Lecture #10

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Second Example

$$P\ (75 \leq X \leq 115)\ =\ ?$$

$$\sigma_x\ =\ 10$$

X

70   80   90   100   110   120   130

How many std. devs. from the mean ?

75??

115??

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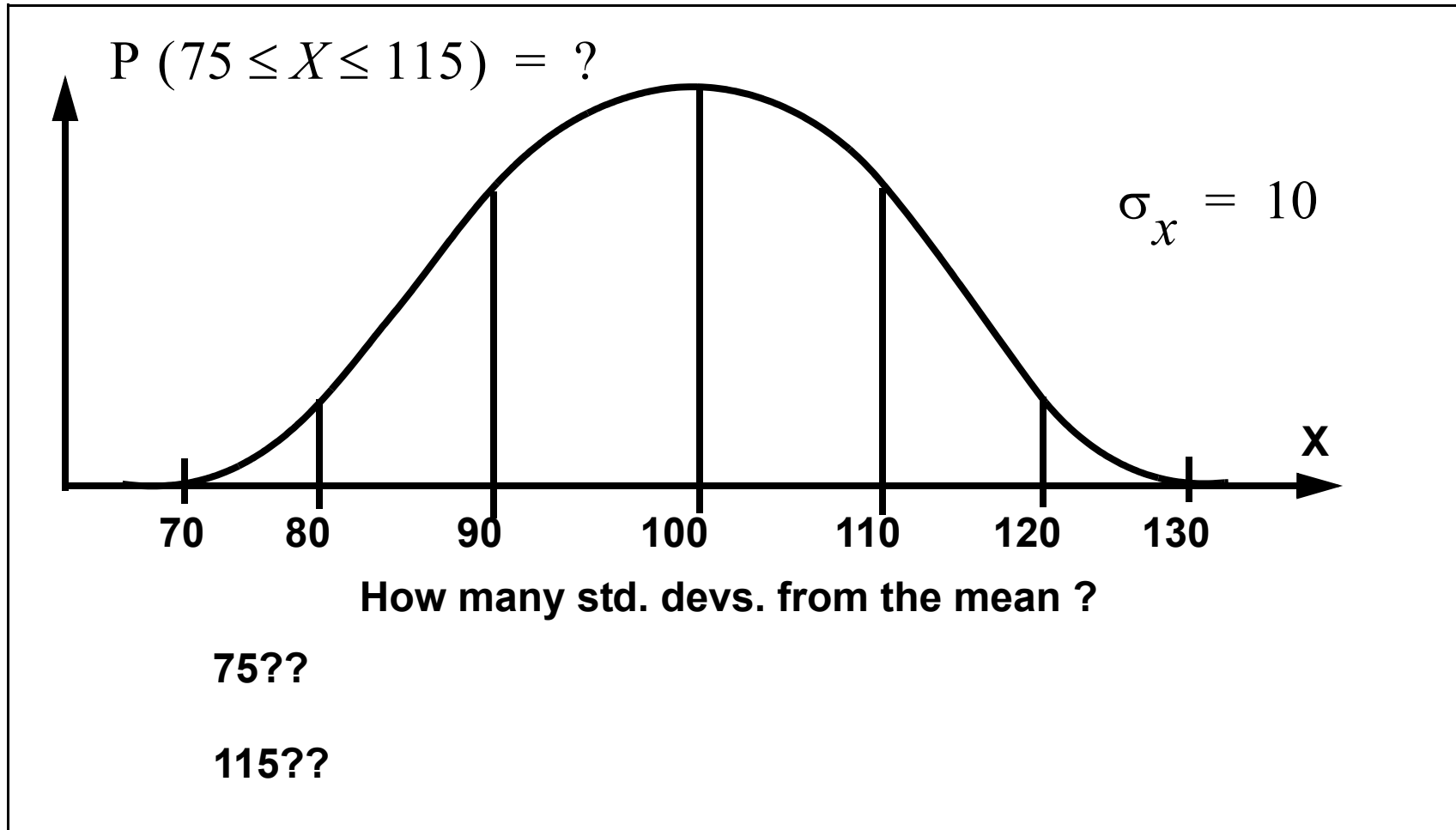
Example # 2 Continued

# of Std. Devs., z	Cum. Prob. - area under curve from $-\infty$ to z, F(z)
-3	0.00135
-2	0.0228
-1	0.1587
0	0.50
1	0.8413
2	0.9772
3	0.99865

Table A.1
lists F(z) for various std. dev., z

$F(-2.5)=0.0062$ $F(1.5)=0.9332$

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Example # 2 Summary
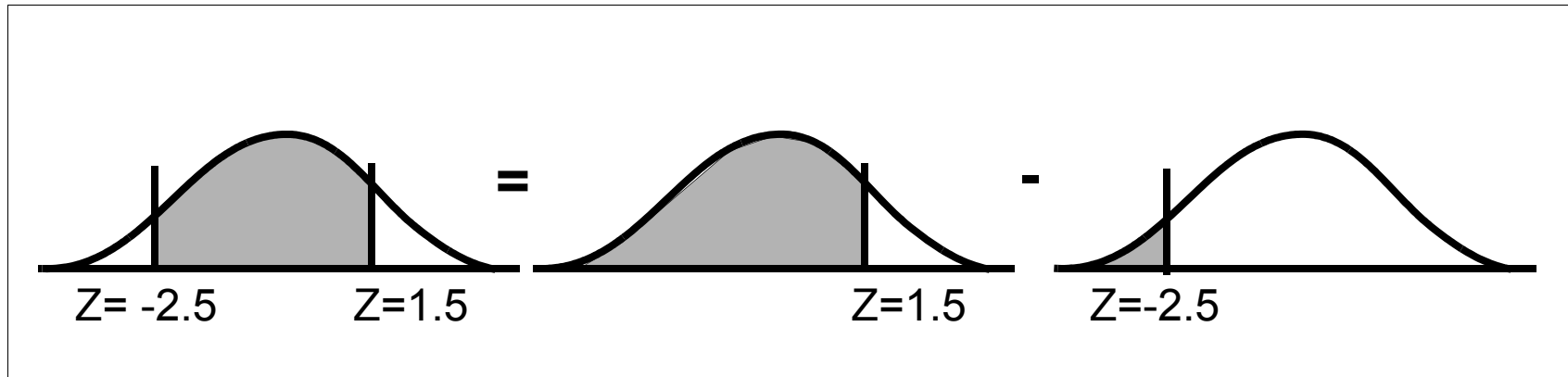
$$P\ (-2.5 \le Z \le 1.5)\ =\ P\ (Z \le 1.5) - P\ (Z \le -2.5)$$

= 0.9332 - 0.0062 = 0.927

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Behavior of Sample Means

How are the $\overline{X}$'s distributed ?

- **Central tendency**
- **Spread**
- **Shape - distribution of sample means**

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Distribution of Sample Means

$$\sigma_{\bar{X}}$$

$$\mu_{\bar{X}}$$

$$\bar{X}$$

$$X$$

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Distribution of Sample Means

$$\mu_{\overline{X}} = E[\overline{X}]$$

$$\sigma^2_{\overline{X}} = Var[\overline{X}]$$

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Central Limit Theorem

Averages (in fact, any linear combination of data) tend to be normally distributed regardless of the distribution of X. Tendency towards normality improves as n increases. If X's are normal, averages are also normal.
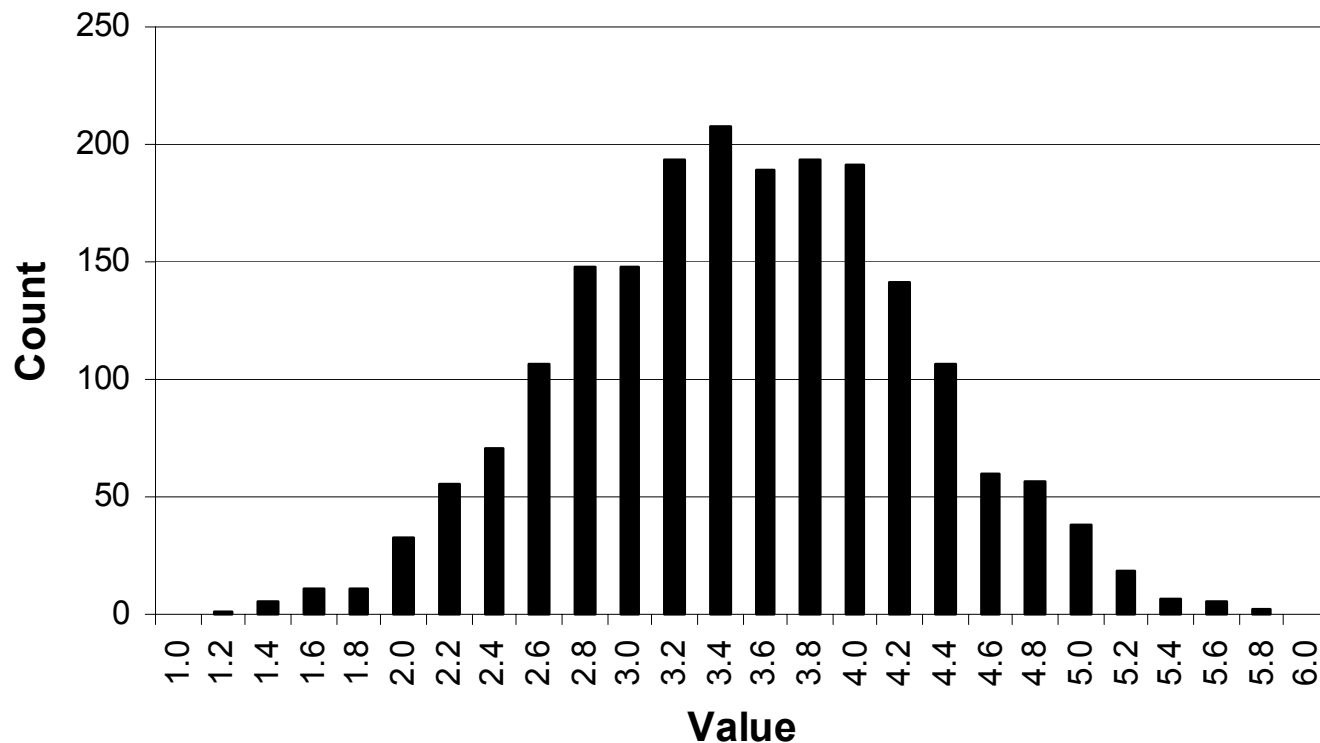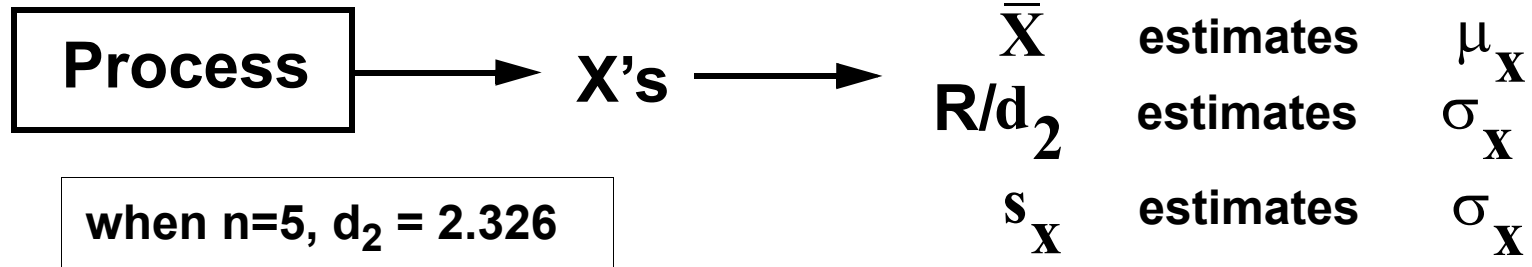
Example
(2000 throws of a die)

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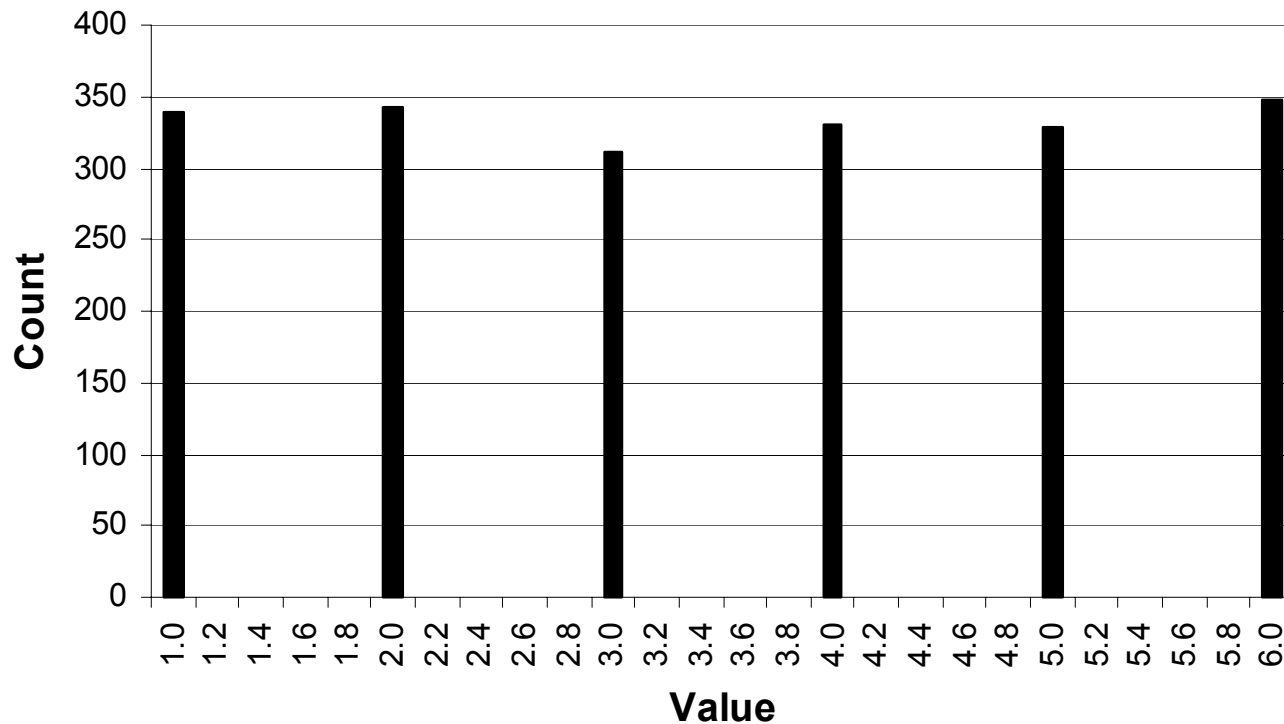
Example
(Throw 2 dice -- find avg)

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Example
(Throw 5 dice 2000 times -- find avg)

Shape of the Distribution of X's

Process $\longrightarrow$ **X's** $\longrightarrow$

$\overline{\overline{X}}$ estimates μ_X

R/d_2 estimates σ_X

when n=5, d_2 = 2.326

s_X estimates σ_X

n=10

?

?

?
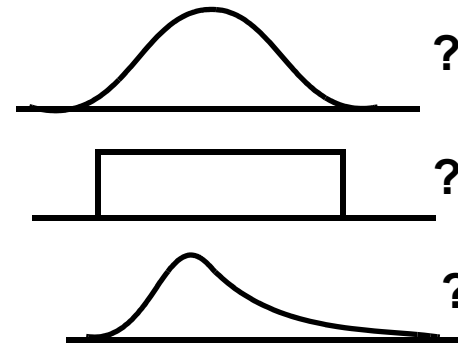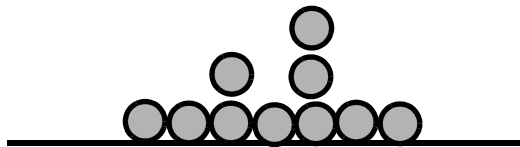
Can't judge distribution shape - not enough data

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Here's Some Data

44, 62, 39, 53, 80, 33, 57, 22, 49, 68

What distribution describes this data?

Are the data normally distributed??
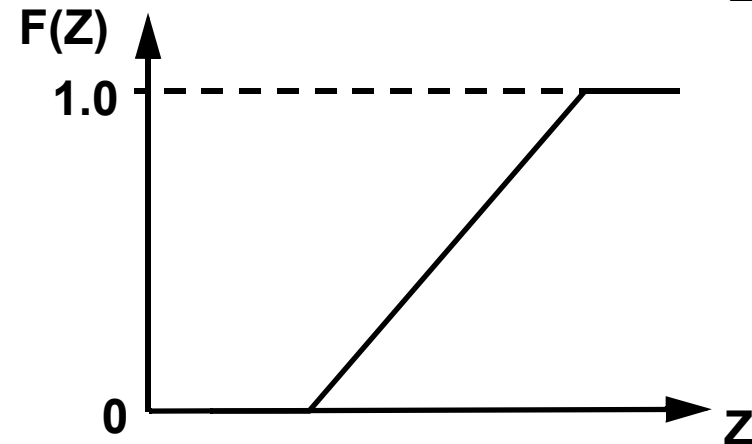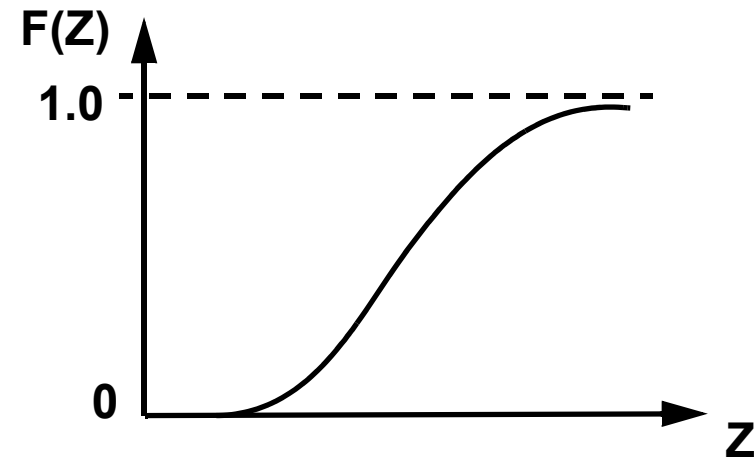
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Cumulative Histogram (Plot)

Rank X's in Ascending Order

i	
Ranked X's	
Cum. Prob., P_i	

Smallest X must represent lowest 10% of underlying distribution

$$P_i = (i-.5)/n$$

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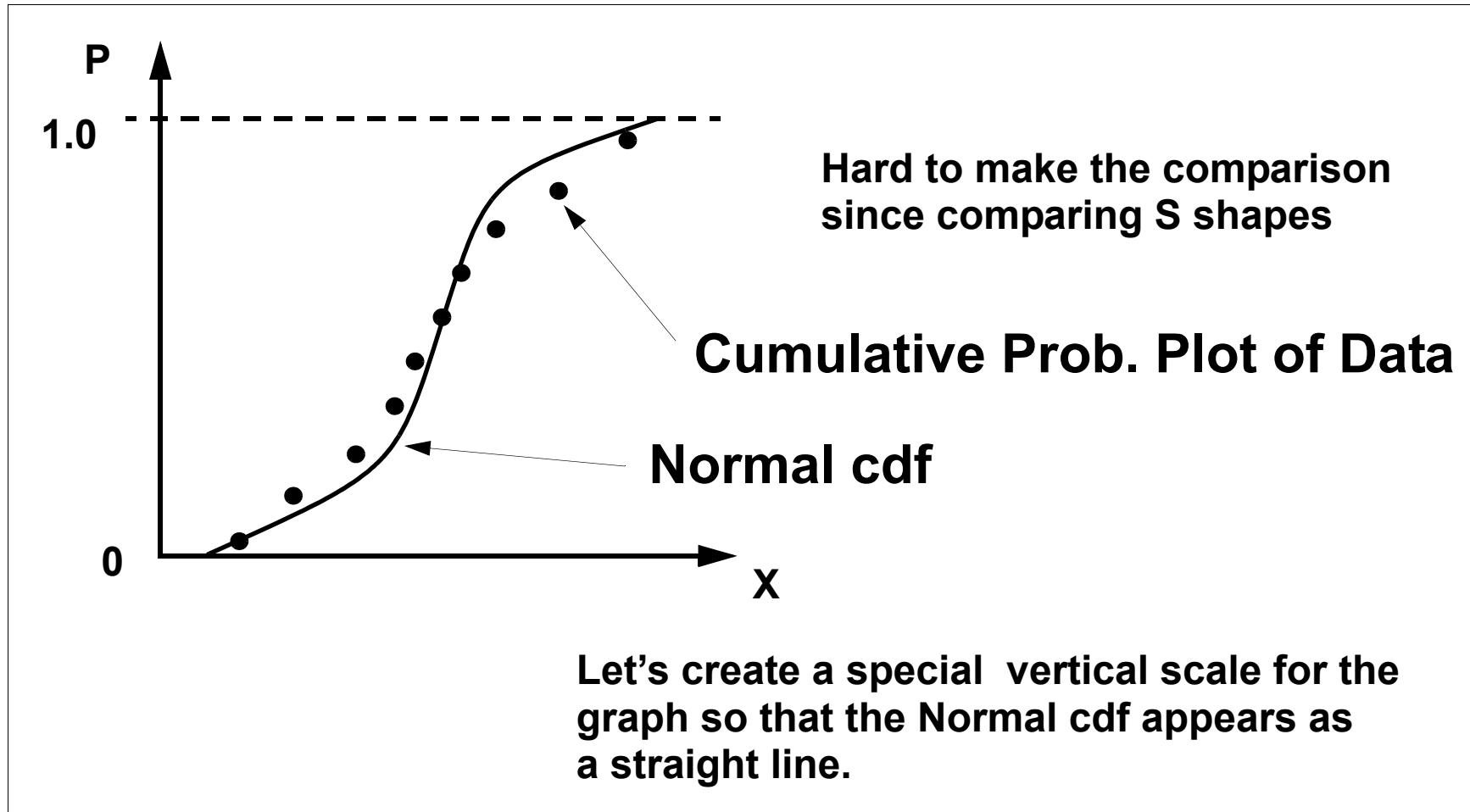# Cumulative Histogram
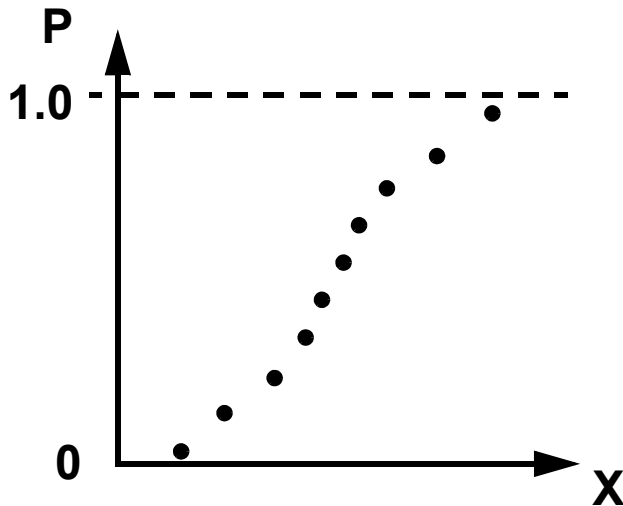
Which "S" shaped cdf
best matches the cumulative
probability plot of the data?

Normal Probability Paper

P

1.0

0

X

Hard to make the comparison
since comparing S shapes

Cumulative Prob. Plot of Data

Normal cdf

Let's create a special vertical scale for the
graph so that the Normal cdf appears as
a straight line.

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Normal Probability Paper

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10, 23, 3, 27, 10,
29, 13, 28, 21, 4,
25, 7, 17, 30, 5

0.999
0.998
0.995
0.990
0.980
0.950
0.900
0.800
0.700
0.600
0.500
0.400
0.300
0.200
0.100
0.050
0.020
0.010
0.005
0.002
0.001

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25, 16, 39, 21, 12, 28, 19, 23

0.999
0.998
0.995
0.990
0.980
0.950
0.900
0.800
0.700
0.600
0.500
0.400
0.300
0.200
0.100
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0.010
0.005
0.002
0.001

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Normal Probability Paper

| 0.999 |
| 0.998 |
| 0.995 |
| 0.990 |
| 0.980 |
| 0.950 |
| 0.900 |
| 0.800 |
| 0.700 |
| 0.600 |
| 0.500 |
| 0.400 |
| 0.300 |
| 0.200 |
| 0.100 |
| 0.050 |
| 0.020 |
| 0.010 |
| 0.005 |
| 0.002 |
| 0.001 |

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