DECENTRALIZED FEEDBACK STRUCTURES OF A VAPOR COMPRESSION CYCLE SYSTEM

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ABSTRACT

In vapor compression cycle (VCC) systems, it is desirable to effectively control the thermodynamic cycle. By controlling the thermodynamic states of the refrigerant with an inner-loop, supervisory algorithms can manage critical objectives such as maintaining superheat and maximizing the coefficient of performance, etc. In the HVAC industry, it is generally preferred to tune multiple single-input-single-output (SISO) control inner-loops rather than a single multiple-input-multiple-output (MIMO) control inner-loop. This paper presents a process by which a simplified feedback control structure amenable to a decoupled SISO control loop design may be identified. In particular, the many possible candidate input-output pairs for decentralized control are sorted via a decoupling metric, the relative gain array number. From a reduced set of promising candidate input-output pairs, engineering insight is applied to arrive at the final pairings successfully verified on a refrigeration test stand.

1. INTRODUCTION

The primary goal of any air conditioning or refrigeration application is to move energy from one location to another. An idealized vapor compression cycle (VCC) system, as shown in Figure 1 and Figure 2, is a thermodynamic system driven by the phase characteristics of the refrigerant that is flowing through it. An ideal VCC system assumes isentropic compression, isenthalpic expansion, and an isobaric process across both the condenser and evaporator coils. The basic control objectives of a VCC system can be conceptualized visually via Figure 2. For example, the difference between \( h_1 \) and \( h_4 \) represents the increase in enthalpy across the evaporator, i.e. the amount of energy \( (Q) \) removed from the cooled space. This is a measure of evaporator capacity. The difference between \( h_2 \) and \( h_1 \) represents the increase in enthalpy across the compressor, i.e. the amount of work \( (W) \) done by the compressor to increase the pressure of the refrigerant vapor. The system coefficient of performance (COP), a measure of system efficiency, is defined as the ratio between these two changes in enthalpy.

\[
COP = \frac{Q}{W} = \frac{h_1 - h_4}{h_2 - h_1}
\]

Figure 1: Schematic of ideal subcritical VCC system.

Figure 2: P-h diagram for ideal subcritical VCC.
Rather than explicitly controlling capacity or efficiency, however, this paper aims to control individual thermodynamic states of the system. Given the assumptions made for an ideal VCC system and the constitutive relationships between pressure, temperature, and enthalpy (i.e. for a given point on the cycle, only two thermodynamic states are required to derive the remaining states at that point), only four thermodynamic states are needed to uniquely define the four critical points of the idealized cycle shown on Figure 2. With an appropriate control architecture, these four points of the VCC could be placed by a higher level planning algorithm [1] so as to achieve an optimal balance between desired capacity and efficiency (see Figure 3). This is a shift from current practice where the actuators are used to control specific control objectives. For example, in [1] the compressor is used to control capacity and the expansion valve is used to control evaporator superheat.

It has been shown that multivariable control techniques [2][3][4] can be used to handle input-output couplings while achieving desired performance objectives. In [2] the system superheat and evaporator saturation temperature were used as the measured outputs in a Linear Quadratic Gaussian (LQG) control approach. However, for industrial practitioners and service engineers in the HVAC industry, it is generally preferred to tune multiple single-input-single-output (SISO) control loops rather than a single multiple-input-multiple-output (MIMO) control loop. Decentralized control structures consisting of SISO control loops will be greatly simplified when the interaction among the individual SISO control loops is sufficiently minimized.

Numerous control schemes have been developed with superheat and evaporation temperature (or pressure) as the feedback signals [2][3][4][5]. These references frequently noted the difficulty of controlling the two outputs with individual SISO control loops due to the physical coupling between superheat and evaporation temperature. Most recently, Keir [6] suggested that a novel choice of output control variables can decouple the dynamics such that a decentralized control approach can be used to meet desired performance objectives. As motivated by [6], this paper focuses on a structured method for identifying control input and output pairs which effectively decouple system dynamics. The output thermodynamic states considered for the control I/O pair analysis are derived from the four labeled points of the idealized cycle in Figure 2.

The rest of the paper is as follows. Section 2 introduces the choice of candidate input control variables and output thermodynamic states and describes the generation of identified models used for the ensuing analysis. The next section describes the use of a decoupling metric to sort through candidate input-output pairings. Section 4 discusses the results of the analysis and narrows the field of candidate I/O pairings to a final set. Finally, Section 5 provides a comparison between similar feedback controller algorithms applied to the best pairings of input parameters and control variables and to traditional I/O pairings.

2. SYSTEM MODELING AND IDENTIFICATION

While first principles models for VCC systems are available, Rasmussen [7] verified that system identification (ID) techniques can be used to construct accurate, low-order, local linear models of VCC systems. In this paper, the basic dynamic response of a VCC is identified using a time domain system ID procedure. Four controllable inputs for a variable-speed VCC are considered: the compressor speed, expansion valve opening, evaporator fan speed, and condenser fan speed (see $u_1$ through $u_4$ in Figure 1). This is often not the case, due to the fact that in many cases, e.g. automotive systems, the condenser air flow is a function of vehicle speed and acts more as a disturbance to the feedback loop. Nevertheless, they are considered here to allow for a thorough analysis of I/O pairs.

There are six thermodynamic states that are considered as output measurements: two pressures and four temperatures. Recall from Figure 2 that, for an idealized cycle, there are two system pressures: $P_2=P_3$ and $P_1=P_4$. These correspond to the pressure inside the condenser and the pressure inside the evaporator, respectively. There are four system refrigerant temperatures: $T_1$, $T_2$, $T_3$, and $T_4$. Again assuming an idealized cycle with saturated refrigerant leaving the condenser, these represent evaporator inlet temperature, condenser outlet temperature, condenser saturation temperature, and evaporator saturation temperature, respectively.

The output responses to step changes in each of the four control inputs, as well as to random Gaussian combinations of all four inputs (see Figure 5) around a set of nominal operating conditions, were collected on an A/C experimental test stand (see Figure 4). For a more detailed description of the experimental system, see [6].

![Figure 4: Experimental test stand at the University of Illinois at Urbana-Champaign.](image-url)
with all four excited inputs and the two pressure measurements as outputs \((P_2 = P_1)\) and a second model was identified with the same inputs and the four temperature measurements as outputs \((T_1, T_2, T_3, T_4)\). As will be described in the next section, only the frequency response between each I/O pair is needed for the decoupling analysis, thereby allowing the flexibility to identify two separate models rather than a single 4-input 6-output model.

For each identified output, the open-loop response is compared against the response as predicted by the identified model. The fit percentages for each output characterizing the goodness of the identification are included in the captions of Figure 6 and Figure 7.

3. I/O PAIRING ANALYSIS

The following section applies a model-based decoupling metric to the above identified models in order to reduce the possible pairs to the most promising I/O pairs.

3.1 Output Combinations

For an idealized VCC, the output measurements \(T_1, T_2, T_3, T_4\), \(P_2=P_1\), and \(P_1=P_4\) may be used to fully characterize the thermodynamic states of the cycle. However, other physically meaningful aspects of the cycle, such as evaporator superheat,

\[ T_{1-4} = T_1 - T_4, \]

may present opportunities for further decoupling. In fact, there is an infinite number of affine combinations, represented by

\[
\tilde{y} = \sum_{i=1}^{n} \alpha_i P_i + \sum_{j=1}^{m} \beta_j T_j
\]

for some \(\alpha_i, \beta_j \in \mathbb{R}\), that could be considered. Even though such affine combinations could be used to thoroughly decouple the individual identified loops, they are not necessarily physically meaningful in terms of the VCC. The inverse and singular value decomposition (SVD) of the DC-gain could be used to determine affine combinations that decouple the individual loops, but such combinations are sensitive to the choice of input and output scaling [9] and disregard the physical significance and units of the resulting combinations. For example, the units of \(\tilde{y} = \alpha P + \beta T\) are ambiguous, and the simple addition of a pressure and temperature does not carry any particular physical significance. Instead, for this investigation, each output is defined as a binary combination of either the temperatures or pressures, representing differences and averages, such as evaporator superheat or pressure differential. Each output combination \(\tilde{y}\) takes the form of (3), where \((\beta = 0, \alpha = \{-1,0,1\})\) or \((\beta = \{-1,0,1\}, \alpha = 0)\).

There is a very large number of potential input-output pairing combinations that may be effective in decoupling the individual loop. The possible binary combinations (3) for either two pressures and four temperatures are \(\sum_{i=1}^{2} 3^{(i-1)} = 4\) and \(\sum_{i=1}^{4} 3^{(i-1)} = 40\), respectively. Consequently, there are \(44!/(44-4)! = 3,258,024\) possible I/O pairs when considering 4 inputs to the system.

3.2 Sorting via a Decoupling Metric

Given the large number of possible input-output pairings, it is essential that a decoupling metric be used to filter them. One metric that can be used to quantify the reduction in coupling is the relative gain array (RGA) technique originally developed by Bristol [10]. The technique is independent of input and output scaling, thereby avoiding the question of appropriate scaling of inputs and outputs.
with different magnitudes. To apply the RGA analysis, a dynamic model of the system must be identified using time domain data as was described in Section 2. The RGA is a steady-state measure of closed loop interactions for decentralized (multiple SISO loop) control. For a non-singular square matrix $P$, the relative gain array is defined by Equation (4), where $\times$ denotes the Schur product.

$$RGA(P) = \Lambda(P) = P \times (P^{-1})^T$$  \hspace{1cm} (4)

The RGA is a good indicator of sensitivity to uncertainty in the input channels, diagonal dominance for state-space systems, and stability levels of decentralized controller designs [9].

Uncertainty in the input channels is indicated by plants with large RGA elements around the crossover frequency, making these plants fundamentally difficult to control. A measure of the diagonal dominance of a plant, $G$, is obtained by calculating the RGA-number, given in Equation (5).

$$RGA\text{-number}(G(j\omega)) = \| \Lambda(G(j\omega)) - I \|_{\text{max}},$$  \hspace{1cm} \text{(5)}

where $\| M \|_{\text{max}}$ for some square matrix $M \in \mathbb{R}^{n \times n}$ is defined as $\sum_{i=1}^{n} \sum_{j=1}^{n} | m_{ij} |$.

Large RGA numbers are a clear indicator that the closed loop performance will be poor when decentralized control schemes are applied [9]. When all four inputs are considered in the RGA analysis, the best achievable RGA-number is 1.4374 (see Figure 8).

When the condenser air flow rate $u_4$ is removed from the group of inputs considered, the best achievable RGA-number is 0.076 (see Figure 9), a 20-fold reduction compared to the minimum RGA-number for the four input case. Based on this result, we consider only inputs $u_1$ through $u_3$ in the analysis presented in Section 4.

### 4. COMPARISON OF COUPLING FOR DIFFERENT I/O PAIRINGS

Applying the RGA number as a decoupling metric, the control designer is presented with a reduced field of the best potential I/O pair candidates. Subsequently, physical intuition and an understanding of vapor compression cycles is applied to further narrow the field for experimental trials. The following tables characterize how frequently each output was associated with an input $u_i$ for the top 500 I/O pairs, as characterized by the RGA-number, at three different frequencies. Note the tables only present the top recurring outputs coupled to each input; therefore the rows do not necessarily total 100%. Moreover, recall that $u_1$, $u_2$, and $u_3$ represent EEV opening, compressor speed, and evaporator airflow rate, respectively.

#### TABLE 1: INCIDENCE OF CANDIDATE PAIRINGS FOR INPUT $U_1$ FOR TOP 500 PAIRINGS

<table>
<thead>
<tr>
<th>freq.</th>
<th>$P_1=P_3$</th>
<th>$T_1$</th>
<th>$P_{3+4}$</th>
<th>$T_{3+4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega=0$ rad/s</td>
<td>64%</td>
<td>26%</td>
<td>6%</td>
<td>0%</td>
</tr>
<tr>
<td>$\omega=0.01$ rad/s</td>
<td>30%</td>
<td>7%</td>
<td>0%</td>
<td>6%</td>
</tr>
<tr>
<td>$\omega=1$ rad/s</td>
<td>0%</td>
<td>10%</td>
<td>2%</td>
<td>7%</td>
</tr>
</tbody>
</table>

#### TABLE 2: INCIDENCE OF CANDIDATE PAIRINGS FOR INPUT $U_2$ FOR TOP 500 PAIRINGS

<table>
<thead>
<tr>
<th>freq.</th>
<th>$P_1=P_4$</th>
<th>$P_2-4$</th>
<th>$T_{2+1}$</th>
<th>$T_{2+4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega=0$ rad/s</td>
<td>26%</td>
<td>6%</td>
<td>9%</td>
<td>11%</td>
</tr>
<tr>
<td>$\omega=0.01$ rad/s</td>
<td>6%</td>
<td>52%</td>
<td>9%</td>
<td>6%</td>
</tr>
<tr>
<td>$\omega=1$ rad/s</td>
<td>2%</td>
<td>6%</td>
<td>3%</td>
<td>0%</td>
</tr>
</tbody>
</table>

#### TABLE 3: INCIDENCE OF CANDIDATES OUTPUTS FOR $U_3$ FOR TOP 500 PAIRINGS

<table>
<thead>
<tr>
<th>freq.</th>
<th>$T_{1+2+3+4}$</th>
<th>$T_{1+2+4}$</th>
<th>$T_{1+3+4}$</th>
<th>$T_{1+2+3+4}$</th>
<th>$T_{1+4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega=0$ rad/s</td>
<td>8%</td>
<td>6%</td>
<td>6%</td>
<td>5%</td>
<td>6%</td>
</tr>
<tr>
<td>$\omega=0.01$ rad/s</td>
<td>8%</td>
<td>5%</td>
<td>6%</td>
<td>3%</td>
<td>4%</td>
</tr>
<tr>
<td>$\omega=1$ rad/s</td>
<td>3%</td>
<td>10%</td>
<td>9%</td>
<td>3%</td>
<td>9%</td>
</tr>
</tbody>
</table>

The statistics presented in Tables 1-3, and an understanding of the vapor compression cycle, are applied to further develop engineering insight for selecting candidate I/O pairings. For example, Table 1 suggests the EEV, input $u_1$, most strongly drives the condenser pressure, $P_3=P_1$, particularly at low frequencies. Figure 10 provides a qualitative metric for assessing how strongly $u_1$ drives each of these candidate output states. Recall that the experimental system was perturbed with a series of step changes in each input; here we show a particular output state against only one input signal at a time to more clearly assess the coupling between that I/O pair.
coupled with both the evaporator pressure, condenser pressure via the EEV runs somewhat counter to the typical used to regulate evaporator superheat. Consequently, regulation of the 11. The open loop response for these I/O pairings is provided in Figure 11.

Note that the response of both \( P_2 = P_3 \) and \( T_1 \) are relatively slow, which is consistent with the RGA analysis which showed a stronger coupling at lower frequencies. This pairing also agrees with the intuition that the EEV controls the refrigerant mass flow rate leaving the condenser. Oftentimes, the EEV is used to regulate the evaporator superheat \( T_{s,2} \), because the electronic expansion valve is viewed as a programmable version of another device, a thermal expansion valve, used to regulate evaporator superheat. Consequently, regulation of the condenser pressure via the EEV runs somewhat counter to the typical VCC control design practice. Gaining potentially unconventional insights is one of the benefits of sorting through such a large set of I/O pairings.

Table 2 suggests that the compressor speed, input \( u_2 \), is tightly coupled with both the evaporator pressure, \( P_1 (=P_4) \), and the difference between the condenser and evaporator pressures, \( P_{2-1} \). In particular, the RGA analysis suggests that \( u_2 \) is more strongly coupled with \( P_1 \) at low frequencies, but that at higher frequencies, it drives \( P_{2-1} \). The open loop response for these I/O pairings is provided in Figure 11.

From Figure 12 both an average of \( T_1 \) and \( T_4 \), as well as an average of all four temperatures, appear to respond similarly to perturbations in \( u_3 \). From a physical perspective, this is particularly interesting because controlling the average evaporator refrigerant temperature would in effect allow the control designer to regulate evaporator superheat which is critical to ensuring that the refrigerant entering the compressor is in the vapor form.

With the field of possible I/O pairings narrowed down to two candidate outputs for each input (see Figure 13), one final metric is needed to better understand if one I/O pair provides an advantage over another. Note that \( T_{1+2+3+4} \) has been replaced with \( T_1 \). While there is some loss in decoupling, considering \( T_1 \) is more practical, from a systems perspective, as a parameter that is meaningful to the vapor compression cycle.

The response of \( P_3 (=P_4) \) is slower than that of \( P_{2-1} \), confirming that there is tighter coupling at lower frequencies between \( u_2 \) and \( P_3 (=P_4) \), but that at high frequencies, the coupling is stronger between \( u_2 \) and \( P_{2-1} \). These two particular I/O parameter pairs were originally explored in [6], wherein the reduction in coupling exhibited by using \( P_{2-1} \) rather than \( P_1 \) was shown to significantly improve the performance of the decentralized two-input-two-output closed loop system. However, only two inputs were considered in [6], and the best choice of I/O pairs might be different when considering three inputs.

Table 3 suggests that the evaporator airflow rate, \( u_3 \), drives some combination of the four temperatures. Both the evaporator outlet and condenser saturation temperatures consistently appear in the most prominent combinations in Table 3, suggesting that an average of those two alone might be a good candidate. Again these candidate I/O pairings can be evaluated qualitatively using experimental data of the open loop response of each to perturbations in \( u_3 \).
Because the RGA analysis is frequency-based, it is important to more closely examine how the coupling between each I/O pair varies with respect to frequency. Tables 1–3 suggested that certain output states were more strongly coupled at either low frequencies or high frequencies, but not necessarily both. Given two possible output states for each control input \( u \), there are eight possible I/O pairing sets that can be used to control the system. Table 4 lists these eight sets. Figures 14 and 15 characterize the RGA-number for each I/O set as function of frequency.

<table>
<thead>
<tr>
<th>Inputs/Outputs</th>
<th>Pair ( y_1 )</th>
<th>Pair ( y_2 )</th>
<th>Pair ( y_3 )</th>
<th>Pair ( y_4 )</th>
<th>Pair ( y_5 )</th>
<th>Pair ( y_6 )</th>
<th>Pair ( y_7 )</th>
<th>Pair ( y_8 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( u_1 )</td>
<td>( P_1 )</td>
<td>( P_3 )</td>
<td>( P_4 )</td>
<td>( T_3 )</td>
<td>( T_5 )</td>
<td>( T_7 )</td>
<td>( T_3 )</td>
<td>( T_5 )</td>
</tr>
<tr>
<td>( u_2 )</td>
<td>( P_1 )</td>
<td>( P_3 )</td>
<td>( P_5 )</td>
<td>( T_3 )</td>
<td>( T_5 )</td>
<td>( T_7 )</td>
<td>( T_3 )</td>
<td>( T_5 )</td>
</tr>
<tr>
<td>( u_3 )</td>
<td>( T_{1+4} )</td>
<td>( T_1 )</td>
<td>( T_{1+4} )</td>
<td>( T_1 )</td>
<td>( T_{1+4} )</td>
<td>( T_1 )</td>
<td>( T_{1+4} )</td>
<td>( T_1 )</td>
</tr>
</tbody>
</table>

Sets \( y_1 (= [P_3, P_{3-4}, T_{1+4}]) \) and \( y_3 (= [P_3, P_{3-4}, T_{1+4}]) \) show the lowest RGA number at low frequencies and no spikes in RGA-number at higher frequencies, making both sets good candidates. Choosing \( y_1 \) as the optimal I/O set, the idea of controlling individual points of the cycle can be revisited. As an illustration, see Figure 16.

While parameters such as evaporator superheat and evaporator pressure might still be preferable in terms of describing desired performance, Figure 16 illustrates how the chosen I/O pairs still allow for these parameters to be controlled indirectly. For example, although a decentralized control approach would be used to control condenser pressure and differential pressure independently, a correct choice of reference signals for each would result in the indirect control of evaporator pressure. A similar argument can be made for evaporator superheat. This reinforces the importance of selecting I/O pairs which sufficiently decouple the system and using an outer control loop to coordinate those thermodynamic states (which can be controlled effectively with individual SISO loops) to achieve desired performance objectives.

5. COMPARISON OF DECENTRALIZED CONTROLLER DESIGNS

To further demonstrate the system performance with the candidate I/O pairings, the set \( y_1 (= [P_3, P_{3-4}, T_{1+4}]) \) is used to present a decentralized proportional-integral-derivative (PID) controller constructed with three individual SISO control loops.

The decentralized feedback configuration used for the controller design of the experimental system is shown in Figure 17. The controllers \( K_1(s) \), \( K_3(s) \), and \( K_5(s) \) are independent PID controllers, since some types of PID algorithms are prevalent in the majority of industrial controllers for these kinds of systems. The output variables are the condenser pressure, \( P_3 \), the difference between condenser and evaporator pressure, \( P_{3-4} \), and the average of evaporator saturation and evaporator outlet temperatures, \( T_{1+4} \).

PID controllers were tuned online to evaluate the closed loop system’s reference tracking characteristics. This highlights the
industrial attractiveness of using PID algorithms, since the controllers can be tuned online during experiments and do not require extensive first principle models. A sequence of step input reference signals was applied to the condenser pressure, $P_3$, and pressure differential, $P_{3-4}$, respectively, while the average evaporation temperature $T_{1+4}$ was regulated at a constant set point. The three controller gains are given in Table 5.

**TABLE 5: TUNED GAINS FOR THREE CONTROLLERS**

<table>
<thead>
<tr>
<th>Input/Controller</th>
<th>Proportional Gain</th>
<th>Integral Gain</th>
<th>Derivative Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>EEV opening $u_1/K_1$</td>
<td>0.25</td>
<td>0.002</td>
<td>2</td>
</tr>
<tr>
<td>Compressor speed $u_2/K_2$</td>
<td>12</td>
<td>0.4</td>
<td>3</td>
</tr>
<tr>
<td>Evap. airflow rate $u_3/K_3$</td>
<td>3</td>
<td>0.05</td>
<td>20</td>
</tr>
</tbody>
</table>

Figure 18 shows the time domain response of the output of each control loop. The condenser pressure controller, $K_1(s)$, and pressure differential controller, $K_2(s)$, have a settling time of about 60 seconds, while the oscillation around the pressure set points is within ± 3 kPa. The average temperature feedback controller, $K_3(s)$, regulates the average temperature within ± 1°C error during the tracking process. The results demonstrate the ability of three controllers to independently track three set points while selecting the set $y_1 = [P_3, P_{3-4}, T_{1+4}]$ as the I/O pairing.

Next we provide a comparison of this candidate I/O pairing set with a baseline I/O pairings for this type of system. Most VCC systems utilize an electronic expansion valve (EEV) to regulate the superheat ($T_{1-4}$) of the system, while the compressor speed is used to control either evaporator temperature or evaporator pressure ($T_4$ and $P_4$, respectively). All three inputs are not used because evaporator superheat is simply a function of evaporator temperature and pressure; that is, each output is uniquely determined by the other two. This suggests an additional advantage of the approach presented in this paper wherein the candidate output set chosen in Section 4 contains three independent parameters, allowing for an additional degree of freedom.

For the I/O pairing $y_{\text{baseline}} = [T_{1-4}, P_4]$ with EEV opening and compressor speed as inputs, two SISO PID controllers were tuned on the experimental test stand. The reference tracking results of these two SISO loops are shown in Figure 20. It is clear that the actuator input signals required by the controllers in Figure 19 result in less-oscillation compared to the EEV and compressor actuation shown in Figure 20. Similar results are also discussed in [6].
6. CONCLUSIONS

This paper demonstrates that an appropriate choice of feedback variables can improve the effectiveness of a decentralized controller on a VCC system in the HVAC industry. Rather than explicitly controlling capacity or efficiency, three individual SISO control loops are used to control specific thermodynamic states of the system while relying on a higher level planning algorithm to optimize the set points for each state such that desired performance and efficiency objectives can be met. An expansive set of I/O pairings was considered, and a decoupling metric and engineering insight were used to choose the best pairs.

It was found that the EEV is best paired with condenser pressure, the compressor speed with differential pressure, and the evaporator fan with average evaporator temperature. The primary advantage of considering such a large set of I/O pairings was that it led to potentially unconventional insights with regards to optimal pairings for decentralized control. It should be noted that the analysis performed here is for a linear system representation. VCC systems are actually very nonlinear systems. Any linear VCC system representation will vary greatly in its parameters about different operating points [12]. However, linearizing the system about any operating point and performing a local analysis will result in a similar type of decoupling benefit as that presented in this paper.

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REFERENCES


APPENDIX A. IDENTIFIED STATE SPACE MODELS

The identified state space \([A,B,C,D]\) system model for two output pressures is given in Equation (6). Note that differential pressure, rather than \(P_2\), was used for the identification.

\[
A = \begin{bmatrix}
-0.06794 & 0.04704 & 0.2834 & -0.0481 & 0.2248 & 1.042 \\
-0.05117 & -0.2241 & -0.03257 & -0.2986 & 1.154 & 1.587 \\
0.2587 & -0.3095 & -1.295 & 0.05653 & -0.4164 & -3.854 \\
-0.245 & -0.22 & 0.7351 & -0.7526 & 2.883 & 6.105 \\
0.6989 & -3.387 & -5.924 & -2.978 & -0.9685 & -5.694 \cdot 10^{-6} \\
-0.5499 & 0.8788 & 2.963 & 0.1192 & 0 & -1.327
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
-0.001744 & 4.635 & -6 & 0.1389 & 0.02843 \\
-0.002344 & -4.6765 & -5 & 0.2562 & 0.06265 \\
0.007939 & -3.87 & -5 & -0.4873 & -0.1302 \\
-0.01288 & -9.731 & -6 & 0.9648 & 0.2149 \\
0.002021 & -0.0004335 & 0.7811 & 0.06284 \\
-0.01574 & 0.0001202 & 0.8895 & 0.2418
\end{bmatrix}
\]

\[
C = \begin{bmatrix}
256.3 & 93.39 & 14.05 & -51.34 & 0 & 0 \\
95.29 & -237 & 49.71 & 68.76 & 0 & 0
\end{bmatrix}
\]

\[
D = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
u = \begin{bmatrix} a_v & \omega & \dot{m}_{c,air} & \dot{m}_{e,air} \end{bmatrix}^T 
\]

\[
y = \begin{bmatrix} P_1 & P_2 & P_3 & P_4 \end{bmatrix}^T
\]

The identified state space \([A,B,C,D]\) system model for the four output temperatures is given in Equation (7).

\[
A = \begin{bmatrix}
-0.005231 & 0.001287 & 0.001944 & 0.001778 \\
-0.004315 & -0.009934 & -0.007708 & -0.01032 \\
-0.002326 & -0.01244 & -0.02806 & 0.01421 \\
-0.00884 & -0.03749 & -0.008136 & -0.09927
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
-0.0002647 & -1.453e-6 & 0.02311 & 0.004852 \\
-0.0003534 & 1.08e-5 & 0.02181 & -0.001921 \\
-0.0006485 & 4.105e-6 & -0.1572 & 0.01233 \\
-0.001648 & 7.498e-5 & 0.1627 & -0.05476
\end{bmatrix}
\]

\[
C = \begin{bmatrix}
140.8 & 9.181 & -13.99 & 2.757 \\
-13.7 & 19.41 & -6.834 & 0.3653 \\
-6.119 & -0.3302 & -2.125 & 3.257 \\
-10.9 & -5.542 & -4.846 & -0.8949
\end{bmatrix}
\]

\[
D = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
u = \begin{bmatrix} a_v & \omega & \dot{m}_{c,air} & \dot{m}_{e,air} \end{bmatrix}^T 
\]

\[
y = \begin{bmatrix} T_1 & T_2 & T_3 & T_4 \end{bmatrix}^T
\]