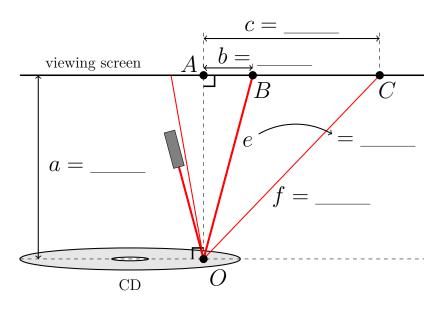
CD Laser Measurement Station

Laser light is a very special kind of light. It's special in a way that we can use it to measure things much smaller than we could otherwise. You can measure the width of a single hair, or you can measure the spacing between the very small tracks on a CD, like we'll do here!

Feel free to write the measured or calculated lengths in either the blanks on the diagram, the blanks in the steps, or both.



Step 1: Measure the distance from the CD to the viewing screen. What is this distance, that we'll call a? (In terms of the labeled points, this is the length of line segment \overline{OA} .)

$$a =$$
____ cm

Step 2: Mark or measure where the two bright spots land on the viewing screen. What are the shorter and longer distances on the screen, b and c? (Or, \overline{AB} and \overline{AC} .)

$$b = \underline{\hspace{1cm}}$$
 cm $c = \underline{\hspace{1cm}}$ cm

Step 3: Measure the lengths of the third sides of the triangles, e and f (or, \overline{OB} and \overline{OC}). Or, use an online right triangle calculator to find them, using a and b to find e, and a and c to find f.

$$e = \underline{\qquad}$$
 cm $f = \underline{\qquad}$ cm

Worksheet version 4; last updated December 6, 2022 by Adam W. Behnke.

Step 4: Find the ratios of the sides of the triangles on the screen to the triangles' longest sides by dividing b by e, and dividing c by f.

$$b \div e = \underline{\hspace{1cm}} = g \hspace{1cm} c \div f = \underline{\hspace{1cm}} = h$$

Step 5: Subtract the first quotient from Step 4 from the second.

$$h-g = \underline{\qquad} = i$$

Step 6: Divide 0.650 µm by the difference from Step 5 to find the calculated track spacing.

$$(0.650 \, \mu\text{m}) \div i = \underline{\qquad} \, \mu\text{m} = d$$

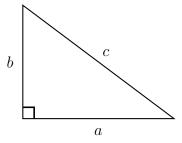
Step 7: What is the "real" track spacing? (The station supervisor will tell you.)

$$D = \underline{\qquad} \mu m$$

A micrometer (μ m) is one thousandth of a millimeter. Lasers can help measure very small things! The 0.650 μ m number used in Step 6 is related to the color of the laser light.

Extra: The Pythagorean Theorem

In Step 3, we used an online calculator to find the length of the last side of a triangle. Since the setup contained a **right triangle**, we were able to use the **Pythagorean theorem** to find the length of the last side! You'll learn about this later on in your math classes.



For a right triangle (as shown above), "right" meaning one of the corners has an angle of exactly 90° (indicated by the square in the lower left), the Pythagorean theorem states that the side lengths a, b, and c are related by

$$a^2 + b^2 = c^2.$$

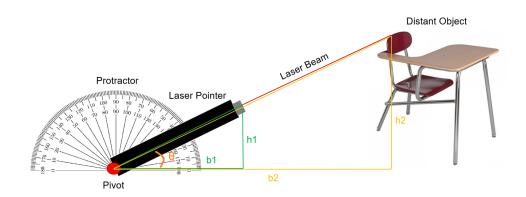
You'll also learn later about the **square root** operation, where "the square root of x" is written as \sqrt{x} . The square root operation undoes the operation of squaring a number. So, if you know the lengths of sides a and b, you can find the length of the remaining side by using

$$c = \sqrt{a^2 + b^2}.$$

Laser Inclinometer Station

How would you measure the size of something that much larger than any ruler or measuring tape? Or the height of a building that is miles away from you? There are many situations in real life where this type of measurement might be necessary. For example, a construction worker might need to measure the height of a tall building, or an air traffic controller might need to figure out how high a plane is flying in the air. In this activity, we will use the properties of triangles, along with a simple tool called an inclinometer to measure the height of distant objects.

To complete this activity, follow the instructions below, and write your measurements and calculations in the blanks below each step.



Step 1: Place the object provided by the instructor on the floor at the point marked "Object". Place the inclinometer at the point marked "Inclinometer". Verify with a measuring tape that these two points are 300 cm apart.

Step 2: Point the inclinometer at the object, and rotate the laser pointer until the beam strikes the top of the object. Once the beam is lined up on the top of the object, lock the laser pointer in place by tightening the nut on the pivot.

Step 3: Measure the distance from the pivot of the inclinometer to the end of the laser pointer along the floor and the height of the tip of the laser pointer from the floor. We'll call these b and h.

$$b = \underline{\hspace{1cm}} cm \qquad \qquad h = \underline{\hspace{1cm}} cm$$

Worksheet version 1; last updated December 6, 2022 by Christopher M. Lacny.

Step 4: The triangle formed by the pivot, the floor, and the tip of the laser pointer is similar to the triangle formed by the pivot, the top point of the object, and the floor. As a result, the ratio of the height to the base of these triangles must be the same. Mathematically, this can be expressed as

$$\frac{h}{b} = \frac{H}{B} \tag{1}$$

where H is the height of the object, and B is the distance between the inclinometer and the object. We can rewrite this formula to sove for H:

$$H = B \cdot \frac{h}{b} \tag{2}$$

Use this relationship, and the fact that B = 300 cm, to compute the height of the object.

$$H = \underline{\hspace{1cm}} \operatorname{cm}$$

Step 5: Measure the angle on the protractor when the laser pointer is pointed at the top of the object.

$$\theta =$$

Step 6: Using trigonometry, the height of the object can be computed with only the angle measurement, rather than having to measure both the base and height of the triangle formed by the laser pointer. The tangent, or tan, function on a calculator tells you the ratio of the height to the base of the small triangle $(\frac{h}{b})$ if you plug in the angle on the protractor. As a result, we can rewrite our formula for the height of the distant object as

$$H = B \cdot \frac{h}{b} = B \cdot \tan(\theta) \tag{3}$$

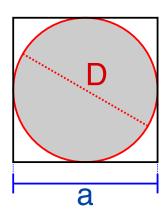
Use a calculator and the formula above to compute the height of the object. (Make sure your calculator is set to degree mode, and recall that B=300 cm!)

$$H = B \cdot \tan \theta =$$
 _____ cm

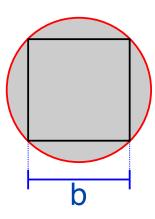
Step 7: Measure the actual height of the distant object with a measuring tape. How does this compare to the two results from the inclinometer?

Step 8 (Optional): If time allows, repeat steps 1-7 with a second object. How accurate were your measurements in both cases? Can you think of any ways that the inclinometer could be improved to measure the height of the objects more accurately?

Estimating Pi Station



D = cr



a = cr

$$A = 4 \times a$$

b = cm

$$B = 4 \times b$$

A = ____ cm

B = cm

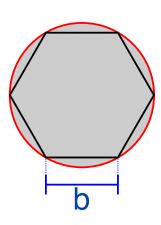
$$C = (A + B) \div 2$$

 $C = \underline{\hspace{1cm}} cm$

$$C = \pi \times D$$

$$\pi = C \div D$$

 $\pi = \underline{\hspace{1cm}}$



$$A = 6 \times a$$

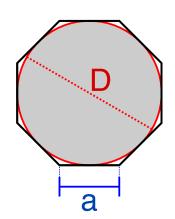
$$B = 6 \times b$$

$$C = (A + B) \div 2$$

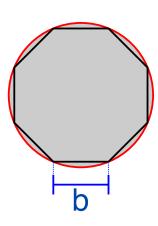
$$C = \pi \times D$$

$$\pi = C \div D$$

$$\pi =$$



$$D = cn$$



$$A = 8 \times a$$

$$B = 8 \times b$$

$$C = (A + B) \div 2$$

$$C = \pi \times D$$

$$\pi = C \div D$$

$$\pi = \underline{\hspace{1cm}}$$

Results: This is where we will write down our estimates for π (pi). Record your estimates in the table below.

Shape Used	Estimate for π
Square	
Hexagon	
Octagon	

How do your estimates compare to the real value of π ?

$$\pi = 3.14159265358979...$$

Name:	Date:					
Monty Hall Station						

Probability can sometimes be weird. Suppose there are three doors, and you pick one. Behind one door is a brand new shiny car, and behind the other two are goats (let's say you don't like goats). Monty Hall, the gameshow host, opens one of the doors you didn't pick, revealing a goat behind it. He then offers you a choice: do you stay with your original door choice, or switch to the unopened door? Which is the right choice? Let's find out.

Strategies	Cars (Wins)	Goats (Losses)	Total Used	% Wins
Switch				
Stay				

Step 1: Decide who will hold the cards first (the other one will play the game). This first player should choose one strategy (either Switch or Stay) and keep using it, tallying up the number of wins and losses in the table. Play the game for about 4 minutes.

Step 2: Let the other team member have a turn. This second player should use the other strategy (the one that the first player didn't choose), tallying up the number of wins and losses in the table. Play the game for about 4 minutes.

Step 3: For each strategy, add up the Wins and Losses tallies to get the total number of times each strategy was used.

Step 4: For each strategy, divide the number of Wins by the Total to get the win percentage. Which strategy has the higher % Wins? Is this a surprising result?