Lecture #12

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Last Class -- Hypothesis Testing

1. State Null and Alternative Hypotheses (H_0 and H_A). Define test statistic.

2. Define, α, the risk level.

3. Collect Data - calculate test statistic.

4. Define reference distribution.

5. Make statistical decision

6. Draw conclusion.

Example #1

MUB claims soup is served at 160 deg. on the average. Soup temps. are know to be normal with a std. dev. of 10.

We want to test this claim!!

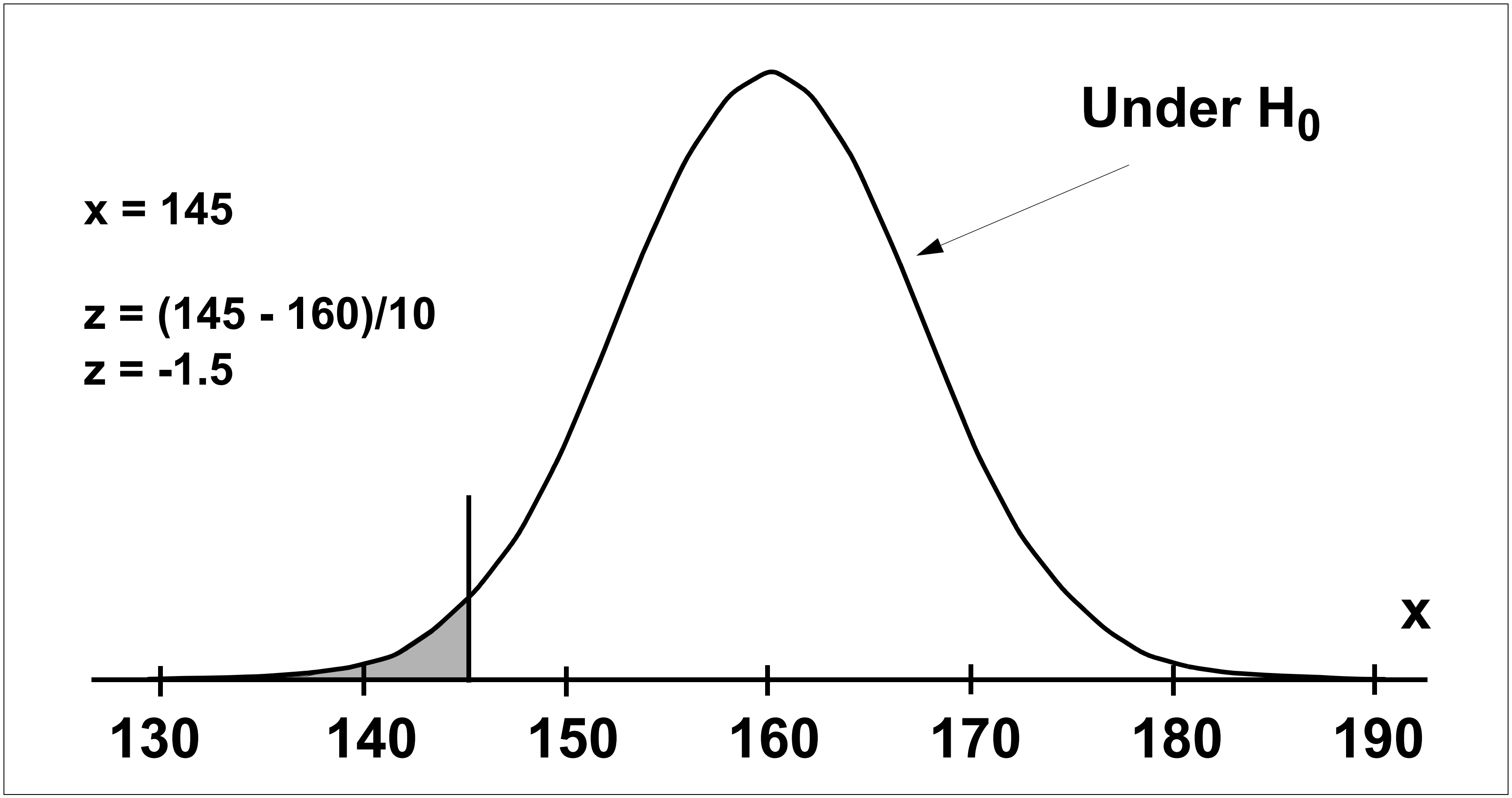
1. $H_0: \mu_x = 160 \qquad H_A: \mu_x \neq 160$

   Select a single bowl at random

2. α **= 0.05**

3. **x = 145 deg.**

4. **Draw the reference distribution under the null hypothesis.**



5.

$$\Pr(X \le 145) = \Pr(Z \le -1.5) = 0.0668$$

Compare 0.0668 with $\alpha/2 = 0.025$. Since the value obtained is not a rare event. Cannot reject H_0. This means that H_0 could be true.

6. The MUB's claim cannot be refuted. It may be true.

Example #2

MUB claims soup is served at 160 deg. on the average. Soup temps. are known to be normal with a std. dev. of 10. X's follow normal distribution.

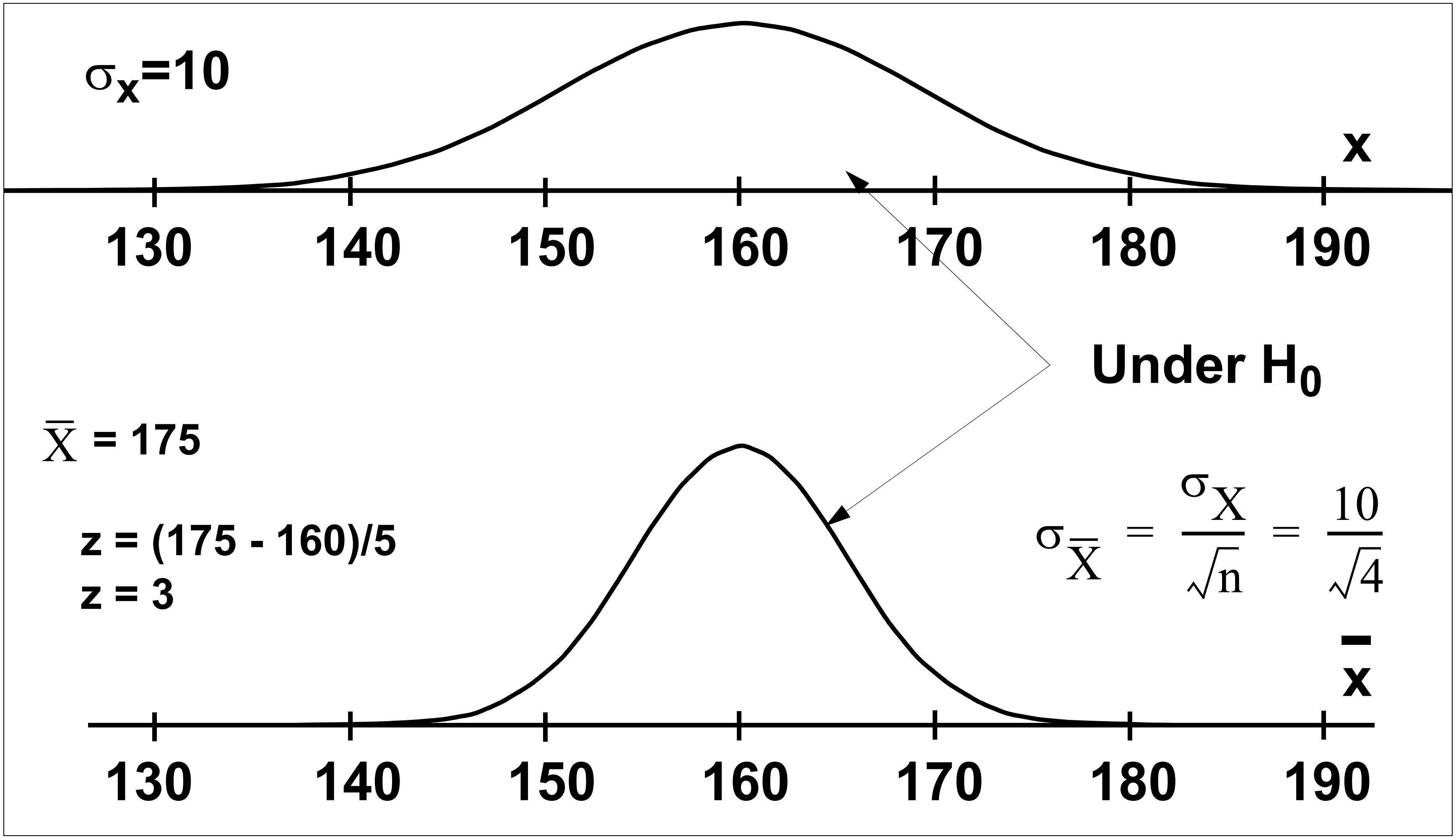
We want to test this claim!!

1. $H_0: \mu_X = 160$ $\quad\quad\quad H_A: \mu_X \neq 160$

   Select 4 bowls at random -- to form a sample

2. α **= 0.05**

3. **Pick 4 bowls (X's)** $\quad\quad \bar{X}$ **= 175 deg.**
4. **Draw reference distribution under null hypothesis.**



σ_x=10
x
130
140
150
160
170
180
190
Under H_0
$\bar{X}$ = 175
z = (175 - 160)/5
z = 3
$\sigma_{\bar{X}} = \frac{\sigma_X}{\sqrt{n}} = \frac{10}{\sqrt{4}}$
$\bar{x}$
130
140
150
160
170
180
190

5.

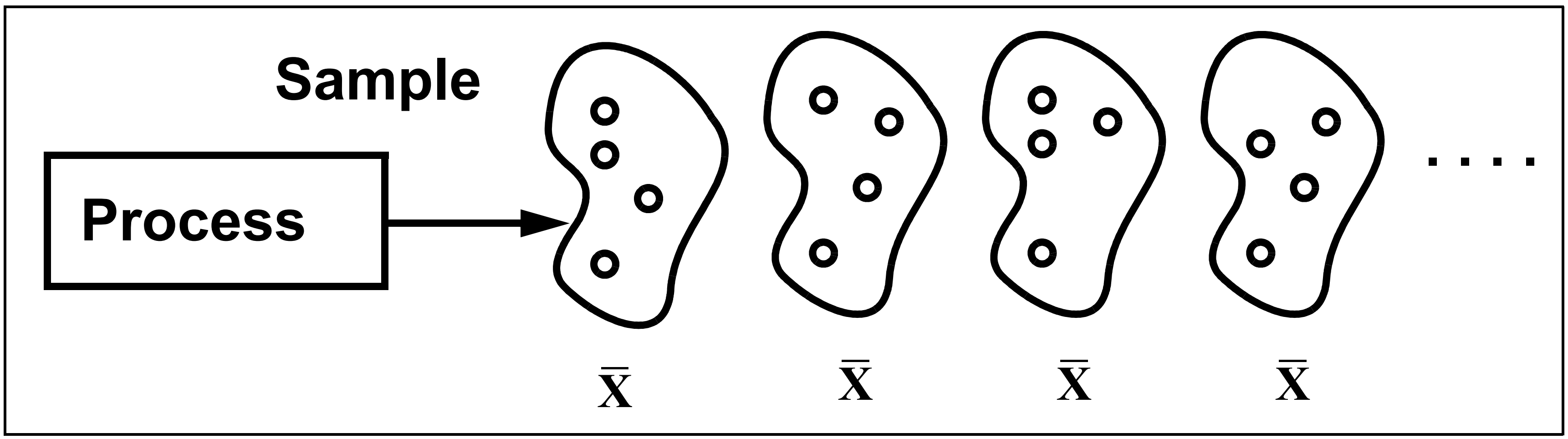
$$\Pr(X \geq 175) = \Pr(Z \geq 3) = 1 - 0.99865 = 0.00135$$

Compare 0.00135 with $\alpha/2$ = 0.025. Since the value obtained is a rare event. Reject H_0.

6. The MUB's claim appears to be false.

Hypothesis Testing - Example #3

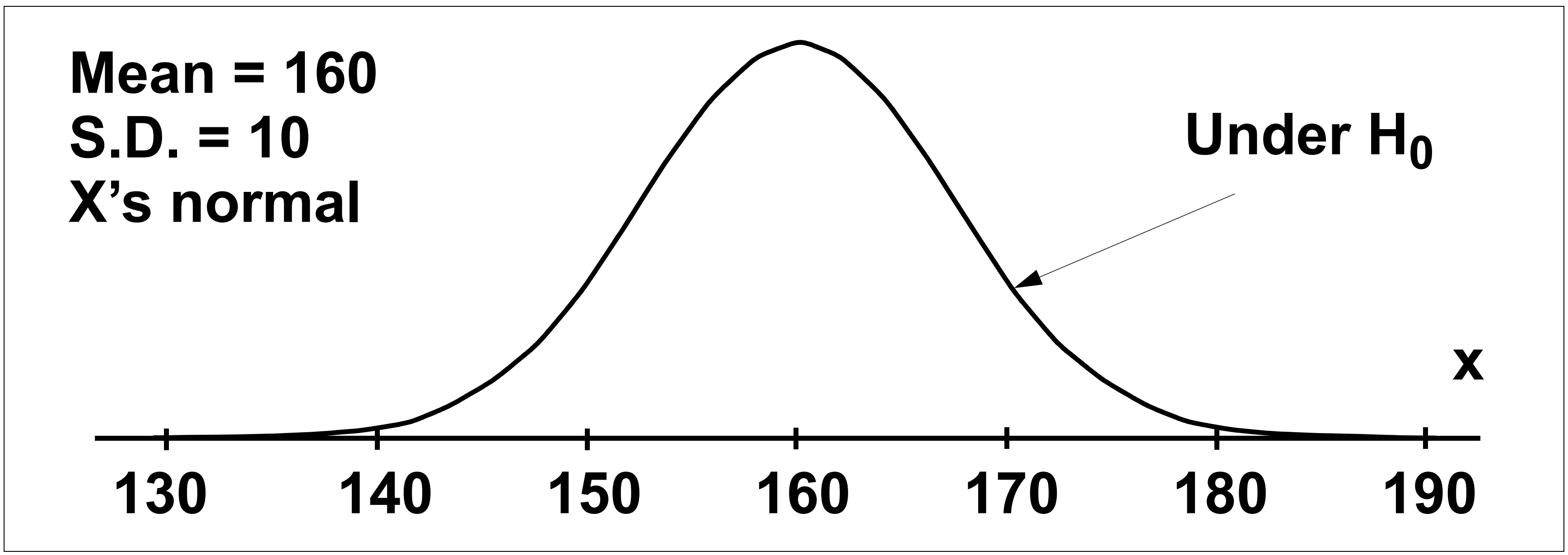
Because of our continuing concern about the temperature of soup at the MUB - let's devise a scheme to continuously monitor it.



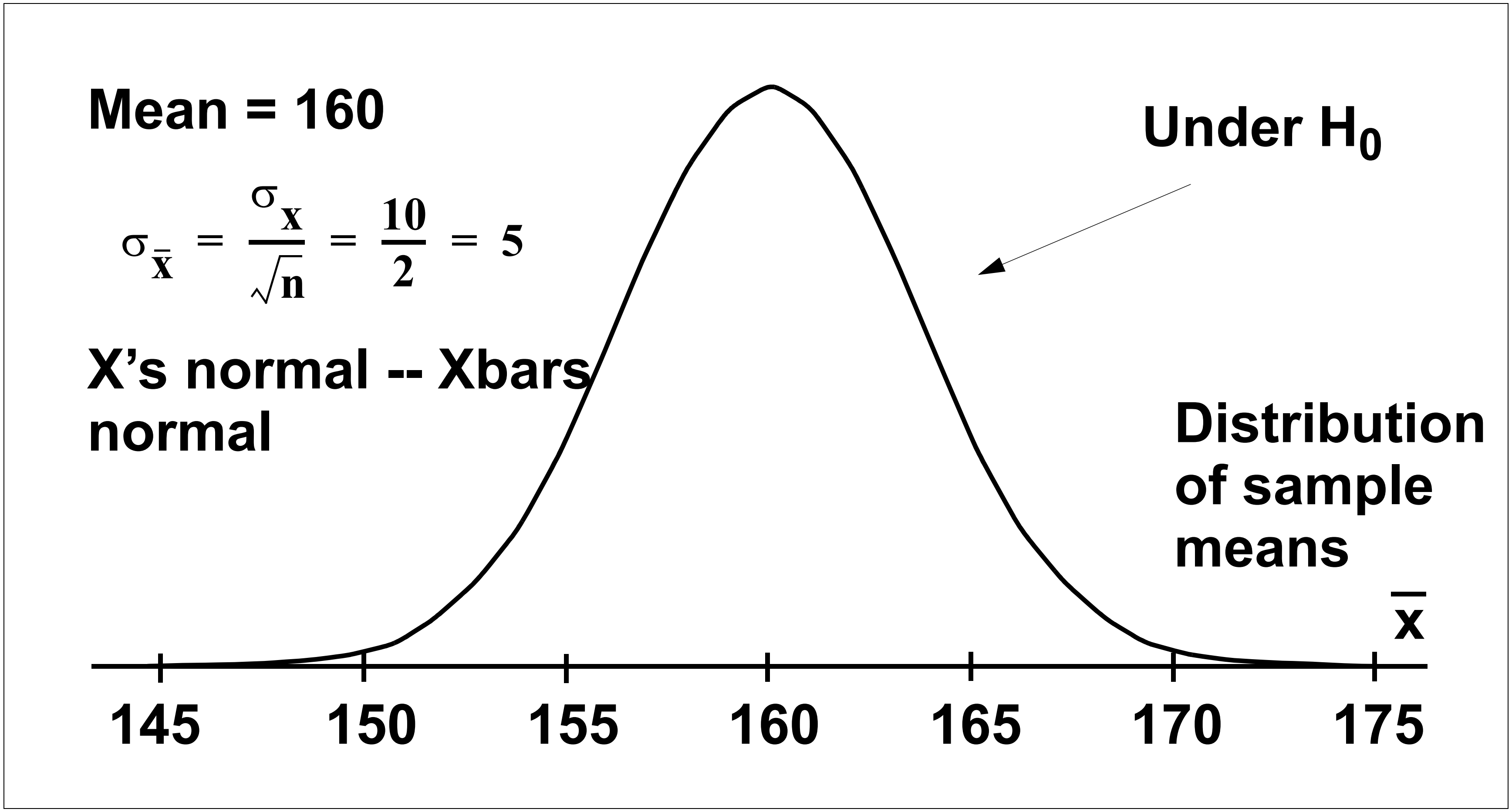
Let's perform a hypothesis test for each $\bar{X}$

Hypothesis Testing of Multiple Sample Means

Recall that the claimed average soup temperature is 160 deg.



Hypothesis Testing -- Example #3



Example #3

$H_0: \mu_x = 160$ $H_A: \mu_x \neq 160$ α = 0.05

So, for every sample mean we obtain from the MUB (say 175), we must:

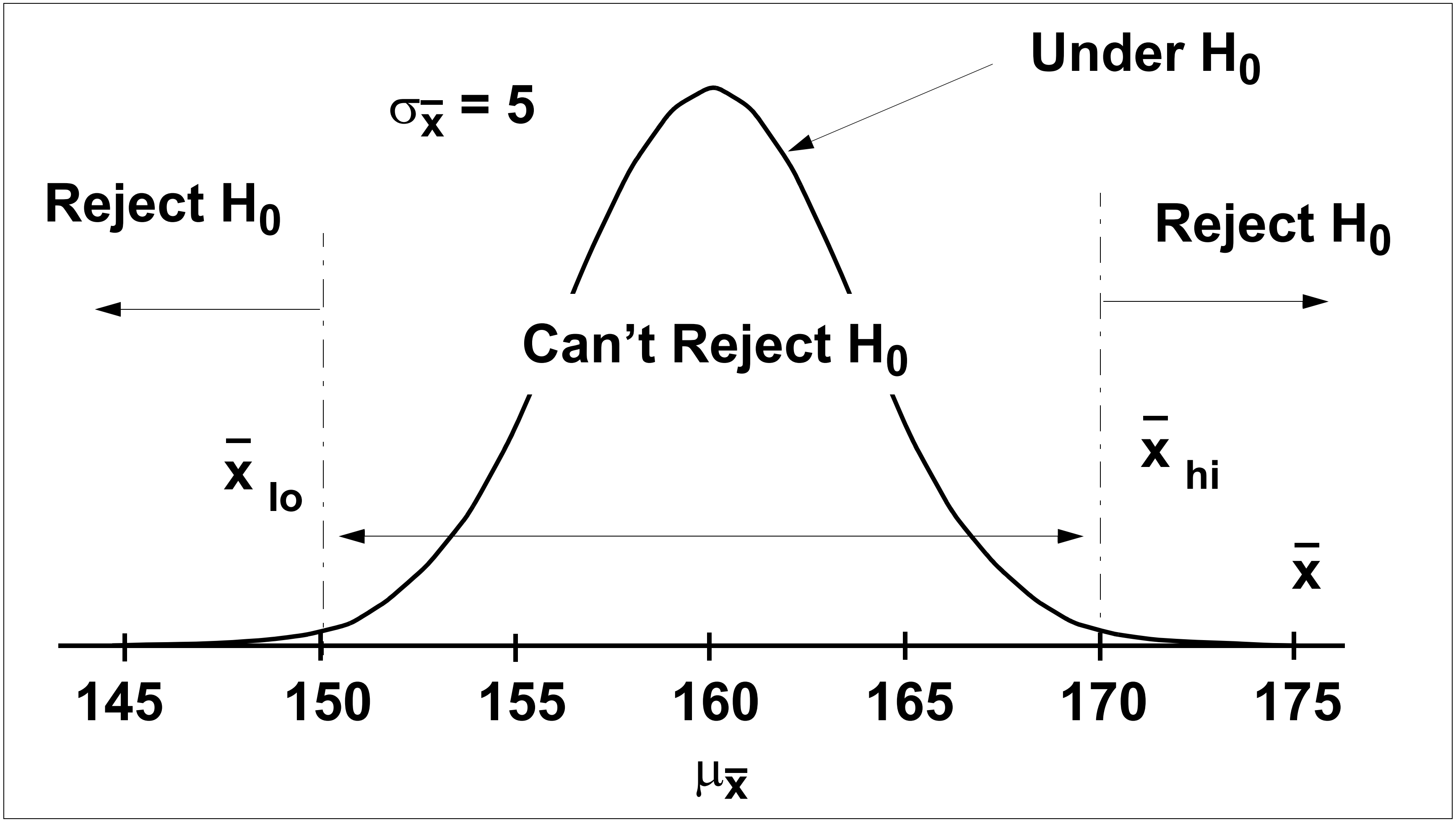
- **Calculate Z**
- **Find probability of getting a value more extreme than that Z**
- **Compare the calculated probability to $\alpha/2$ - reject claim if calc. prob. is less than $\alpha/2$.**

Example #3 - comments

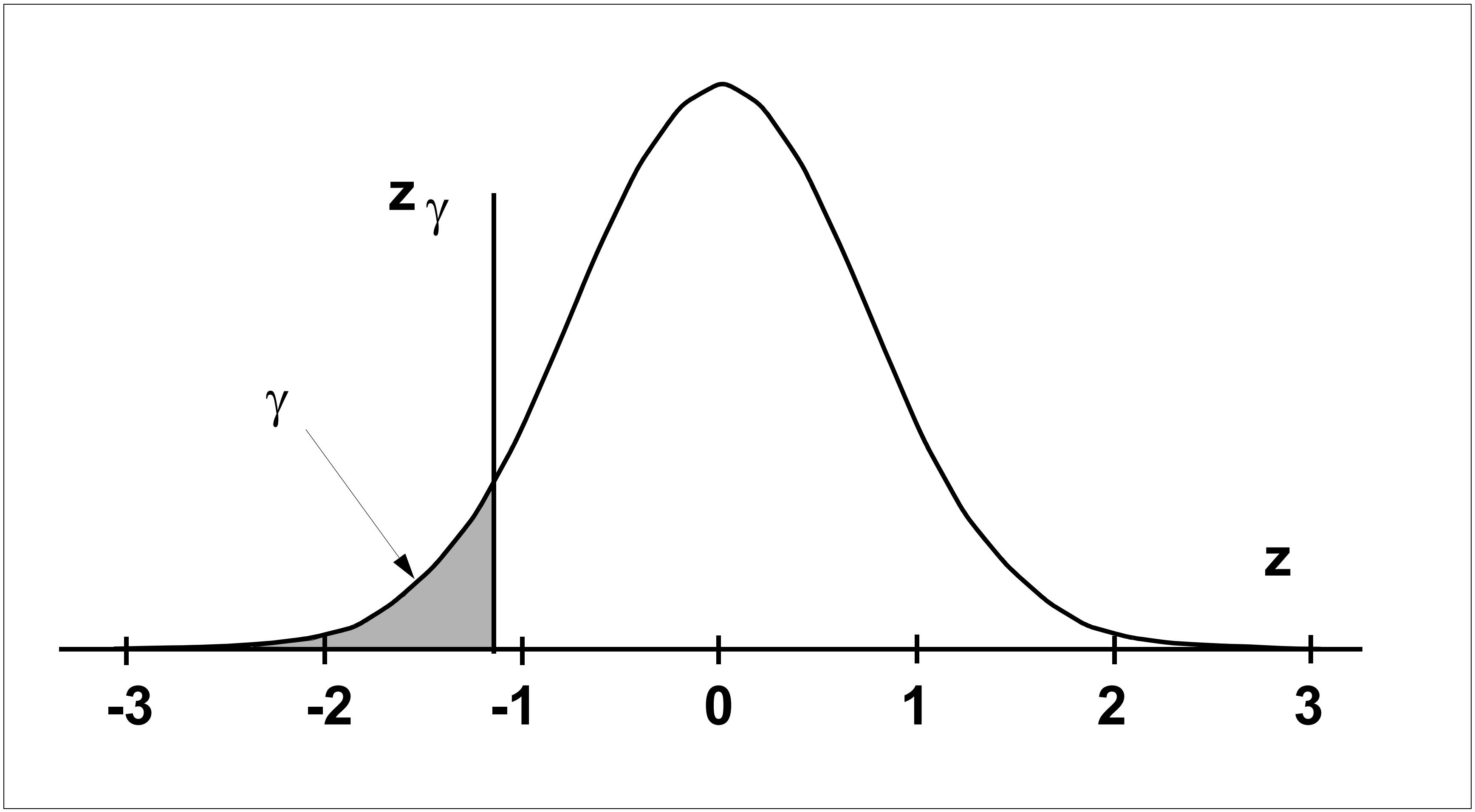
What a hassle! There is no way we wish to calculate a separate Z for each sample mean and then look up the probability for each Z.

Another approach: let's define limits for distribution of sample means - if an Xbar is beyond the limits - reject claim. If an Xbar is within limits, can't reject claim.

Rejection Limits



Notation



Rejection Limits - continued

We want to pick values for $\bar{X}_{lo}$ and $\bar{X}_{hi}$.

Any sample mean larger than $\bar{X}_{hi}$ should fail the hypothesis test, i.e., the probability of a value this large or larger is less than α/2.

So, let's place $\bar{X}_{hi}$ so that it places α/2 in the tail of the distribution.

$$\Pr(\bar{X} \geq \bar{X}_{hi}) = \alpha/2 = 0.025$$

$\bar{X}_{hi}$ corresponds to $Z_{0.975}$

Example #3 -- continued

$\bar{X}_{hi}$ **corresponds to** $Z_{1-\alpha/2} = Z_{0.975} = 1.96$

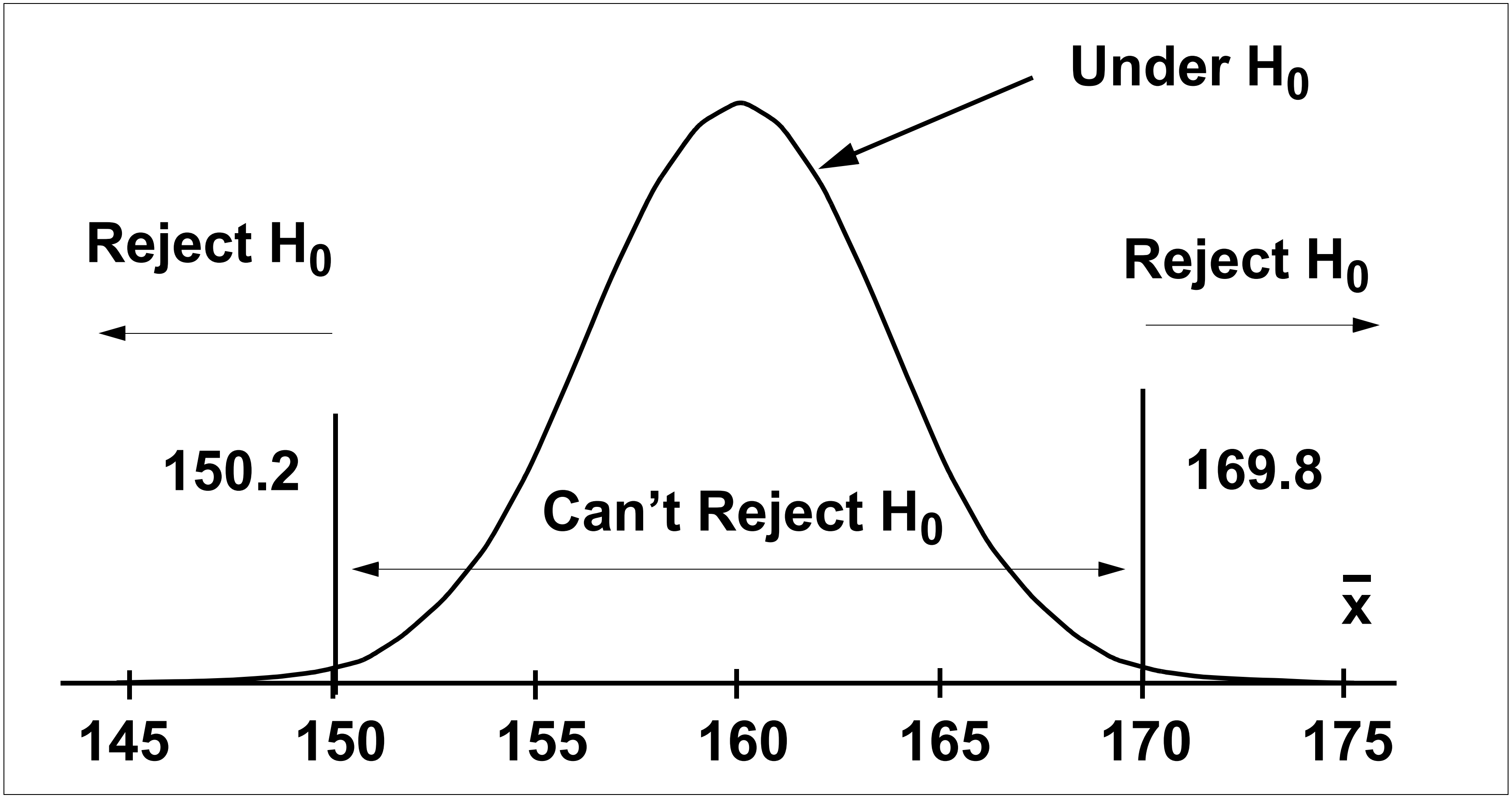
$\bar{X}_{lo}$ **corresponds to** $Z_{\alpha/2} = Z_{0.025} = -1.96$

Given the above,

$$\frac{\bar{X}_{hi} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = Z_{1-\alpha/2} \qquad \frac{\bar{X}_{lo} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = Z_{\alpha/2}$$

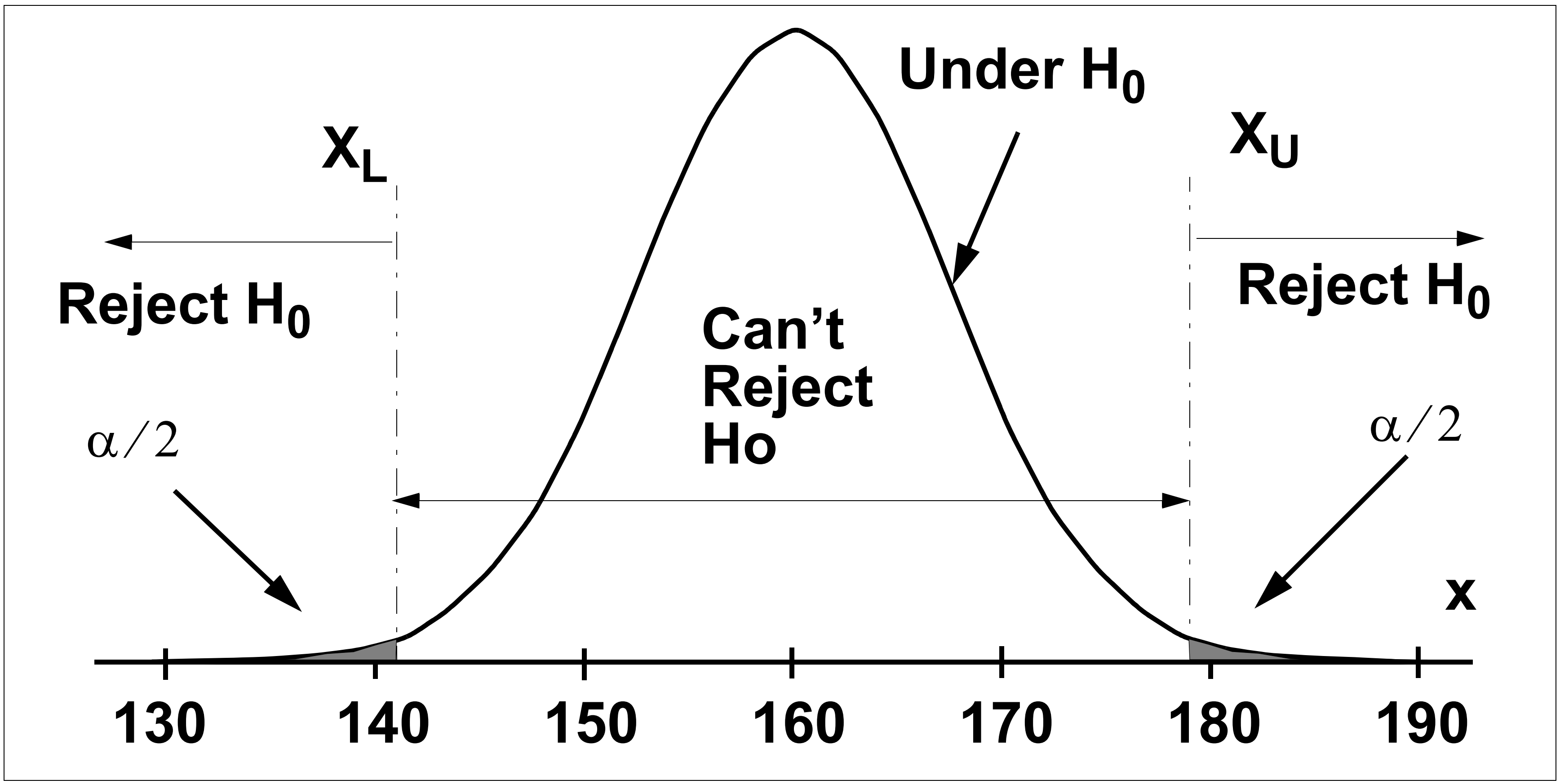
Solving for $\bar{X}_{lo}$ and $\bar{X}_{hi}$ gives 150.2 and 169.8

Rejection Limits



Decision Errors in Hyp. Testing

(Reconsider the Dist. of Individuals - X's)



Hypothesis Testing - Cutoff Values

$$Z_{\alpha/2} = Z_{0.025} = -1.96$$

$$Z_{1-\alpha/2} = Z_{0.975} = 1.96$$

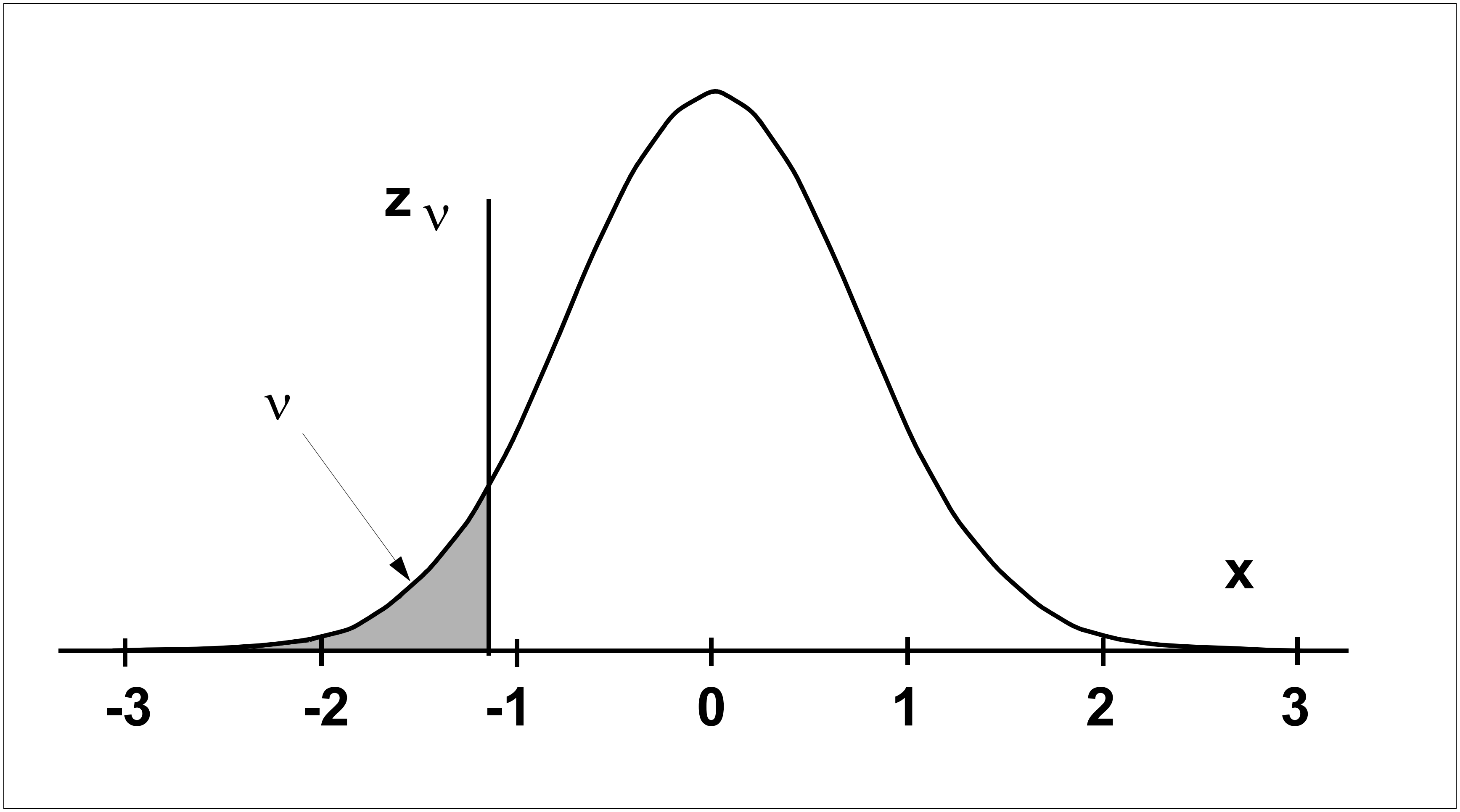
$$Z_{\alpha/2} = (X_L - \mu_x)/\sigma_x = (X_L - 160)/10 = -1.96$$

-19.6+160=X_L=140.4

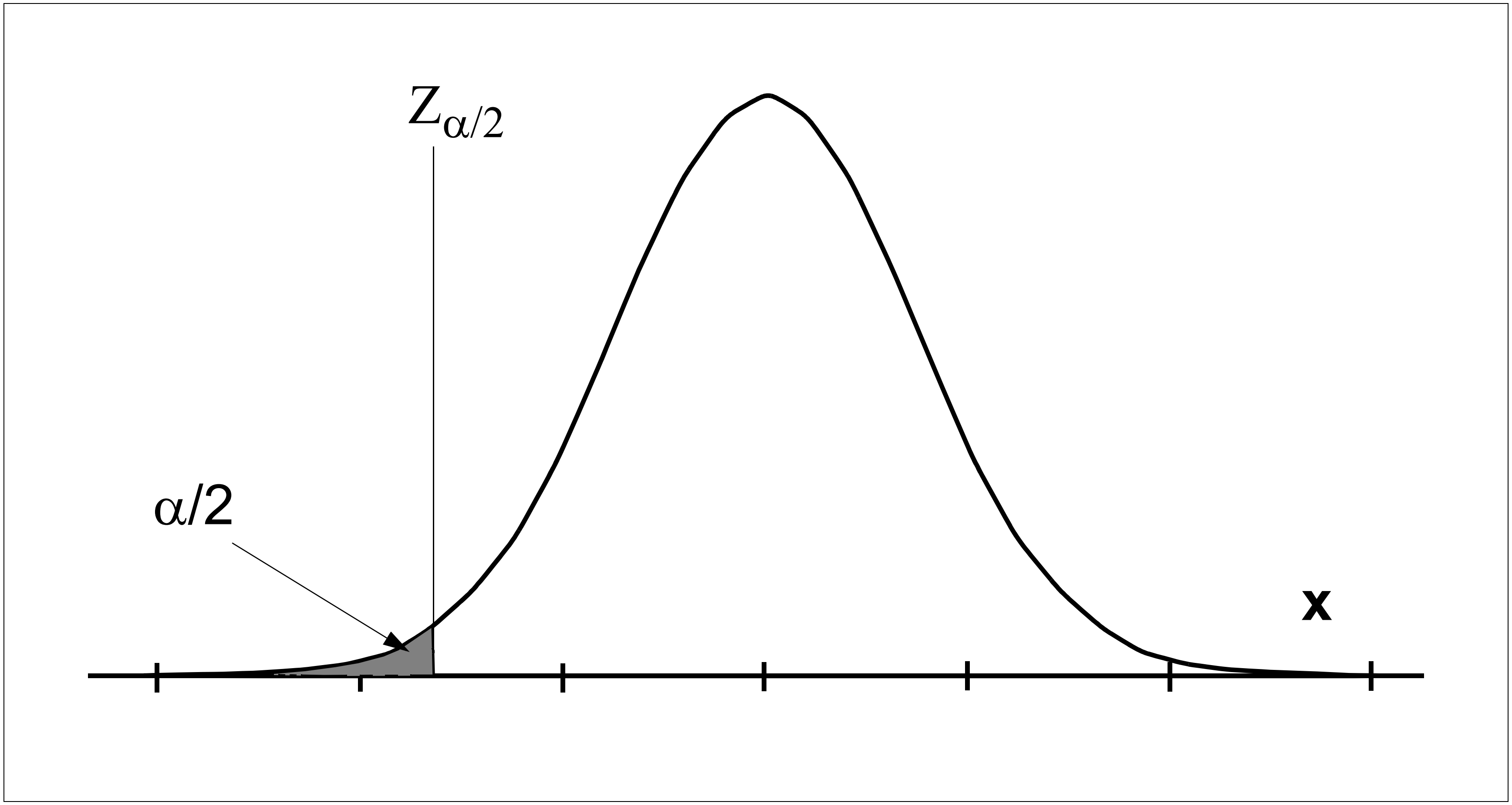
$$Z_{1-\alpha/2} = (X_U - \mu_x)/\sigma_x = (X_U - 160)/10 = 1.96$$

19.6+160=X_U=179.6

Notation



Alpha Error



MUB Soup Example

Lets revisit the MUB Soup example we did in the last class and investigate the risk associated in the hypothesis testing

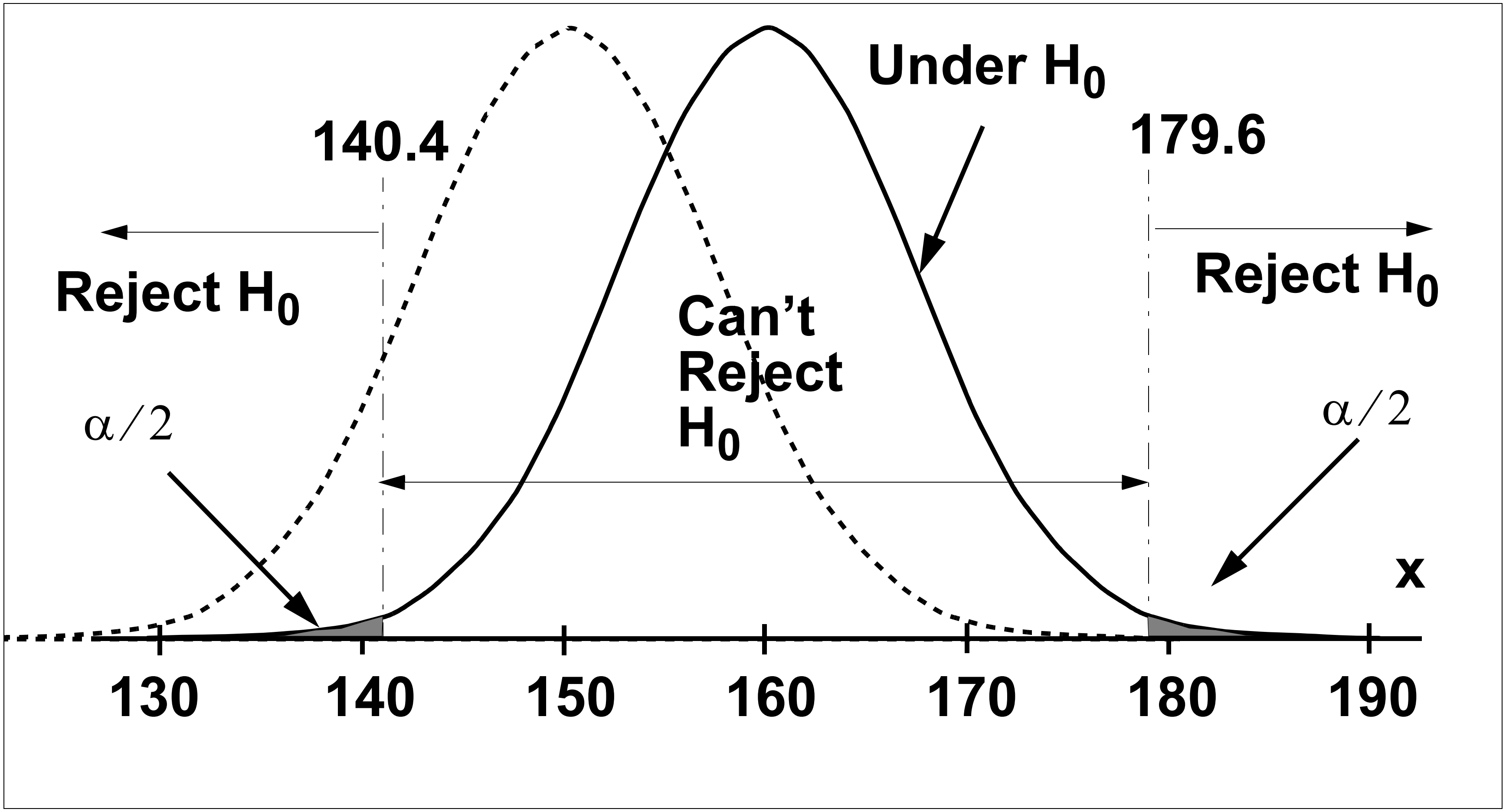
1. $H_0: \mu_x = 160$ $H_A: \mu_x \neq 160$

2. α **= 0.05**

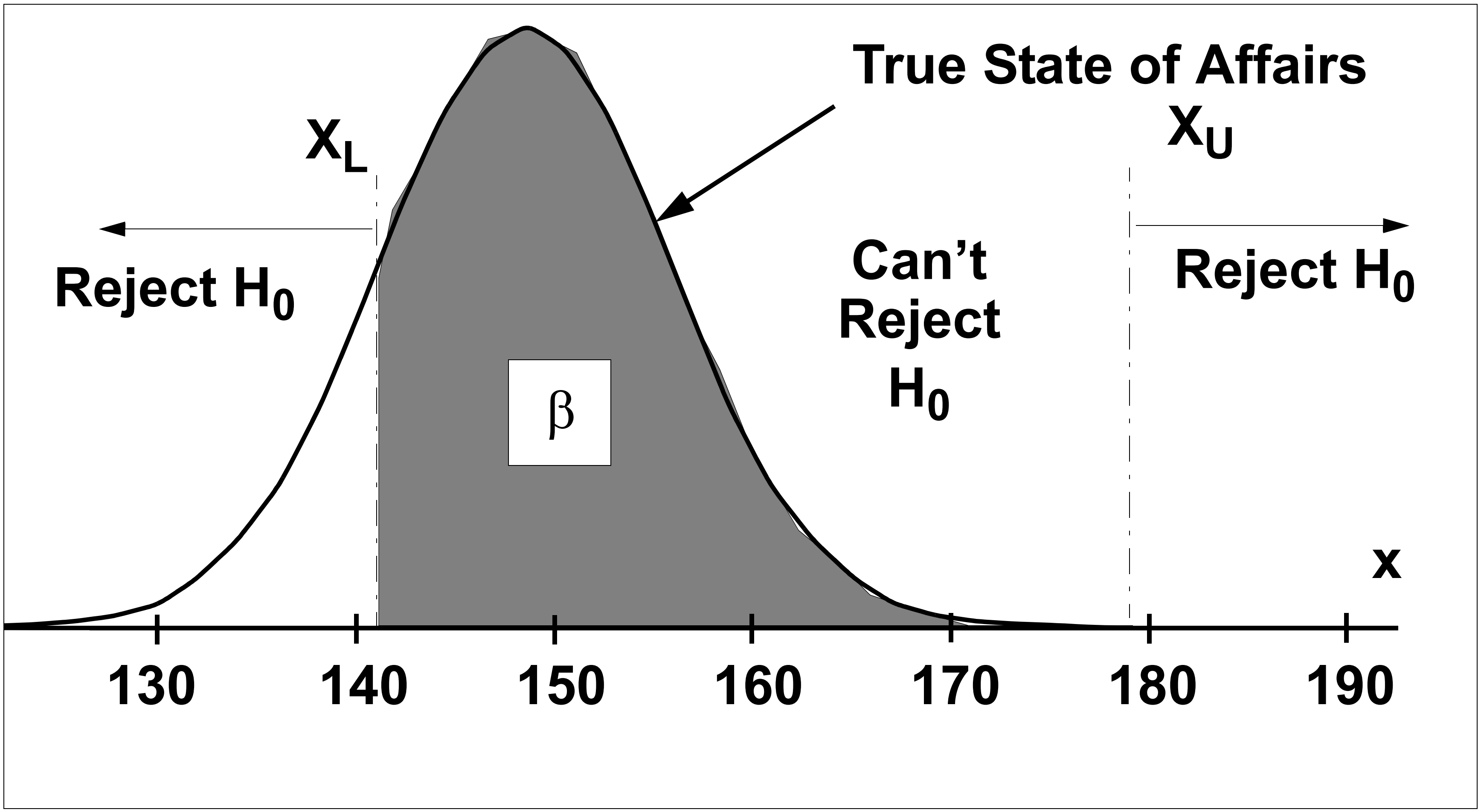
X_L **=140.4**

X_U **=179.6**

Example (Cont.)



Beta Error



True State of Affairs
X_L
X_U
Reject H_0
Can't Reject H_0
Reject H_0
β
x
130
140
150
160
170
180
190

Risk & Errors - Summary

α = Producer's Risk

Wrongly Reject Ho: Type 1 Error

β =Consumer's Risk

Don't Reject Ho when we should: Type 2 Error
"Wrongly Accepting Ho"

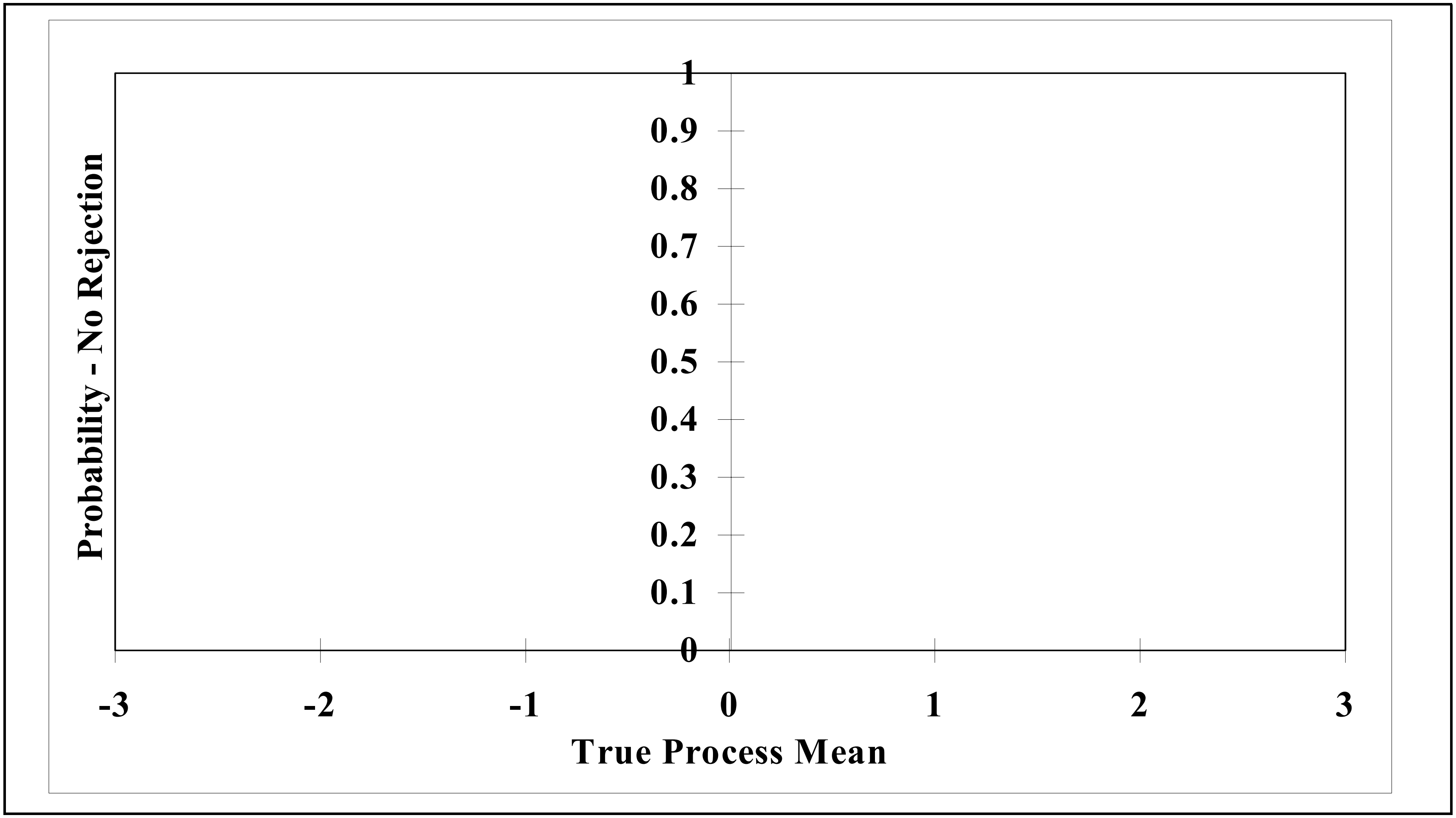
Decision Errors (Summary)

Outcome from Statistical Test of Hypothesis	True State: Claim True	True State: Claim False
Can't Reject Ho	No Error	β Risk Type 2
Reject Ho	α Risk Type 1	No Error

Judicial System Analogy

Outcome from Trial -- Jury's Decision	True State: Innocent	True State: Guilty
Not Guilty	No Error	β Risk Type 2
Guilty	α Risk Type 1	No Error

Let's Make a Graph



Example (cont.)

Lets do some calculations

Assume: True Mean: 160 deg.

$$Pr\left(140.4 \leq X \leq 179.6\right) = Pr\left(-1.96 \leq \mathbf{Z} \leq 1.96\right) = 0.95$$

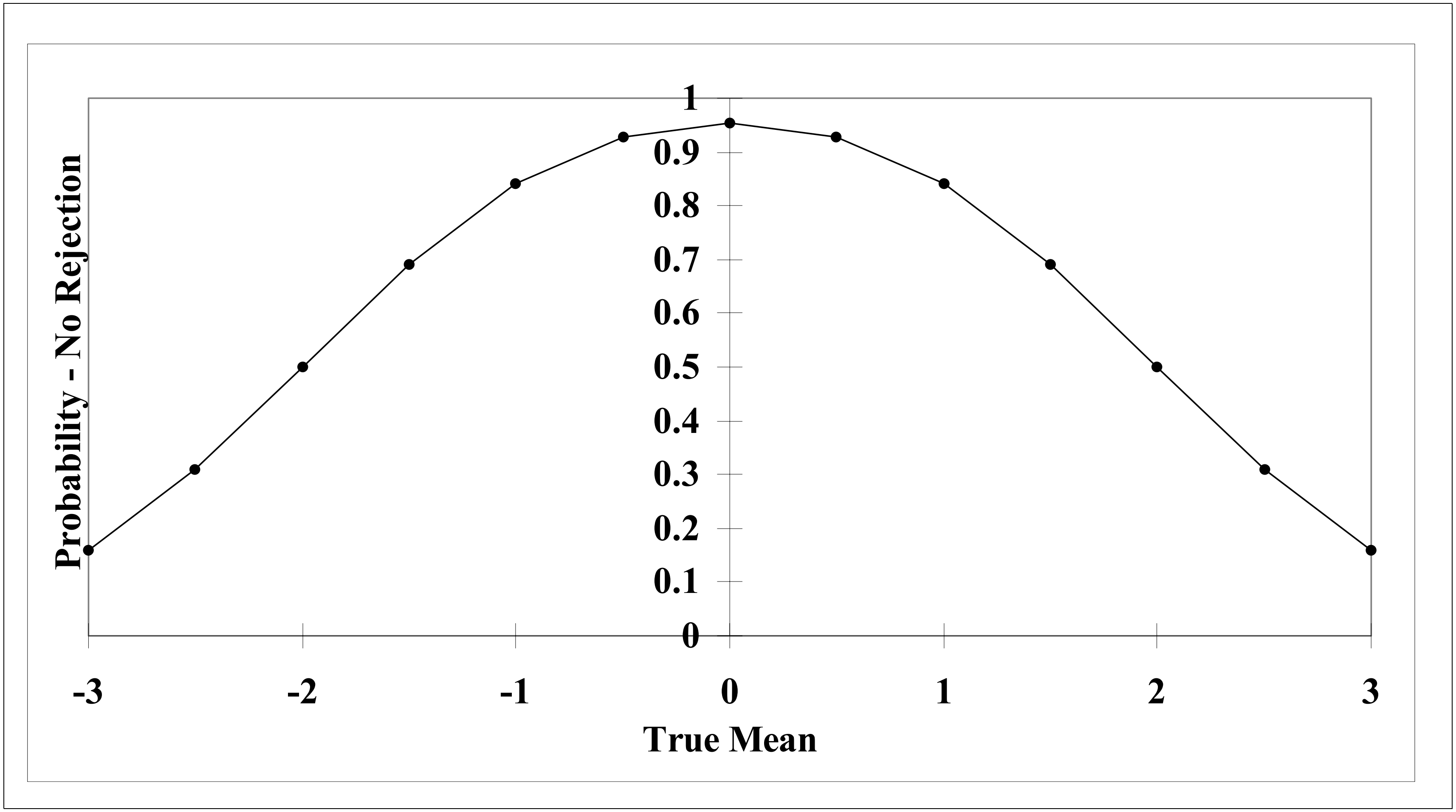
If True Mean is: 150

$$Pr\left(140.4 \leq X \leq 179.6\right) = Pr\left(-0.96 \leq \mathbf{Z} \leq 2.96\right) = 0.83$$

If True Mean is: 140

$$Pr(140.4 \leq X \leq 179.6) = Pr(0.04 \leq Z \leq 3.96) = 0.48$$

Completed Graph



Example #2

Blood & Gore are running a Halloween gift store, they claim that the revenue for the store is $100k per month with a standard deviation of $20k. An audit committee from IRS(Internal Revenue Services) has come to check their accounting record and see if their claim is right.

Part 1: For the first time, the committee randomly selected four month record and found that the revenue is $120k. Assuming a risk level of 0.05, evaluate the claim.

Part 2: Calculate the upper and lower limits on X i.e., X_L, X_U

Example #3

Part 3: Draw the graph for β risk.