to } \alpha/2$$

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Test 2 examples - 2 out of 3 Points in Zone A or Beyond
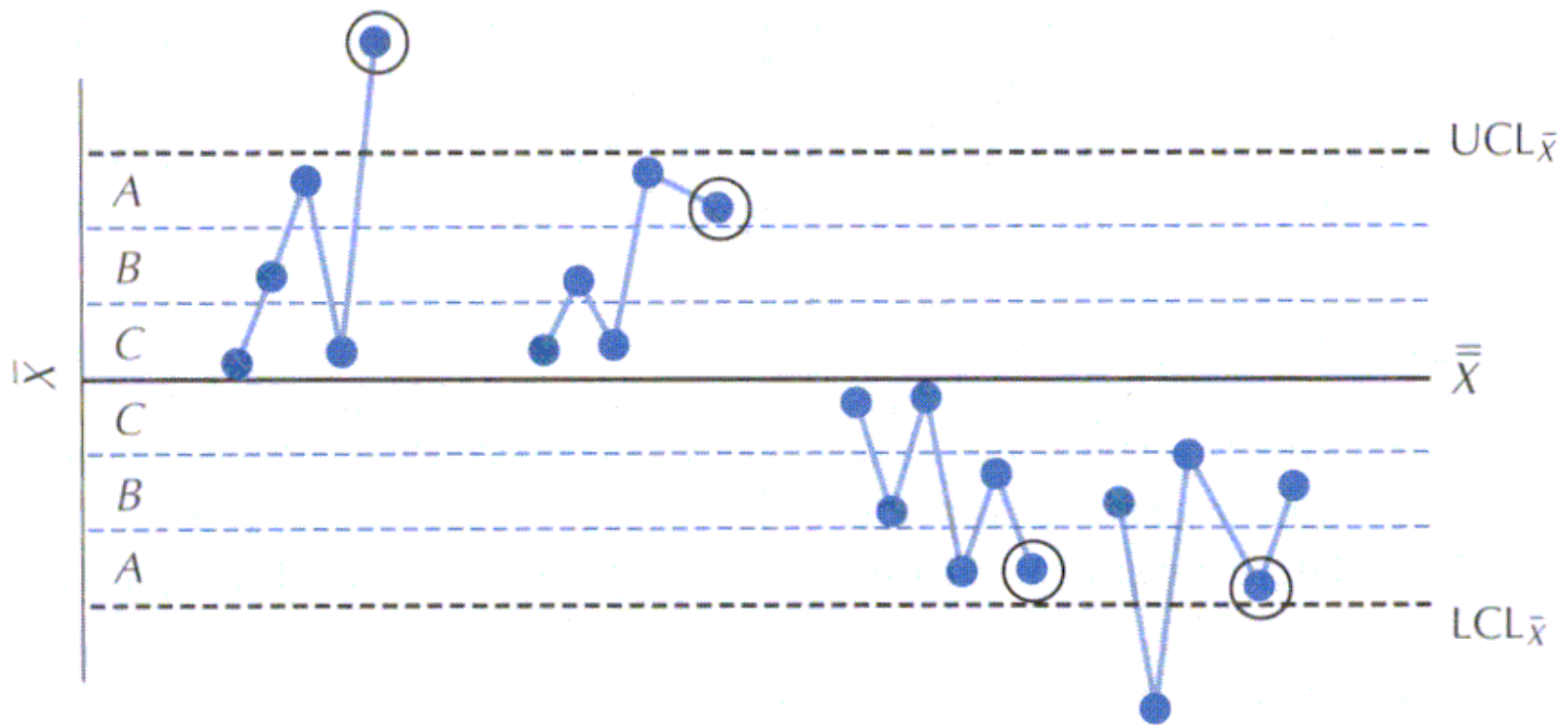
Additional Comments on Test 2

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Test 3 examples - 4 out of 5 Points in Zone B or Beyond

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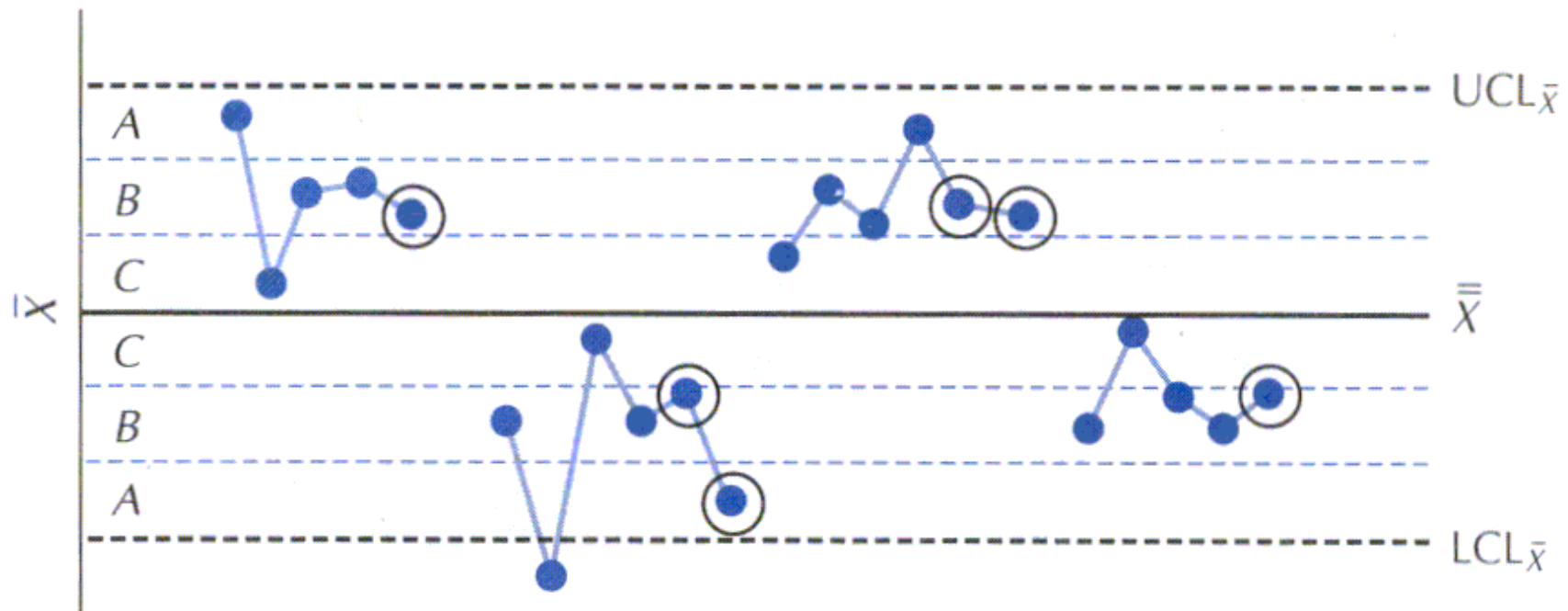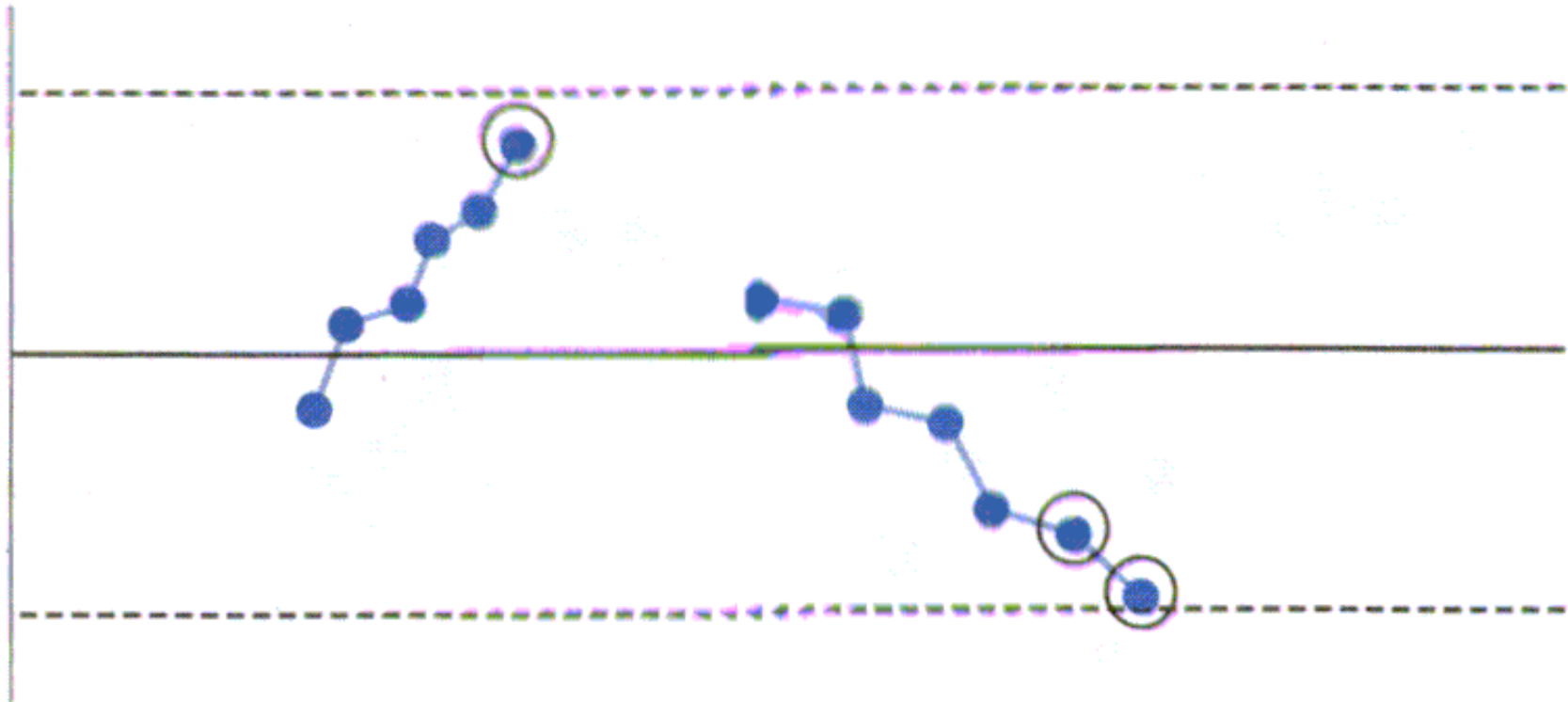
Test 4 examples - Runs Above or Below the Centerline
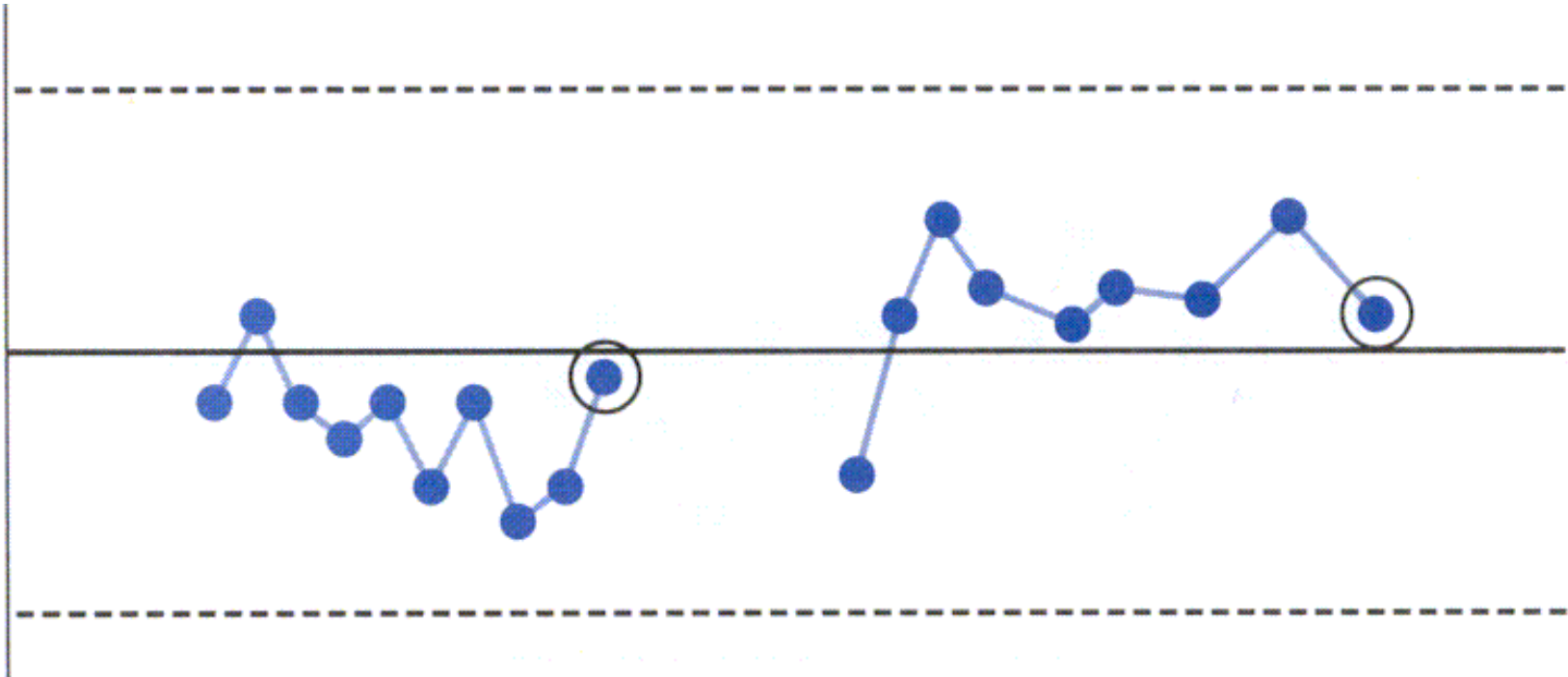
Probability Above / Below CL = 0.5

Prob (8 in a row above) = $(0.5)^8$ = 0.0039
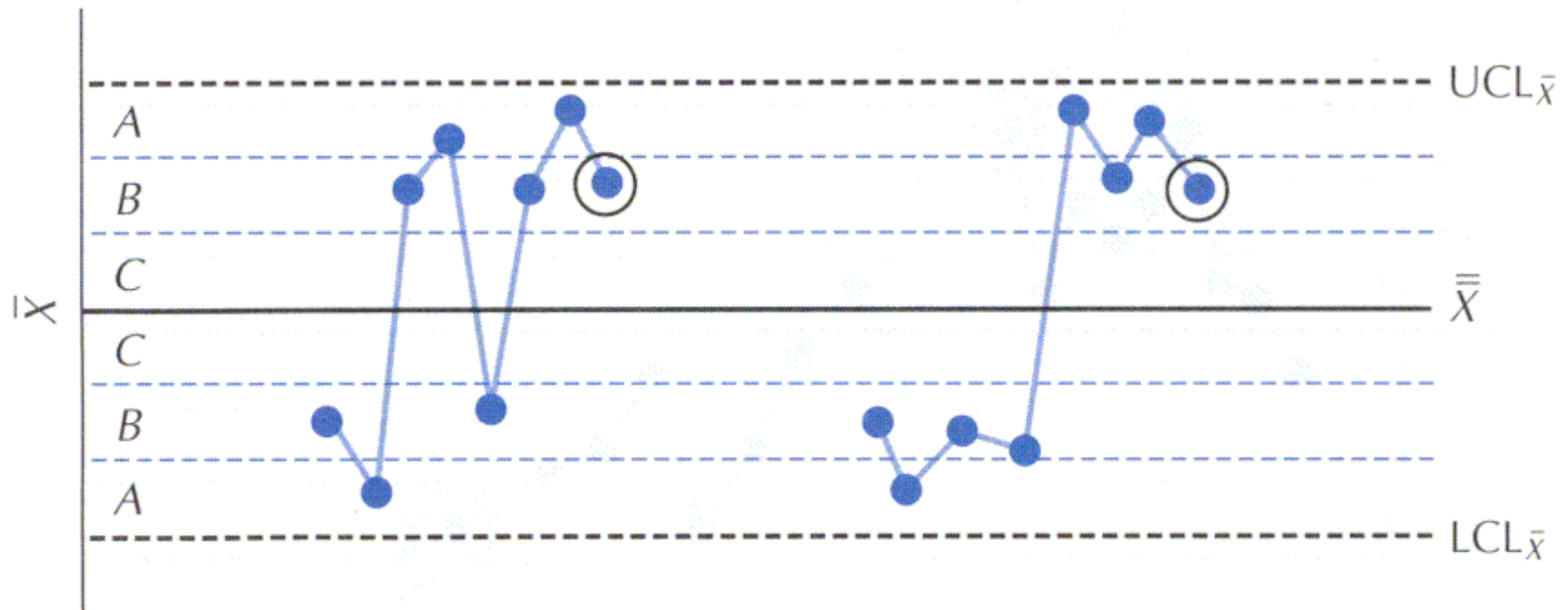
FYI... $(0.5)^9$ = 0.00195 $(0.5)^{10}$ = 0.000977

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Test 5 examples - Linear (Upward / Downward) Trend

Test 6 examples - Oscillatory Trend

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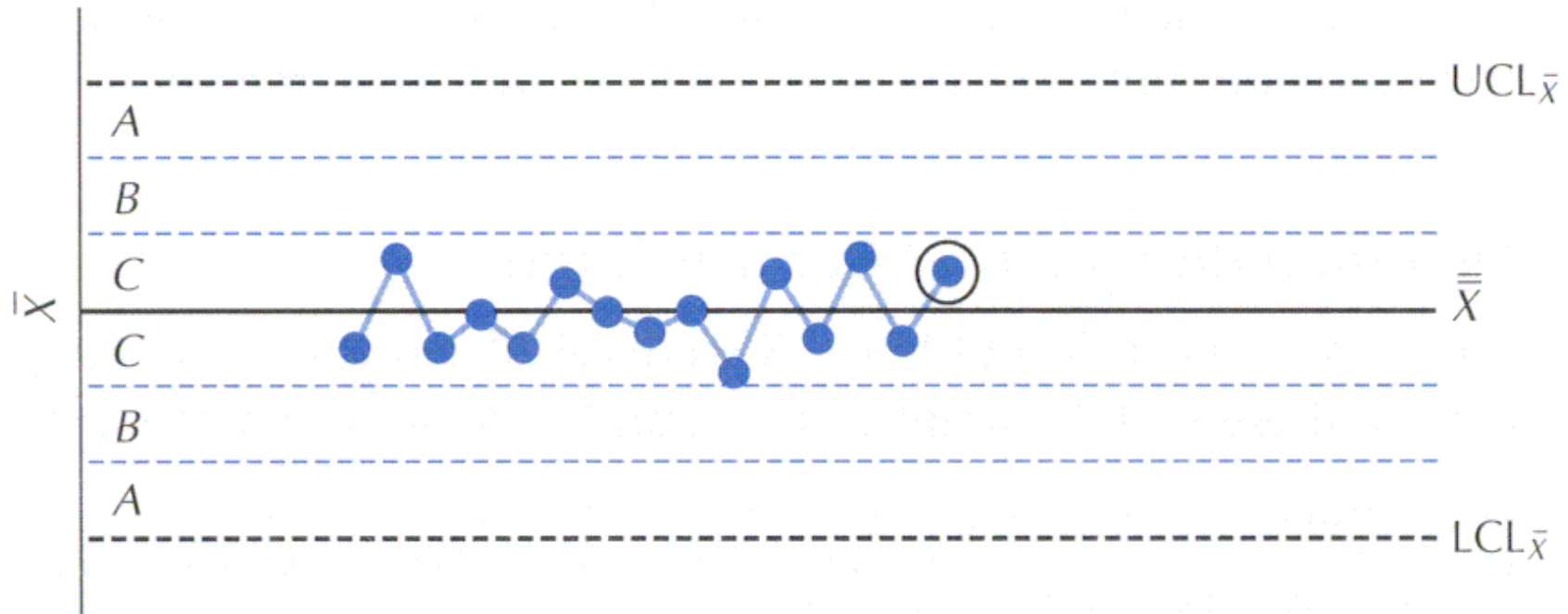**Michigan Technological University**
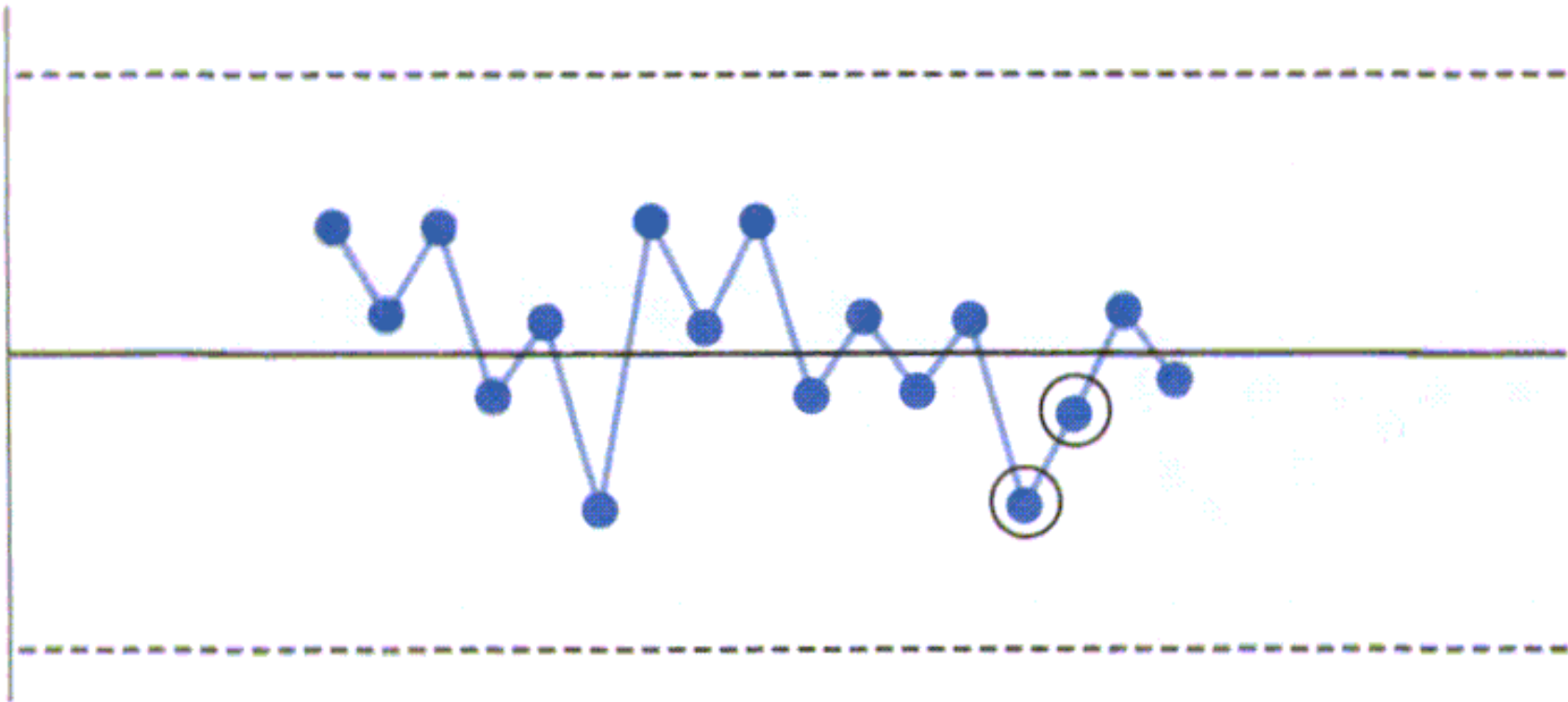
Test 7 examples - Avoiding Zone C

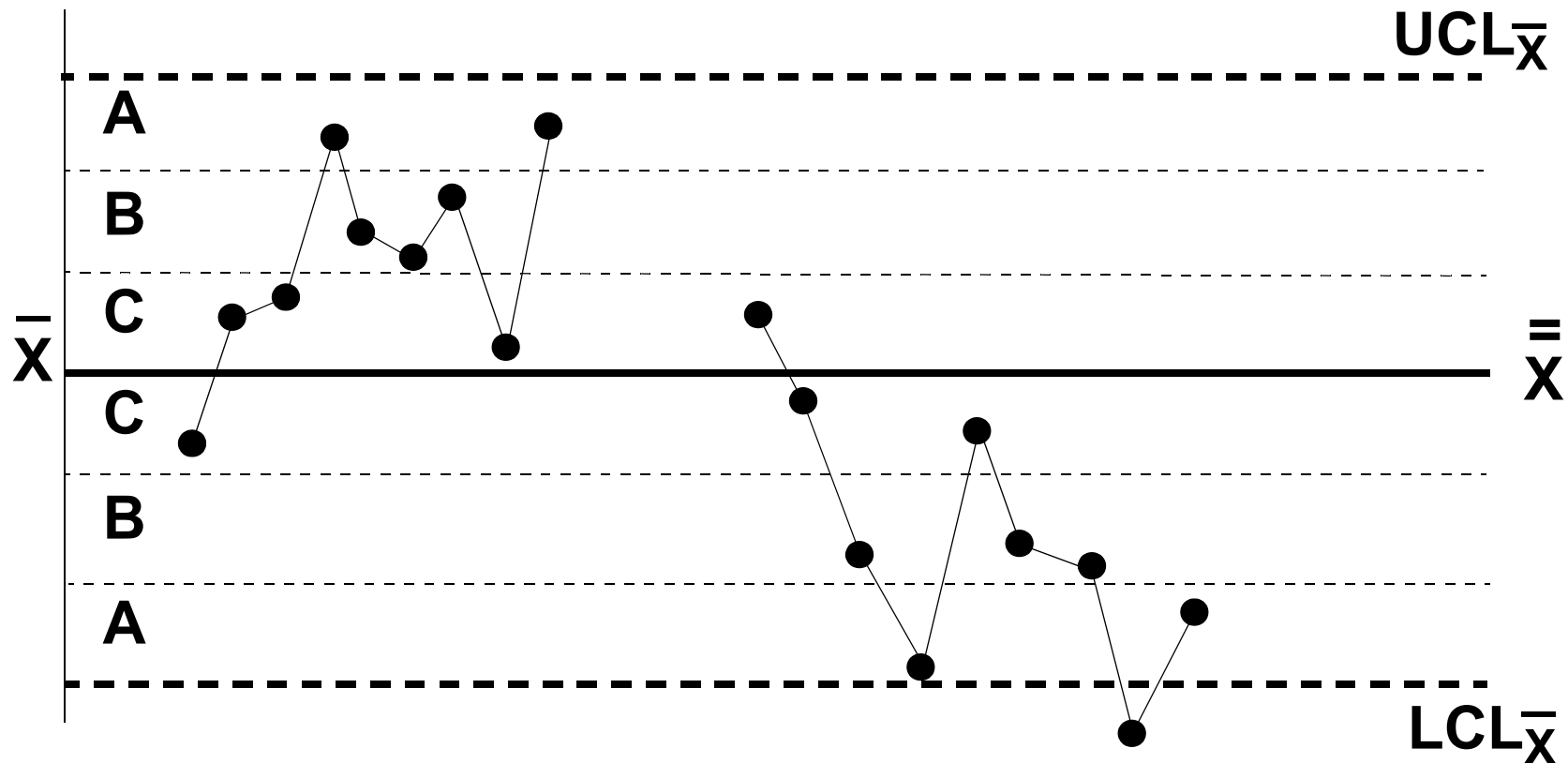**Prob (one point outside zone C) = 1-0.68 = 0.32**

$(0.32)^8 = 0.00011$ **-- very small**

Test 8 examples - Run in Zone C

Prob (being in zone C) = 0.68

Prob (15 in a row in zone C) = $(0.68)^{15}$ = 0.0031

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Example - Simultaneous Application of More Than One Test for Out-of-Control Conditions

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