Dynamic Modeling and Performance Analysis of Sensible Thermal Energy Storage Systems

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ABSTRACT

In this paper we consider the problem of dynamic performance evaluation for sensible thermal energy storage (TES), with a specific focus on hot water storage tanks. We derive transient performance metrics, from second law principles, that can be used to guide real-time decision-making aimed toward improving demand response. We show how the transient nature of the metrics can be used not only to influence control variables within the system, but also to mitigate adverse effects of disturbances during operation. To evaluate these metrics in the context of hot water storage tanks, a thermal stratification model is needed. We derive a reduced-order model which allows the simulation of tank thermal stratification during all modes of system operation. The proposed performance metrics are analyzed in simulation using the dynamic tank model. The results highlight key trade-offs captured by the metrics that can be incorporated into future optimal control design for sensible TES systems.

1. INTRODUCTION

Motivation and Problem Definition: Within thermodynamic processes, waste heat losses occur on a large scale due to equipment inefficiencies and limitations. It has been estimated that anywhere from 20-50% of energy consumption in U.S. industrial applications alone is ultimately released as waste heat (Waste Heat Recovery - Technology and Opportunities in U.S. Industry, 2008; Semkov, Mooney, Connolly, & Adley, 2014). As such, there exists a great opportunity to develop and improve strategies for waste heat recovery. Waste heat recovery is most commonly carried out in centralized power plants through heat recovery steam generators where the thermal energy is immediately used to generate additional electricity (Giaccone & Canova, 2009). However, with increasing penetration of distributed power generation through micro-combined heat and power (micro-CHP) systems, opportunities for waste heat recovery at the residential scale are growing (Barbieri, Melino, & Morini, 2012). In these systems, the recovered heat is typically used to heat water that is stored in a hot water storage tank for domestic use. The use of a thermal energy storage (TES) system enables the recovered energy to meet future thermal demand. However, in order to design optimal control strategies to achieve demand response, dynamic performance metrics for TES systems are needed.

Gaps in Literature: Many researchers have studied the development of efficiency metrics to quantify the performance of hot water storage tanks using second law-based analyses. Most efforts, however, use finite time exergy-based metrics, rather than rate-based, metrics (Han, Wang, & Dai, 2009; Li, 2016; Fernández-Seara, Uhía, & Sieres, 2007a, 2007b). The disadvantage is that, while these types of metrics give a global perspective of performance which can be used in making offline design decisions, they fail to give an idea of how well energy is being used at any given time for online decision-making. Few researchers to date have performed thermodynamic optimization of equipment involving highly transient processes (Pizzolato, Verda, & Sciacovelli, 2015; Sciacovelli, Verda, & Sciubba, 2015). In order to optimize performance of a sensible TES in real-time, we desire transient second-law based performance metrics which can be used to embed advanced control strategies into system operation.

As previously mentioned, a common type of sensible TES system is a hot water storage tank. Dynamic modeling of hot water storage tanks has been studied by numerous researchers (Kleinbach, Beckman, & Klein, 1993; Han et al., 2009). Recently, researchers have also developed control-oriented dynamic models for hot water storage tanks.
with simultaneous sink and source mass flow loops (Powell & Edgar, 2013). Many hot water storage tanks utilize an immersed coil heat exchanger as a means of heat absorption, and dynamic models incorporating this feature are available within the literature (Cadafalch, Carbonell, Consul, & Ruiz, 2015). However, these models are typically computationally expensive due to the increase in dynamic states that results from the dynamic modeling of the heat exchanger. For eventual control design, we require a reduced-order model of a hot water storage tank with an immersed coil heat exchanger.

**Contribution:** We define a general set of dynamic performance metrics for the different operation modes of sensible TES systems. These metrics are then analyzed in simulation using a control-oriented dynamic hot water storage tank model; key benefits are demonstrated with respect to real-time decision-making and demand response for hot water storage tank operation.

**Outline:** In Section 2 we present a background on performance analysis of TES systems and thermal stratification in hot water storage tanks. Section 3 describes the sensible TES model proposed in this work, while Section 4 details transient performance metrics for evaluating system performance. Simulation results demonstrating the dynamic behavior of a hot water storage tank are presented in Section 5, followed by a set of case studies wherein we analyze the performance metrics developed in Section 4. This is followed by concluding remarks in Section 6.

## 2. BACKGROUND

In this section, we present an overview on the importance of performance metrics for improving demand response in sensible TES systems. We then introduce thermal stratification and its impact on the performance of sensible TES in hot water storage tanks.

### 2.1 Performance Metrics for Sensible Thermal Storage

In electrochemical batteries, state-of-charge (SOC) is an important concept that enables the control engineer to design algorithms that effectively utilize the stored electrical energy (Cheng, Divakar, Wu, Ding, & Ho, 2011). In the context of thermal energy storage, little attention is paid to quantifying SOC; instead, performance and efficiency metrics typically offer a steady-state or aggregate perspective of the behavior of the system (Han et al., 2009; Pizzolato et al., 2015). These steady-state metrics are often used with the intent of influencing offline design factors such as storage sizing. From the perspective of real-time control, we require metrics that not only quantify the current SOC but also the charging and/or discharging behavior of the system, so that energy management strategies may be designed and implemented.

### 2.2 Thermal Stratification in Hot Water Storage Tanks

Thermal stratification in storage tanks is a phenomenon that results when a density gradient is present within the tank. The gradient causes the warmer, less dense water to rise to the top of the tank while the cooler, higher density water sinks to the bottom of the tank. The result is the presence of a high temperature section of water toward the top of the tank and a low temperature section of water at the bottom, as depicted in Fig. 1.

In the mixing region between the hot and cold sections, there exists a sharp temperature gradient called a *thermocline*. Thermal stratification in storage tanks is a very desirable characteristic. It is estimated that the efficiency of the heat storage can be increased by up to 20% when the tank is kept highly stratified with a thin thermocline region (Han et al., 2009). Therefore, modeling the dynamic evolution of the thermocline region has important implications on improving system performance. In order to most accurately quantify the thermal dynamics in a hot water storage tank, partial differential equations are needed in multiple dimensions. Alternatively, a lumped parameter assumption can be combined with a spatial discretization in order to derive a one-dimensional (1D) model of the system. A 1D model was introduced in the 1990s (Kleinbach et al., 1993). More recently, Cadafalch et al. presented a method for incorporating an immersed coil heat exchanger into the analysis (Cadafalch et al., 2015). In the following section, we present a control-oriented, reduced-order 1D model for a thermally stratified hot water storage tank with an immersed coil heat exchanger.
3. HOT WATER STORAGE TANK MODEL

In this section we present the governing equations for modeling the thermal stratification in a cylindrical hot water storage tank. A schematic of the system is shown in Fig. 2. Hot waste heat water at a temperature $T_{in}$ enters an immersed hot water coil situated at the lower portion of the tank. The waste heat water is then pumped vertically through the coil until it exits the tank at $T_{out}$. Heat is transferred from the warm water flowing through the coil to the colder water in the storage tank. We define three different modes of operation for the system. We define charging to be the heat addition mode, or the mode during which waste heat water is being pumped through the coil in order to heat the water in the tank. Conversely, we define discharging to be the heat rejection mode, or the mode during which the stored hot water is being pumped out of the tank and replaced with cold water. The hot water is removed from the tank at a flow rate $\dot{m}$ which acts as a disturbance on the system. Moreover, the water in the tank is replenished by an equal flow of water defined as $\dot{m}_{cw}$ which enters at the bottom of the tank. Finally, we define a third mode of operation to be simultaneous charging and discharging of the tank.

3.1 Governing Equations

In this section we derive a control-oriented model of the thermal storage tank dynamics that can be used for model-based control design. The model presented here builds upon work presented by previous researchers (Kleinbach et al., 1993; Powell & Edgar, 2013; Cadafalch et al., 2015). The tank is discretized vertically into $n$ nodes (Kleinbach et al., 1993), with the top node defined as node one. An illustrative view of a discretized control volume is shown in Fig. 3.

Each node is allowed to exchange heat with its surroundings in several ways. Nodes can absorb heat from the coil, $\dot{Q}_{coil}$, and lose heat to the ambient by conduction through the tank walls, $\dot{Q}_{cond,wall}$. Additionally, each node is allowed to exchange heat with its bordering nodes, modeled as $\dot{Q}_{j-1}$ and $\dot{Q}_{j+1}$. While in discharge mode, water flows upward through each control volume. Each node is assumed to remain in steady-state with respect to mass flow. The bottom node is sized such that it contains the cold water flow inlet valve. Conservation of energy is used to derive a system of
n ordinary differential equations that can be solved numerically, yielding the temperature stratification in the storage tank as a function of time. For the jth node, the energy balance equation is

$$m_j c_v \frac{dT_j}{dt} = Q_{\text{coil},j} - Q_{\text{cond.wall},j} + \dot{Q}_{j-1} - \dot{Q}_{j+1} + m_j c_v (T_{j+1} - T_j).$$  \hspace{1cm} (1)$$

One by one, nodal temperature differentials $\frac{dT_j}{dt}$ can be solved independently. Values for heat loss through the walls are given by $Q_{\text{cond.wall},j} = \frac{k_{\text{wall}} A}{w_i} (T_j - T_0)$, where $k_{\text{wall}}$ is a tuning parameter, $w_i$ is the approximate wall thickness, and $T_0$ is the temperature of the ambient environment. Logic is used to determine whether each node is within the coil region which in turn dictates whether or not there is heat transfer between the coil and that particular node. The internal heat transfer rate terms $\dot{Q}_{j-1}$ and $\dot{Q}_{j+1}$ are solved using a finite difference scheme.

### 3.2 Calculation of Internal Heat Transfer Between Nodes

To quantify the internal heat transfer rates, $\dot{Q}_{j-1}$ and $\dot{Q}_{j+1}$, a finite difference scheme is used (Powell & Edgar, 2013). Fig. 4 helps to illustrate the process. The scheme yields the following expressions for the internal heat transfer rates,

$$\dot{Q}_{j-1} = -k_{j-1} A \frac{T_{j-1} - T_j}{z_{j-1} - z_j}, \hspace{1cm} (2)$$

$$\dot{Q}_{j+1} = -k_{j+1} A \frac{T_j - T_{j+1}}{z_j - z_{j+1}}. \hspace{1cm} (3)$$

The $z$ terms represent heights of the vertical midpoints of the nodes with respect to a zero datum at the tank’s bottom. In Eqns. (2)-(3), the $k$ terms are tuning parameters in the model and are used to handle temperature inversion within the model (Powell & Edgar, 2013). Temperature inversion is a phenomenon which occurs when warmer, less dense water exists below cooler, heavier water. In this work, temperature inversion exists due to the presence of the immersed heat coil in the lower portion of the tank. The coils heat the water at the bottom of the tank; this hot water then rises toward the top of the tank. One-dimensional models are incapable of naturally accounting for temperature inversion. Hence, an alternative way of incorporating the phenomenon is necessary.

At any point in simulation, if the temperature of node $j$ is either lower than the node below or higher than the node above, the $k_{j-1}$ and $k_{j+1}$ terms are increased by several orders of magnitude in order to force heat to be transferred upward, thereby resolving the temperature inversion problem. The algorithm for the variation of the coefficients is given by

$$k_{j-1} = \begin{cases} k_{j-1} A |T_j - T_{j-1}|, & \text{if } T_j > T_{j-1} \\ k_{j-1}, & \text{otherwise} \end{cases} \hspace{1cm} \text{and} \hspace{1cm} k_{j+1} = \begin{cases} k_{j+1} A |T_j - T_{j+1}|, & \text{if } T_j < T_{j+1} \\ k_{j+1}, & \text{otherwise} \end{cases} \hspace{1cm} (4)$$

where $Δ$ is a tuning parameter whose magnitude is several orders higher than the magnitude of the $k$ term itself.

### 3.3 Calculation of Coil Heat Transfer

In previous work, Cadafalch et al. present a method of incorporating a transient analysis of the immersed heat exchanger (Cadafalch et al., 2015). In this work, we are interested in a reduced-order model of the interaction between the tank water and the coil in order to obtain a computationally efficient control-oriented model. To calculate the heat transfer rate between the coil and the tank nodes within the coil region, we treat the coil as a separate system acting at steady-state and assume a linear temperature profile from the tank coil inlet to the tank coil outlet as shown in Fig. 5.

In Fig. 5, the value of $T_{\text{in}}$ is known. At any time $t$, the value of $T_{\text{out}}$ is assumed to be equal to the temperature of the water of the surrounding node. This assumption is based upon the fact that 1) the mass flow rate inside the coil is slow (3-7 liters per minute) and 2) the amount of water in the coil is small in comparison to the overall amount of water in the tank. Assuming the coil to be at steady-state, the value for the heat transfer rate of the $j^{th}$ coil slice is calculated
Figure 5: Schematic for calculating heat transfer rates due to immersed coil.

as $Q_{\text{coil},j} = m_{\text{coil}}c_v(T_{\text{coil,in},j} - T_{\text{coil,out},j})$. The values for $T_{\text{coil,in},j}$ and $T_{\text{coil,out},j}$ are determined using a linear reduction in temperature from $T_{\text{in}}$ at the coil inlet to $T_{\text{out}}$ at the coil outlet.

4. TRANSIENT PERFORMANCE METRICS

In this section, we present transient second-law based performance metrics for the modes of operation of the hot water storage tank.

4.1 Charging Performance Measure

To quantify transient performance during charging mode, we require a metric to compare the rate at which useful work (i.e., exergy) is supplied to the system to the rate at which useful work is lost. Overall, the metric should inform how efficiently exergy is being stored within the system. Therefore, we believe the best way to characterize performance during a charging mode is to use the standard exergetic efficiency metric, given by

$$\psi_C = 1 - \frac{\dot{X}_{\text{dest}}}{\dot{X}_{\text{sup}}},$$

where $\dot{X}_{\text{dest}}$ represents the total rate of exergy destruction in the tank, $\dot{X}_{\text{sup}}$ represents the total rate of exergy being supplied to the tank, and the subscript $c$ denotes charging. The supplied exergy rate is calculated based on the rate at which exergy is transferred via heat transfer from the immersed coil to the tank water. Within each node in the coil region, there is some amount of exergy transfer via heat transfer. The total rate of supplied exergy to the system is defined as the summation of all nodal supplied exergy rates as shown in Eqn. (6).

$$\dot{X}_{\text{sup}} = \sum_j \dot{X}_{\text{sup},j} = \sum_j \left(1 - \frac{T_o}{T_j}\right) \dot{Q}_{\text{coil},j}$$

To determine the rate of exergy destruction in the tank, an entropy balance is applied to each node. For the $j^{th}$ node, the entropy accounting equation is given by

$$\frac{d}{dt} S_j = \sum_i \frac{\dot{Q}_{i,j}}{T_{b,i}} + m_j(s_{j+1} - s_j) + S_{\text{gen},j}.$$  

(7)

For the heat transfer terms, boundary temperatures are taken to be as given in Table 1. The left hand side of Eqn. (7) is rearranged to yield

$$\frac{dS_j}{dt} = \frac{m_j c_v}{T_j} \cdot \frac{dT_j}{dt}.$$  

(8)
Table 1: Boundary temperatures for heat transfer rates in entropy accounting equation

<table>
<thead>
<tr>
<th>Heat transfer term</th>
<th>Boundary temperature</th>
</tr>
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<tbody>
<tr>
<td>(Q_{\text{coil},j})</td>
<td>(T_j)</td>
</tr>
<tr>
<td>(Q_{j+1})</td>
<td>(\frac{1}{2} (T_j + T_{j+1}))</td>
</tr>
<tr>
<td>(Q_{j-1})</td>
<td>(\frac{1}{2} (T_j + T_{j-1}))</td>
</tr>
<tr>
<td>(Q_{\text{cond,wall},j})</td>
<td>(T_o)</td>
</tr>
</tbody>
</table>

Equation (7) can be solved for \(S_{\text{gen},j}\). Then the corresponding exergy destruction rate for the \(j^{th}\) node is \(\dot{X}_{\text{dest},j} = T_o \dot{S}_{\text{gen},j}\). The total rate of exergy destruction in the tank is found by summing all of the nodal exergy destruction rates: \(\dot{X}_{\text{dest}} = \sum_j \dot{X}_{\text{dest},j}\).

4.2 Discharging Performance Measure

For any mode involving a discharge of thermal energy, we seek to quantify performance by analyzing the rate at which useful work is being recovered relative to the rate at which the stored thermal energy is changing. We define the discharge performance metric to be

\[
\psi_d = \frac{\dot{X}_{\text{rec}}}{\dot{X}_{\text{sto}}} ,
\]

where \(\dot{X}_{\text{rec}}\) is the rate at which exergy is being recovered from the TES, \(\dot{X}_{\text{sto}}\) is the rate at which exergy is being stored, and the subscript \(d\) denotes a discharging mode. In order to determine the rate at which exergy is stored, exergy accounting equations are used on each node. For the \(j^{th}\) node, the rate of change of exergy is given by

\[
\frac{d}{dt} X_j = \sum_i \left( 1 - \frac{T_o}{T_{b,i}} \right) \dot{Q}_{i,j} + \dot{m}_t (x_{j+1} - x_j) - T_o \dot{S}_{\text{gen},j} ,
\]

where the boundary temperatures correspond to those given in Table 1 and \(S_{\text{gen},j}\) is calculated according to Eqn. (7).

We define the total rate of exergy stored in the system to be

\[
\dot{X}_{\text{sto}} = \left| \sum_j \frac{dX_j}{dt} \right| .
\]

We define the rate of exergy recovered during discharge to be the rate of flow exergy exiting the top of the tank with respect to a setpoint state whose temperature is \(T_{\text{set}}\). The setpoint state acts as a threshold for undesirable system performance during a discharging mode. We take the rate of recovered exergy to be

\[
\dot{X}_{\text{rec}} = \dot{m}_t (x_1 - x_{\text{set}}) = \dot{m}_t \left[ c_v (T_1 - T_{\text{set}}) - T_o c_v \ln \left( \frac{T_1}{T_{\text{set}}} \right) \right] .
\]

The value of \(T_{\text{set}}\) is a user-defined parameter which serves to quantify the instant at which useful work is no longer being recovered from the system. Once the temperature of the water at the top of the storage, \(T_1\), is less than \(T_{\text{set}}\), the recovery rate will become a negative quantity, thereby changing the sign of the performance metric.

5. SIMULATION RESULTS

In this section, we first present the dynamic evolution of tank thermal stratification during the simultaneous charging and discharging operation mode. Next, we demonstrate how the dynamic performance metrics capture the relative
effect of control input variables and disturbance input variables on system performance using the dynamic storage tank model. The physical parameters used in these simulation studies are based on an experimental micro-CHP system at Purdue University; values are provided in Table 2. A discretization of 30 nodes is used in all simulation studies, yielding 30 dynamic states.

**Table 2: Baseline values used for hot water storage tank model simulations**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>tank height</td>
<td>1.3 m</td>
<td>coil volumetric flow rate</td>
<td>2 lpm</td>
</tr>
<tr>
<td>tank diameter</td>
<td>0.4 m</td>
<td>tank volumetric flow rate</td>
<td>2 gpm</td>
</tr>
<tr>
<td>coil inlet temperature</td>
<td>45°C</td>
<td>tank wall thickness</td>
<td>0.051 m</td>
</tr>
<tr>
<td>dead state temperature</td>
<td>20°C</td>
<td>height of coil inlet</td>
<td>0.15 m</td>
</tr>
<tr>
<td>take initial temperature</td>
<td>20°C</td>
<td>height of coil outlet</td>
<td>0.58 m</td>
</tr>
<tr>
<td>cold water supply temperature</td>
<td>20°C</td>
<td>set point temperature</td>
<td>30°C</td>
</tr>
</tbody>
</table>

**5.1 Dynamic Thermal Stratification Behavior**

The tank is initially charged for two hours. During this time, hot waste heat water from an integrated power generation system flows through the coil, thereby exchanging heat with the tank. At $t = 120$ minutes, the tank enters the simultaneous charging/discharging mode in which hot water is removed from the tank at a specified flow rate $\dot{m}_d$ while heat continues to be added to the system by the coil. The temperature evolution of the thermal stratification for this mode, over a period of 30 minutes, is shown in Fig. 6.

![Figure 6: Illustration of thermal stratification during 30 minute simultaneous charge/discharge.](image)

In Fig. 6a, the charge/discharge has been ongoing for ten minutes. A portion of the hot water above the coil region has exited the tank and been replenished with the cold water supply. However, the coil waste heat water continues to heat the cold water as it is drawn upwards through the tank. In Fig. 6b, it is clear that the hottest water in the tank, which was located above the coil at a temperature of 40°C, has been removed from the tank. Finally, in Fig. 6c, all of the water above the coil region is at a temperature below 30°C. At this point, the tank has reached its steady-state thermal gradient; water being removed from the tank is heated from the cold water supply temperature of 20°C to about 27°C before it leaves the tank. The mixing of the water being heated by the coil, and the cold supply water flowing into the tank, results in a widened thermocline.

**5.2 Analyzing System Performance**

In this section, we present results for performance characterization of the different modes of system operation. Performance is analyzed with respect to two types of variables: (1) control variables and (2) disturbance variables.
5.2.1 Control Variables and Charging Performance

Within the system, one variable over which we have control is the volumetric flow rate of hot waste heat water through the immersed coil. In order to examine the charging performance of the system, a three hour charge is simulated for coil flow rates varying from one to five liters per minute (lpm). In each case, charging performance is quantified according to Eqn. (5) and the results are compared in Fig. 7.

![Figure 7](image1)

Figure 7: Simulation results depicting effect of varying coil flow rate on charge performance.

Figure 7(a) shows that initially, higher flow rates yield better performance. However, as time progresses, the higher flow rates actually have an adverse effect on system performance. As the tank heats up, there is a decrease in the temperature differential between the waste water in the coil and the water in the tank. This leads to a decrease in the heat transfer rate between the coil and the tank, and consequently, there is less useful work being supplied to the tank. This aspect of system performance can be seen more clearly through Fig. 8 which shows how the coil outlet temperature varies as a function of time. Based on the assumptions presented in Section 3, the coil outlet temperature at any time is equal to that of the surrounding water in the tank. In Fig. 8, we see that for high coil flow rates, the tank heats up, or charges, more quickly. However, the tank then reaches its maximum state of charge more quickly as well, and system performance subsequently degrades as the coil cannot add any additional charge to the system. Once the maximum charge is reached, a higher amount of useful work is lost relative to the amount being supplied via the immersed heat exchanger coil.

![Figure 8](image2)

Figure 8: Simulation results depicting effect of varying coil flow rate on coil outlet temperature.

These results suggest that in order to maximize the charging rate while also maximizing charging efficiency, a variable coil flow rate could be used. For example, the coil flow rate could be set high toward the beginning of the charging period but later decreased as the derivative of the efficiency curve becomes negative. In this way, the performance
metric for charging modes can be used in real-time to make control decisions to help improve global system performance.

5.2.2 Disturbance Variables and Discharging Performance From a controls perspective, we are also interested in understanding the effect of measured disturbances on system performance. The primary disturbance input is the volumetric flow rate of water exiting the tank during a discharge mode. While we cannot control the timing or duration of discharge modes, we can use the performance metric in Eqn. (9) to inform real-time control decisions in order to limit adverse effects from an energy utilization standpoint.

To investigate the effects of different discharge flow rates on the system, we charge the tank from its cold state for two hours. The hot waste heat water is then turned off and a discharge mode is allowed to commence for 30 minutes at a specified tank flow rate. Fig. 9 compares the discharge performance for tank flow rates ranging from one to five gallons per minute, or 3.8-11.4 lpm. The figure shows that higher tank flow rates allow better performance at first. However, as the state-of-charge decreases, performance at higher flow rates degrades quickly. This makes intuitive sense; as hot water is removed from the tank, cold supply water is being added to the tank thereby reducing the overall state-of-charge. This is further illustrated by Fig. 10.

![Figure 10: Simulation results depicting effect of varying tank flow rate on exergy storage rate and available storage.](image)

In Fig. 10b, we see that for higher flow rates, the absolute value of the rate of change of exergy, \( |\frac{dX}{dt}| \), or the exergy storage rate, quickly decreases. The result is that the available exergy within the system decreases rapidly, seen in Fig. 10a. As the high flow rates deplete the system of its charge, cold water is pumped into the system and the available thermal energy decreases further. The system rapidly approaches steady state in which cold water exists uniformly throughout the tank and useful work is unavailable. For smaller tank flow rates, the exergy storage rate in the system decreases much less rapidly, signaling that a higher amount of charge remains available within the storage. The simulations presented in this section allow real-time insight into system performance for various modes of operation. Performance metrics can be used to design control algorithms aimed at improving demand response and to mitigate adverse effects on performance with respect to disturbances.

6. CONCLUSION

In this paper we defined a set of dynamic performance metrics that are generalizable to a range of thermal energy storage systems. These metrics were then analyzed in the context of a hot water storage tank with an immersed coil heat exchanger, a common type of sensible TES system. We derived a reduced-order model for simulating thermal stratification within the hot water storage tank. More specifically, a simplified model for the interaction between the tank’s immersed coil heat exchanger and the surrounding water reduced the overall number of dynamic states needed to describe the transient behavior of the system. Using the dynamic hot water storage tank model, we demonstrated
the sensitivity of the performance metrics to changes in both control input variables and disturbance input variables.  
The performance metrics presented for each mode of operation will enable future work aimed at designing advanced 
control algorithms for improved system performance and demand response.

**NOMENCLATURE**

<table>
<thead>
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<th>Symbol</th>
<th>Description</th>
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<tr>
<td>$A$</td>
<td>area</td>
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<tr>
<td>$s$</td>
<td>entropy</td>
</tr>
<tr>
<td>$c_v$</td>
<td>specific heat</td>
</tr>
<tr>
<td>$T$</td>
<td>temperature</td>
</tr>
<tr>
<td>$k$</td>
<td>heat transfer coefficient</td>
</tr>
<tr>
<td>$m$</td>
<td>mass flow rate</td>
</tr>
<tr>
<td>$w_i$</td>
<td>tank wall thickness</td>
</tr>
<tr>
<td>$x$</td>
<td>specific exergy</td>
</tr>
<tr>
<td>$\psi$</td>
<td>performance metric</td>
</tr>
<tr>
<td>$z$</td>
<td>height in tank</td>
</tr>
<tr>
<td>$\dot{Q}$</td>
<td>heat transfer rate</td>
</tr>
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**Subscript**

- $j$ node $j$
- $b$ boundary $j-1$ node above $j$
- $c$ charge $j+1$ node below $j$
- $d$ discharge
- $sup$ supplied
- $dest$ destroyed
- $t$ tank
- $gen$ generated
- $0$ dead state

**REFERENCES**


