Maximum-Likelihood Speed Estimation using Vehicle-Induced Magnetic Signatures

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Abstract—Modern traffic management systems require accurate vehicle detection, speed estimates, and link travel times for congestion detection, traveler information, ramp metering, optimization of traffic signal timing, and planning. Current speed estimation methods report speeds that are averaged over at least 30 seconds. This is necessary in some cases because the estimates tend to be noisy or in other cases because the algorithms are not intended to deliver individual vehicle speeds. This paper develops an algorithm based on communication theory and compares the results to conventional algorithms. The maximum-likelihood algorithm proposed in this paper provides significantly improved speed estimates that can be used to produce histograms of vehicle speeds instead of the speed averages currently available.

I. INTRODUCTION

Modern traffic management systems require accurate vehicle detection, speed estimates, and link travel times for congestion detection, traveler information, ramp metering, optimization of traffic signal timing, and planning [1]. A variety of technologies are in use for vehicle detection including pneumatic tubes [2], magnetometers (e.g., microloops) [3], mutual inductive coupling sensors (i.e., inductive loops), video cameras [4], and microwave radar [5]. Speed estimators can be roughly classified into four types: 1) Methods that perform joint detection and speed estimation from radar return Doppler shifts, 2) Occupancy-based methods with a priori information about average effective vehicle length (AEVL) [6], 3) Methods using measurements of the travel time between closely spaced sensors (i.e., speed trap), and 4) Methods based on Processing of video streams.

Often speed estimates are averaged over a span of at least 30 seconds [7], [8]. This is necessary in some cases because the estimates tend to be noisy and in other cases because the algorithms are not intended to deliver individual vehicle speed estimates (e.g., occupancy/AEVL methods).

This paper will focus on the use of micro-loops at speed trap installations which have a lead and lag sensor as shown in Figure 1. The each sensor measures the arrival times and the speed is estimated by dividing the known distance between the sensors by the difference between arrival times.

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Fig. 1: Speed Trap Configuration

A slight variation of this method is recommended in the Traffic Control Systems Handbook [9]. In this case, difference between the departure time from the lead and lag sensors is averaged with the difference between arrival times to the lead and lag sensors. The known distance is then divided by this average of delay estimates.

This paper focuses on developing a better estimator of individual vehicle speeds. Since vehicle speeds are individually estimated, a histogram of all speeds can be generated. This flexibility allows any statistic of interest to be computed along with estimates of reliability.

The paper is organized as follows: Section II includes the description of the microloop sensor and the problem statement. Section III develops the maximum-likelihood estimator of vehicle speeds. Experimental results from the conventional algorithm from the Traffic Control Systems Handbook and the maximum-likelihood speed estimator developed in this paper are presented in Section IV. Finally, conclusions are discussed in Section V.

II. SENSOR DESCRIPTION AND PROBLEM STATEMENT

Figure 2 shows a conceptual block diagram of the microloop sensor and associated detector electronics, which produce vehicle signatures and vehicle detections. We use this block diagram to explain the source of the data used to test the algorithms developed in Section III of this paper. For a detailed description of the sensor electronics we refer the reader to the U.S. Federal Highway’s Traffic Detector Handbook [10] and the patent literature [11], [12], [13]. Additional related work can be found in [14], [15].

The micro-loop sensor is the inductance element of an LC tank, which sets the frequency of a sinusoidal oscillator. Because of significant parasitic inductances associated with long lead-in cabling from the sensor under the road to the detector in the roadside cabinet and parasitic capacitances associated with coupling to the Earth, it is impossible to precisely set the resting frequency [14]. Furthermore, the resting...
frequency varies with time and environmental parameters. Therefore, the nominal resting frequency must be estimated along with the instantaneous frequency of the oscillator. When the instantaneous frequency differs sufficiently from the resting frequency, the detector indicates the presence of a vehicle.

The measurement and comparison of instantaneous and resting oscillator frequencies can be implemented in many ways (e.g., use of a “dummy sensor” [16], [17], [18], periodic sampling of sensors while no vehicle is present [19], use of a moving average low pass filter to set a DC voltage [20], etc.). The particular hardware used in this research implements the calculation according to the concept presented in Figure 2. A 32 MHz crystal oscillator provides the reference for estimating the oscillation frequency. At a particular discrete-time index \( k \), the period estimator produces an estimate of the instantaneous oscillation period as a multiple of the reference 32 MHz period (i.e., \( 31.25 \text{ ns} \)): \( \hat{T}_k = \hat{N}_k T_{32\text{MHz}} \). The period estimator produces this estimate by averaging over a number of periods of the oscillating waveform. Most detectors allow the user some control over the length of the estimation window, which is used for noise averaging. The detector circuitry also produces an estimate of the resting period by averaging instantaneous period estimates over a relatively long window and including only those instantaneous period estimates which do not correspond to a vehicle detection. Finally, the detector produces two time series at its output. The first is a binary signal indicating the presence of a vehicle (this is known as the “call” function). The second is the signature time series \( \Delta N_k = \hat{N}_k - N_{\text{ref}} \) consisting of differences between the instantaneous period estimate and the resting period estimate.

Since an LC oscillator’s frequency is inversely proportional to the square root of the inductance (i.e., \( f = K/\sqrt{L} \)) [21] we can show that for small perturbations \( \Delta L = L_{\text{ref}} - L \) in the sensor’s inductance relative to the reference inductance the relative change in frequency is proportional to the relative change in inductance. Furthermore, the relative change in period count is approximately proportional to the relative change in frequency or inductance assuming that 32 MHz is significantly larger than the micro-loop oscillation frequency (in fact, it is more than 100 times larger for the sensor used to make our measurements). Therefore,

\[
\frac{\Delta N}{N_{\text{ref}}} \approx -\frac{\Delta f}{f_{\text{ref}}} = \frac{1}{2} \frac{\Delta L}{L_{\text{ref}}}.
\]

\[\tag{21}\]

### III. ALGORITHM DEVELOPMENT

This section describes the speed estimation method proposed by this paper. It begins by showing that correlation is the maximum-likelihood estimator for vehicle speed estimation from a speed trap. It then develops a segmentation algorithm using a likelihood ratio detection test. The last part of the algorithm development will describe how these methods are used to generate a speed estimate from the cross-correlation function of the segments.

#### A. Correlation as the Optimal Maximum-Likelihood Speed Estimator

The problem of estimating the delay between two vehicle signatures is equivalent to the well understood problem of estimating the delay between a transmitted signal and a noisy received version of the transmitted signal. Since maximum-likelihood estimators have the functional invariance property, the maximum-likelihood speed estimate is \( \hat{v} = \frac{d}{t} \) where \( t \) is the maximum-likelihood delay estimate and \( d \) is the distance between the sensors. The sampling rate is assumed to be above the Nyquist frequency and therefore equivalent to the continuous time problem. The details follow.

Let \( s_i(t) \) be the counts output from sensor \( i \) at time \( t \) where \( t = 0 \) when the vehicle is first detected. Even though the magnetic field at the sensor is a function of the vehicle’s relative position, the data can be equivalently indexed in time if a constant velocity is assumed. This assumption is reasonable due to the close spacing of the sensors.

The received signal at each of the sensors is then modeled as \( r_1(t) = s_1(t) + n_1(t) \) where \( n_1(t) \) is Additive White Gaussian Noise (AWGN) with variance \( \sigma^2 \) and independent between the two sensors. Also, \( s_1(t) = s_2(t - t_0) \) where \( t_0 \) is the travel time between the sensors. We can then write \( r_2(t) = r_1(t - t_0) + n(t) \) where \( n(t) \) is \( n_1(t) + n_2(t - t_0) \). If \( r_1(t) \) is then interpreted as the transmitted signal and \( r_2(t) \) is the received signal embedded in AWGN with variance \( 2\sigma^2 \) then this is the standard delay estimation problem except that the noise has twice the variance. It is well established that the maximum-likelihood estimator for this delay problem is the delay that maximizes the cross-correlation of the two signals, \( r_1(t) \) and \( r_2(t) \) [22]. This method assumes that there is a constant delay between \( r_1(t) \) and \( r_2(t) \).

#### B. Segmentation Algorithm

A segmentation algorithm is a process by which a group of contiguous data points in a data stream are associated with a passing vehicle. Although the shape of the signal produced from a passing vehicle is unknown, it is assumed...
that a passing vehicle will cause a shift in the mean. A
two-sided likelihood ratio detection test is an appropriate
choice for detection of a mean shift. This test, which is
equivalent to a two-sided t test, detects a vehicle when the
ratio of the estimated total variance, \( \sigma^2 \), to the estimated
noise variance, \( \sigma_n^2 \), exceeds a threshold, \( \gamma \) [23]. This test
statistic is \( T(x) = \frac{\sigma}{\sigma_n} \). Since each segment should include
the signal from both the lead and lag detectors, \( T(x) \) will be
computed on the sum of the lead and lag signals. This
performs well when the vehicles are reasonably spaced, but
may tend to group vehicles on congested systems. While this
may affect estimates of the number of vehicles, the speed
estimates should remain stable since closely spaced vehicles
must travel at the same speed.

The total variance, \( \hat{\sigma}^2 \), is estimated at each point by finding
the sample variance of a window of points centered on the
point of interest as shown in Equation 1.

\[
\hat{\sigma}^2[n] = \frac{1}{2N_1} \sum_{i=n-N_1}^{n+N_1} (x_i - \bar{x})^2 \quad (1)
\]

Since the variance should be minimized when no vehicle
is present, the noise variance is estimated by taking the
minimum variance over a larger window:

\[
\hat{\sigma}_n^2[n] = \min_{n-N_2 \leq i \leq n+N_2} \hat{\sigma}^2[i] \quad (2)
\]

where \( N_2 >> N_1 \).

Each group of contiguous data points where \( T(x) > \gamma \) are then identified as a segment. The segment is then
extended one second before and after the detected region
or half way to previous detection and next detection if the
detections are closely spaced. If \( T(x) \) does not fall below
\( \gamma/2 \) between consecutive segments, the segments are joined.
Also, segments with less than 50 data points are discarded.
The lead and lag signals of each segment are used to generate
a speed estimate as is described in Section III-C.

C. Applying the Correlation Algorithm

After the segments are chosen, the correlation coefficient is
calculated at each possible delay. As shown in Section III-A,
the delay that maximizes the correlation coefficient is
the maximum-likelihood delay estimate. The calculation of
the optimal delay estimate, \( \hat{k} \) and the maximum correlation
coefficient, \( \hat{\rho} \) are shown in Equation 3 and Equation 4.

\[
\hat{k} = \arg \max_k \sum_{m=0}^{n} s_1[m]s_2[m+k] / \|s_1\| \|s_2\| \quad (3)
\]

\[
\hat{\rho} = \max_k \sum_{m=0}^{n} s_1[m]s_2[m+k] / \|s_1\| \|s_2\| \quad (4)
\]

Due to maximum-likelihood’s functional invariance prop-
erty, the maximum-likelihood estimate of speed is \( \hat{v} = \frac{d}{kT} \)
where \( T \) is the sampling period.

IV. RESULTS

The results section begins by describing the data collection
site and data set used. It briefly describes two algorithms
that will represent conventional speed estimation methods.
Conventional Algorithm A is a method described in the
Traffic Control Systems Handbook [9]. Several simple improve-
ments are then added to develop Conventional Algorithm
B. After describing the conventional algorithms, the speed
estimates from these methods will be compared with the
proposed maximum-likelihood algorithm speed estimates.

A. Data Collection Procedure

A single detector card was used to capture the counts data
stream from the lead and lag center lane micro-loops on I-
70 at mile marker 67.3 in Indianapolis, IN, from 10:57 AM
to 1:10 PM on August 14th, 2007. No significant weather
traffic pattern changes were observed.

B. Conventional Algorithm Implementation

speed estimation method that uses only the call output. The
call (or vehicle detection) output is described in Section II.
This call function is simulated here by thresholding the test
statistic \( T(x) \) described in Section III-B. Each lead sensor
detection is matched with the following lag sensor detection
and the arrival and departure times of each vehicle are
recorded as \( t_1 \) to \( t_4 \) as shown in Figure 3.

The speed estimate, \( \hat{v} \), is then the distance, \( d \), divided
by the average travel time, \( \Delta t_{ave} \) where \( \Delta t_{ave} = \frac{1}{2}(t_1 - t_2 + (t_4 - t_2)) \). In this paper, this method is referred to as
Conventional Algorithm A.

Additions are made to this conventional method to address
two weaknesses. The less serious of the two occurs when
the vehicles are detected earlier or later than they should
have been detected. An extreme example of this problem is
shown in Figure 4. Another problem arises when a vehicle
is detected by one sensor, but not the other which causes
vehicles to be matched incorrectly.

To reduce the effects of these problems, two restrictions
are applied. There must be only one lag detection between
two consecutive lead detections and lead and lag occupancies

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**Figure 3:** Arrival and Departure Times of the vehicle is designated by the test statistic crossing a chosen threshold.
Fig. 4: If the presence time of a vehicle differs greatly between the lead and lag sensor, it can adversely affect the speed estimate. In this particularly poor example, the arrival times predict the vehicle travelling backwards and the departure times predict the vehicle travelling forward.

Fig. 5: The number of speed estimates is relatively constant between the three methods for a large range of thresholds.

Fig. 6: The average speed estimate from the maximum likelihood algorithm is more robust to changes in threshold than the other two methods.

Fig. 7: The standard deviation of vehicle speed estimates is affected by the actual variation in vehicle speeds and the noise in the estimates. Since all three methods measure the same vehicles, the maximum-likelihood algorithm must have much less noisy estimates of vehicle speed.

must differ by less than 15 milli-seconds [24]. Conventional Algorithm A with these additional restrictions is called Conventional Algorithm B in this paper.

C. Algorithm Results

All three algorithms rely on a chosen threshold to detect the presence of a vehicle. The results are therefore shown over a range of thresholds and then a more detailed comparison follows at a single threshold. A closer look at the algorithm’s performance is provided by focusing on a specific speed estimate of Vehicle A in Figure, 8, 9, and 10.

The threshold's effect on the number of estimates is shown in Figure 5. Most of the false detections are eliminated as the threshold, $\gamma$, exceeds three. Also, the number of vehicles remains fairly constant as $\gamma$ varies between three and six.

The average velocities are graphed against the chosen threshold in Figure 6. While all three yield reasonable average speed estimates, the maximum-likelihood method is much more robust to changes in threshold.

Figure 7 shows that Conventional Algorithm A yields speed estimates with a standard deviation of at least 30 mph that varies wildly with changes in threshold. Conventional Algorithm B removes some of the uncertain behavior and is able to consistently generate estimates with a standard deviation of about 10 mph. The maximum-likelihood algorithm reduces this uncertainty to a consistent 5 mph.

The following graphs show a more detailed view of the speed estimates from the three algorithms when $\gamma = 5$. The results from Conventional Algorithm A are shown in Figure 8. While most of the speed estimates are clustered in the 60 mph - 80 mph range, there are also scattered estimates up through 120 mph and down to almost 0 mph. Figure 9 shows the results from Conventional Algorithm B. The extra filtering in this algorithm removes many of the outliers. Figure 10 shows the results of the maximum-likelihood speed estimator. The estimates are tightly compressed between 60 mph and 80 mph. There are very few outliers.

Lead and lag counts streams from an example vehicle (Vehicle A) are shown in Figure 11. From the shape of the signatures Vehicle A is most likely two vehicles that are detected as one vehicle because they are closely spaced. These streams have been chosen to show an example of the
Fig. 8: Speed Estimates from Conventional Algorithm A with a threshold of $\gamma = 5$.

Fig. 9: Speed Estimates from Conventional Algorithm B with a threshold of $\gamma = 5$.

Fig. 10: Speed Estimates from the Maximum-likelihood Algorithm with a threshold of $\gamma = 5$.

Fig. 11: The signatures that generated the outlier speed estimate designated Vehicle A in Figure 9.

Fig. 12: Illustration of the lead and lag ratio of standard deviation signals from the signatures shown in Figure 11. It is clear from this figure that the arrival times are too close which is interpreted as a shorter travel time or faster speed.

maximum-likelihood algorithm improved performance over conventional algorithms.

Figure 12 demonstrates that the arrival times are much closer than they should be. Averaging with the difference in departure times helps, but the overall estimate is still raised almost 20 mph. Figure 13 shows the lead signature delayed by the amount estimated in the conventional algorithm. Ideally, the lead and lag signatures would line up exactly. Since the speed estimate is too fast, the lead signature is slightly ahead of the lag signature. In the conventional algorithm, there is no way to fix the speed estimate. The only option would be to throw the estimate away. Figure 14 is the same as Figure 13 except that the time delay estimated from the maximum-likelihood algorithm is used. The shifted lead signature lines up almost exactly with the lag signature since it is the accurate speed estimate.

V. Conclusion

This richer data set can then be analyzed in the context of well defined communication theory delay problems from which the maximum-likelihood algorithm proposed is developed.

The results section demonstrates several ways that the maximum-likelihood algorithm is more reliable than conventional methods. The average maximum-likelihood speed estimates are robust to changes in threshold and also have half of the standard deviation in the speed estimates generated. Since it is unclear what the true standard deviation of speeds should be, it is reasonable to propose that the mean and standard deviation estimated by the maximum-likelihood method approach the true mean and standard deviation of vehicle speeds. It is concluded that the maximum-likelihood algorithm is a significantly more reliable method for speed estimation at speed trap micro-loop installations.

Fig. 13: The lead and lag signature from Figure 11 except that the lead signature has been delayed by the time estimated by Conventional Algorithm B

Fig. 14: The lead and lag signature from Figure 11 except that the lead signature has been delayed by the time estimated by the Maximum Likelihood Algorithm, by comparing

practice there are many algorithms similar to these conventional algorithms that suffer from similar limitations. For this reason, most speed estimates are reported on average. The maximum-likelihood algorithm is proposed as an alternative speed estimation algorithm to produce speed estimates with sufficient precision to report estimates for individual vehicles.

As expected, the maximum-likelihood algorithm performs better than the conventional algorithm because it uses the entire waveform generated by the vehicle instead of just the call function. This richer data set can then be analyzed in the context of a well defined communication theory delay problem from which the maximum-likelihood algorithm proposed in this paper is developed.

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REFERENCES