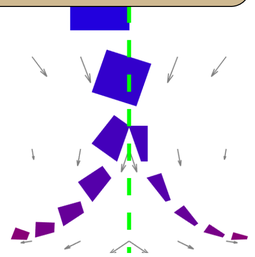
Hybrid Zonotopes: A Mixed Integer Set Representation for Analysis of Hybrid Systems

Research Assistants: Trevor J. Bird
Principal Investigator: Dr. Neera Jain
Contact email: *bird6@purdue.edu, neerajain@purdue.edu*

Problem Statement

- Reachability analysis is a set-based method for guaranteeing performance criteria.
- The reachable sets of hybrid systems – those that have continuous and discrete dynamics, e.g., a controller that turns a component on or off – are inherently nonconvex.
- Existing methods rely on collections of convex sets with worst-case exponential growth in complexity. To stifle this growth, state of the art algorithms use over-approximations leading to conservative results at best and trivial solutions at worst.



Research Objective:
Develop a nonconvex set representation for exact and scalable analysis of hybrid systems

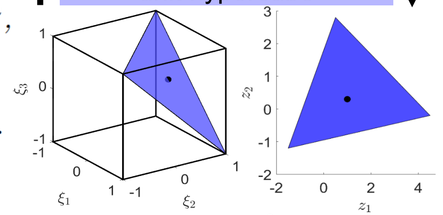
Approach

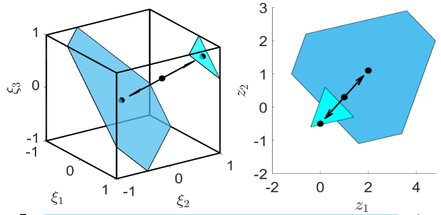
Definition 1 *The set* $\mathcal{Z}_h \subset \mathbb{R}^n$ *is a hybrid zonotope if there exists* $G^c \in \mathbb{R}^{n\times n_g}$, $G^b \in \mathbb{R}^{n\times n_b}$, $c \in \mathbb{R}^n$, $A^c \in \mathbb{R}^{n_c\times n_g}$, $A^b \in \mathbb{R}^{n_c\times n_b}$, *and* $b \in \mathbb{R}^{n_c}$ *such that*

$$\mathcal{Z}_h = \left\{ \begin{bmatrix} G^c & G^b \end{bmatrix} \begin{bmatrix} \xi^c \\ \xi^b \end{bmatrix} + c \;\middle|\; \begin{bmatrix} \xi^c \\ \xi^b \end{bmatrix} \in \mathcal{B}_\infty^{n_g} \times \{-1,1\}^{n_b}, \; \begin{bmatrix} A^c & A^b \end{bmatrix} \begin{bmatrix} \xi^c \\ \xi^b \end{bmatrix} = b \right\}.$$

Constrained zonotopes with n_g continuous factors, ξ^c, may be constructed to represent convex polytopes with up to 2^{n_g} features. Introducing n_b binary factors, ξ^b, the hybrid zonotope is equivalent to the nonconvex union of up to 2^{n_b} constrained zonotopes.

Affine Image of constrained unit hypercube





Affine image of constrained unit hypercube with 2^{n_b} shifts

By modeling systems in their mixed logical dynamical form, the hybrid dynamics may be propagated as hybrid zonotopes through an identity that is computed algebraically and scales linearly.

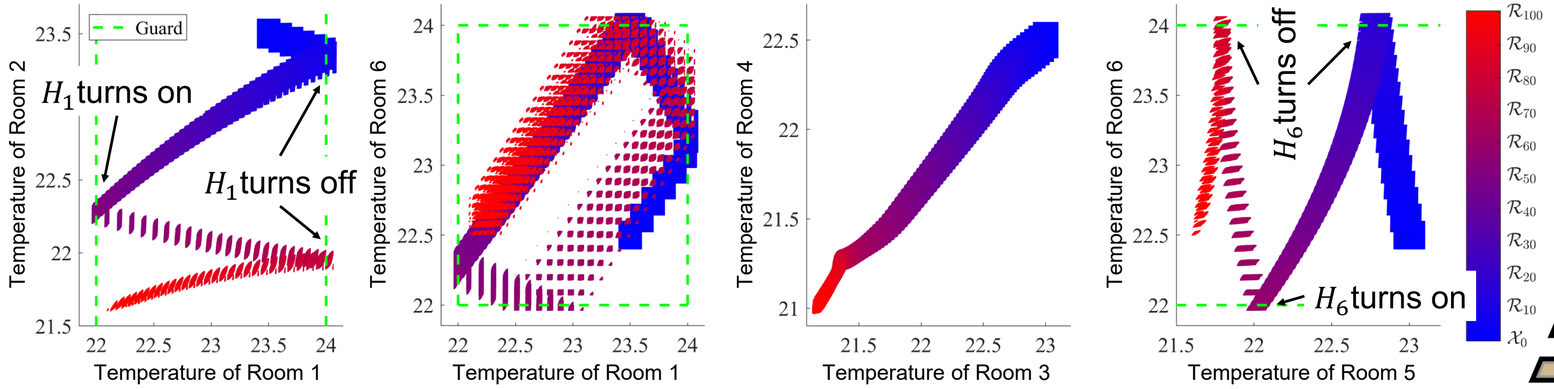
$$\begin{aligned} x_+ &= Ax + B_u u + B_w w + B_{aff} \\ \text{s.t. } & E_x x + E_u u + E_w w \le E_{aff} \\ & x \in \mathbb{R}^{n_{xc}} \times \{0,1\}^{n_{xl}} \\ & u \in \mathbb{R}^{n_{uc}} \times \{0,1\}^{n_{ul}} \\ & w \in \mathbb{R}^{n_{rc}} \times \{0,1\}^{n_{rl}} \end{aligned}$$

Results

Room 1 $T_1[k]$ $H_1[k]$	Room 4 $T_4[k]$
Room 2 $T_2[k]$	Room 5 $T_5[k]$
Room 3 $T_3[k]$	Room 6 $T_6[k]$ $H_6[k]$

$u[k] = T_{amb} \in [0,0.1]$

Reach set of the temperature of six adjacent rooms. Two corner offices (1 and 6) have thermostat controlled heaters turning on when the temperature is below 22C and off when above 24C. Computed in 5.8 seconds.



Acknowledgements

This work was conducted in collaboration with Professors Justin Koeln at UT Dallas and Herschel Pangborn at Penn State.

This material is based upon work supported by the National Science Foundation under Award No. DGE-1333468. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.

PURDUE UNIVERSITY | The Ray W. Herrick Laboratories