# FUNDAMENTALS OF NANOELECTRONICS

*Basic Concepts*

1. The New Perspective
2. Energy Band Model
3. What & Where is the "Voltage"?
4. Heat & Electricity: Second Law & Information

3.1. Introduction

3.2. A New Boundary Condition

3.3. Quasi-Fermi Levels (QFL's)

3.4. Current from QFL's

3.5. Landauer Formulas

3.6. What a Probe Measures

3.7. Electrostatic Potential

3.8. Boltzmann Equation

3.9. Spin voltages

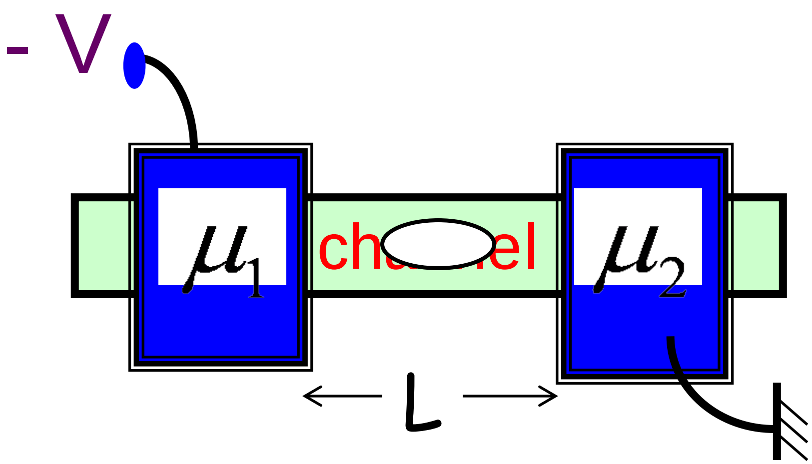
3.10. Summing up ..

Supriyo Datta

*Spring 2015*

http://nanohub.org/groups/lnebook

# 3.1a Introduction




$$\frac{R_B}{2} \qquad R_B\frac{L}{\lambda} \qquad \frac{R_B}{2}$$

Resistance is associated with

- ~~Joule Heating: $I^2R$~~

- Voltage drop: $IR$

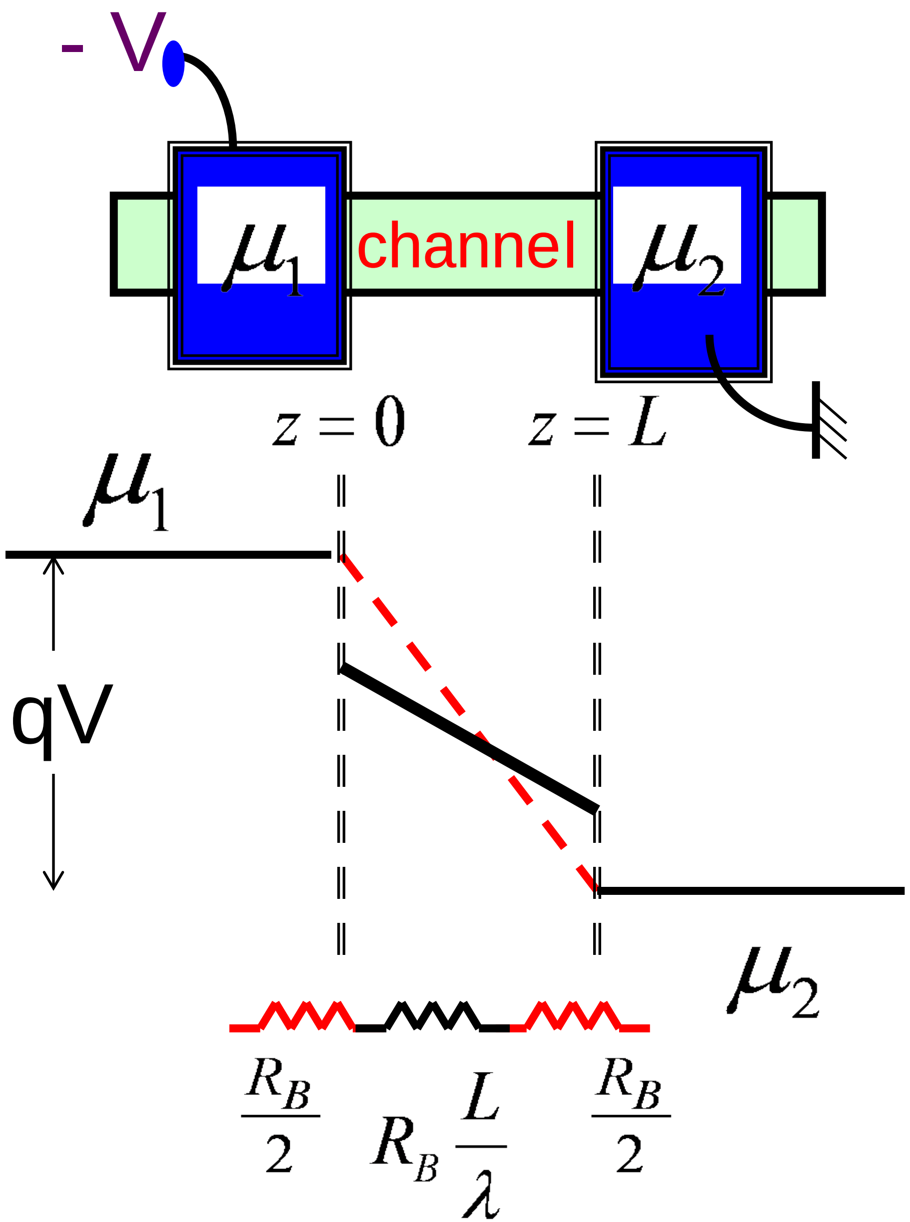
$$I = G_0 V$$

$$R_0 = \frac{1}{G_0} = R_B + R_B\frac{L}{\lambda}$$

$$G_0 = G_B \frac{\lambda}{L+\lambda}$$

$$R_B \equiv \frac{1}{G_B}$$

# 3.1b Introduction




*Where is the voltage drop?*

*What is Voltage ?*

*Common answer*

Electrostatic Potential

$$\times \quad J \sim \sigma_0 F \sim \frac{\sigma_0}{q}\frac{dU}{dz}$$

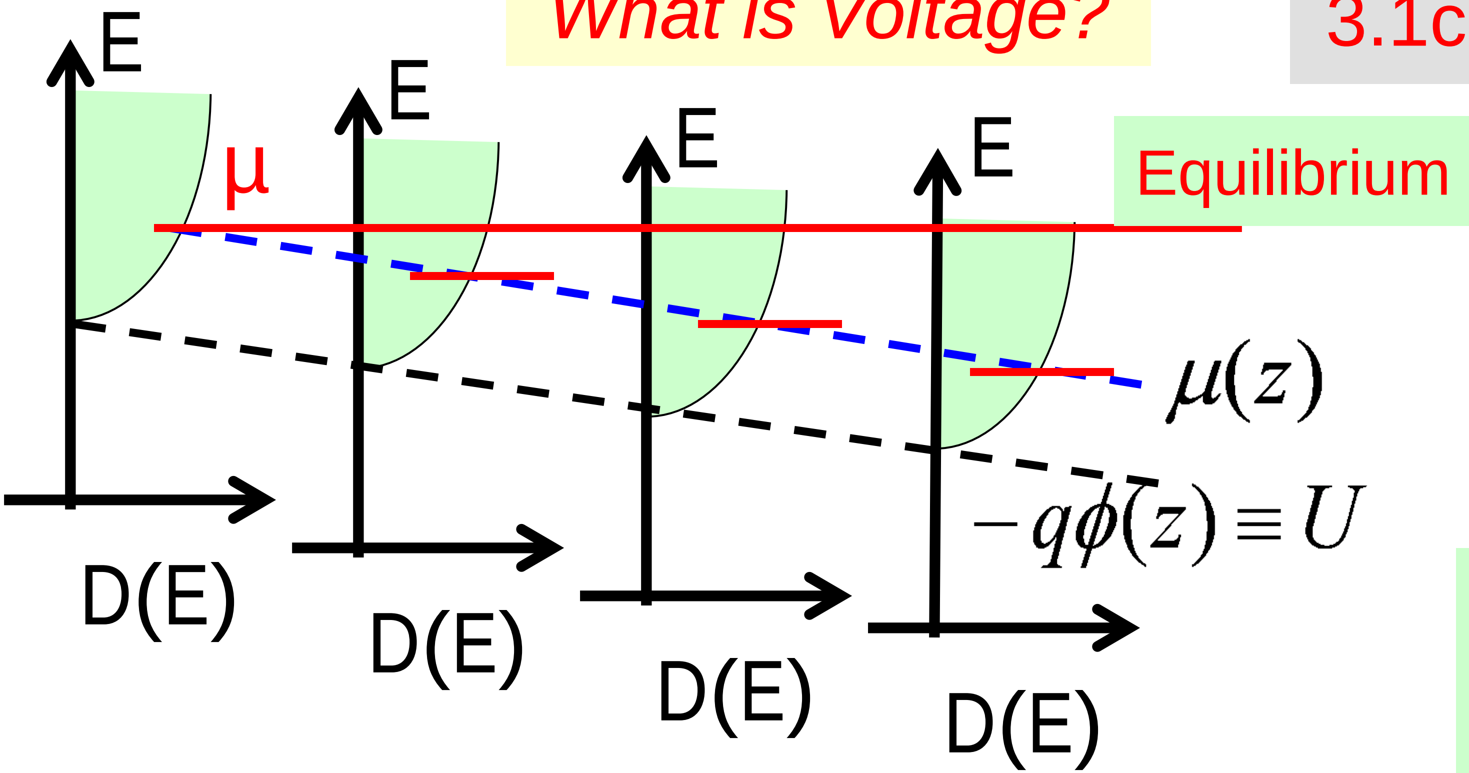
*Correct answer*

Electrochemical Potential

$$J \sim \frac{\sigma_0}{q}\frac{d\mu}{dz}$$

# What is Voltage?

## 3.1c Introduction




*μ is difficult to define under non-equilibrium conditions*

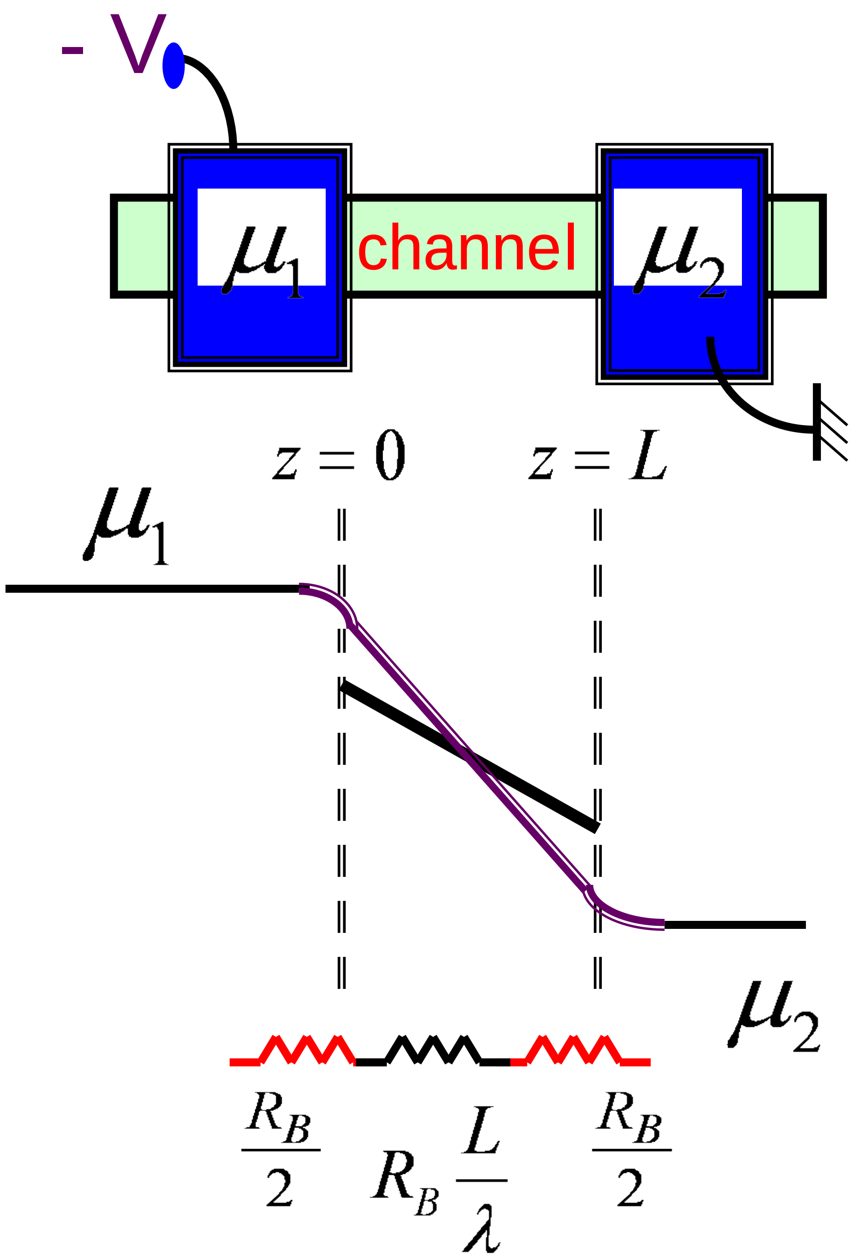
$$-\frac{\sigma_0}{q}\frac{d}{dz}(\mu - U)$$

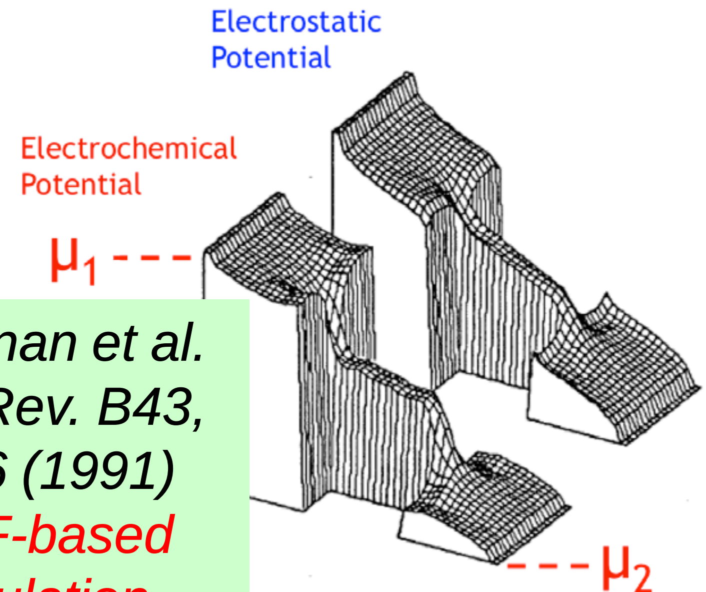
*Electron Current*

$$J = -\frac{\sigma_0}{q}\frac{dU}{dz} - q\frac{dn}{dz}$$

*DRIFT* *DIFFUSION*

$$\boxed{\rightarrow \quad J = -\frac{\sigma_0}{q}\frac{d\mu}{dz}}$$



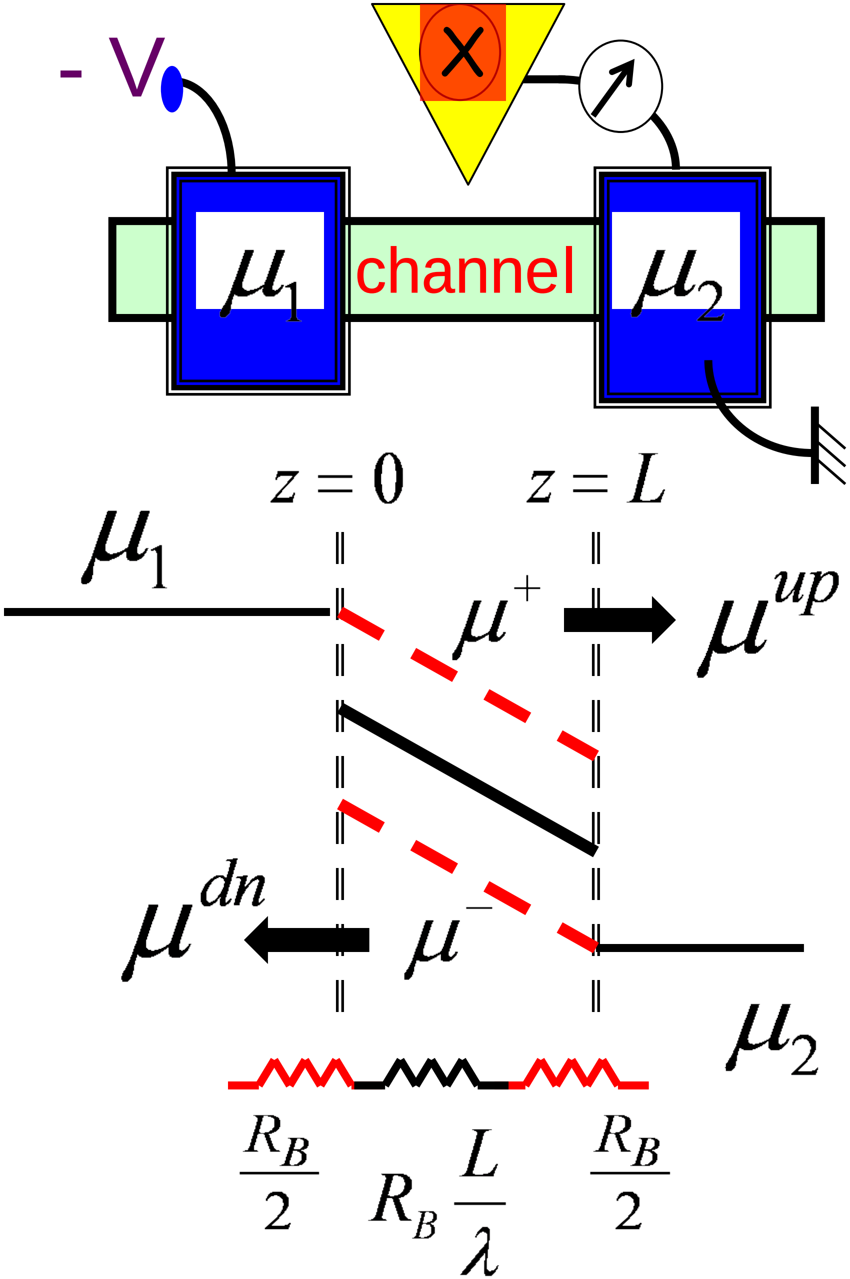




*McLennan et al. Phys. Rev. B43, 13846 (1991)*
*NEGF-based Calculation*

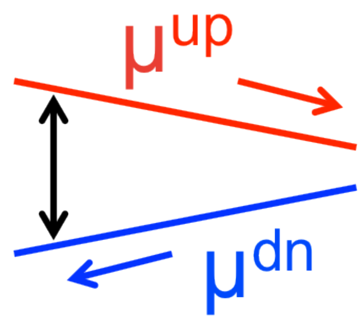
*μ is difficult to define under non-equilibrium conditions*

**Boltzmann Equation**

$$J = -\frac{\sigma_0}{q}\frac{d\mu}{dz}$$




*Spin Potential*



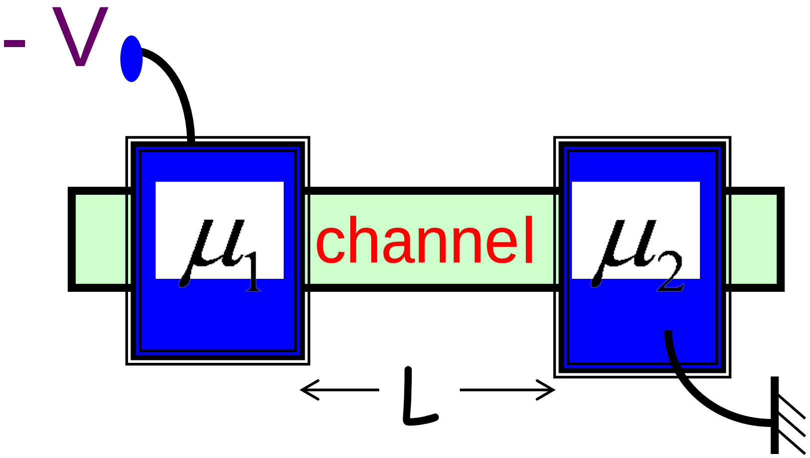

Topological Insulators
*Spin-momentum locking*

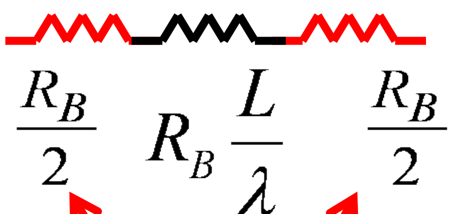
*Quasi-Fermi Levels (QFL's)*

*Boltzmann Equation*

$$J = -\frac{\sigma_0}{q}\frac{d\mu}{dz}$$

# Coming up next ..




$$R_0 = \frac{1}{G_0} = R_B + R_B \frac{L}{\lambda}$$

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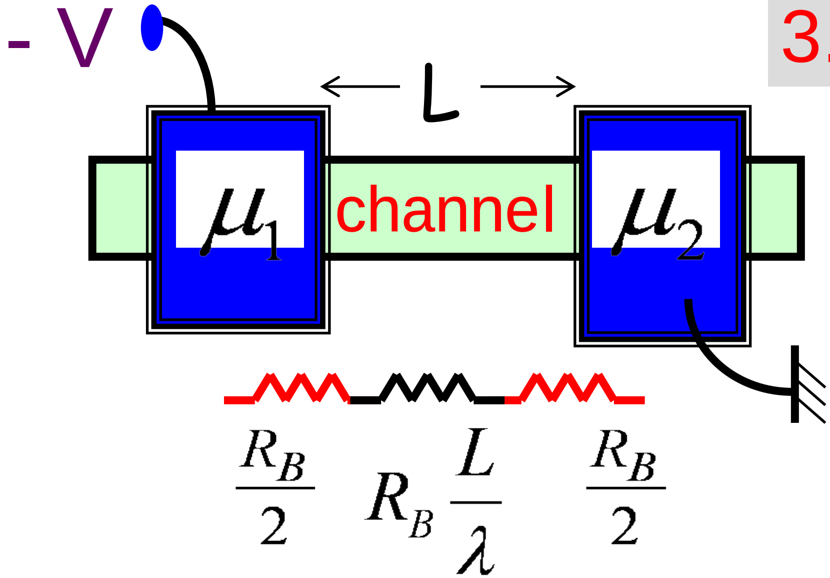
*Spring 2015*

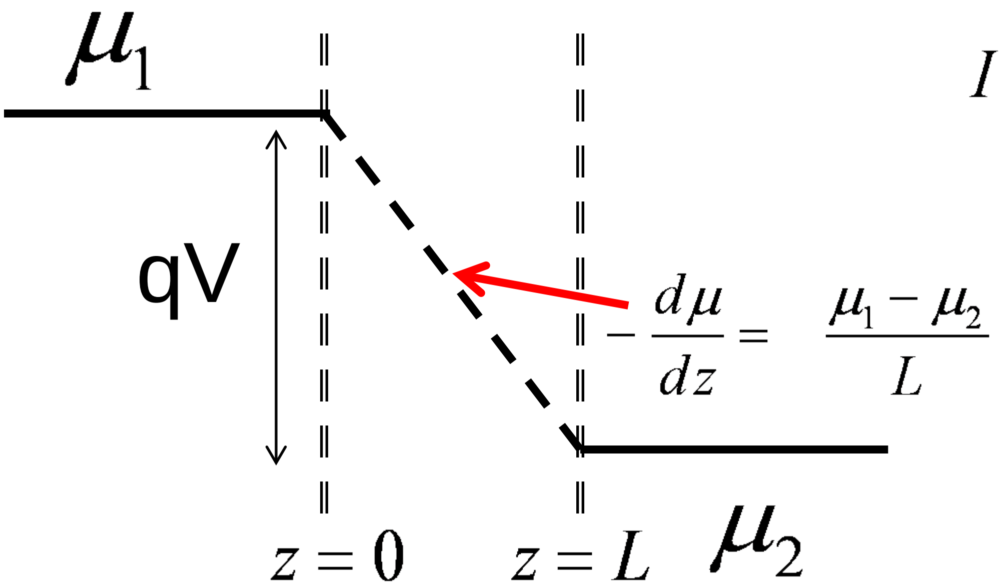
# 3.2a A New Boundary Condition




$$R_0 = R_B + R_B\frac{L}{\lambda}$$

Resistance is associated with

- Voltage drop: IR



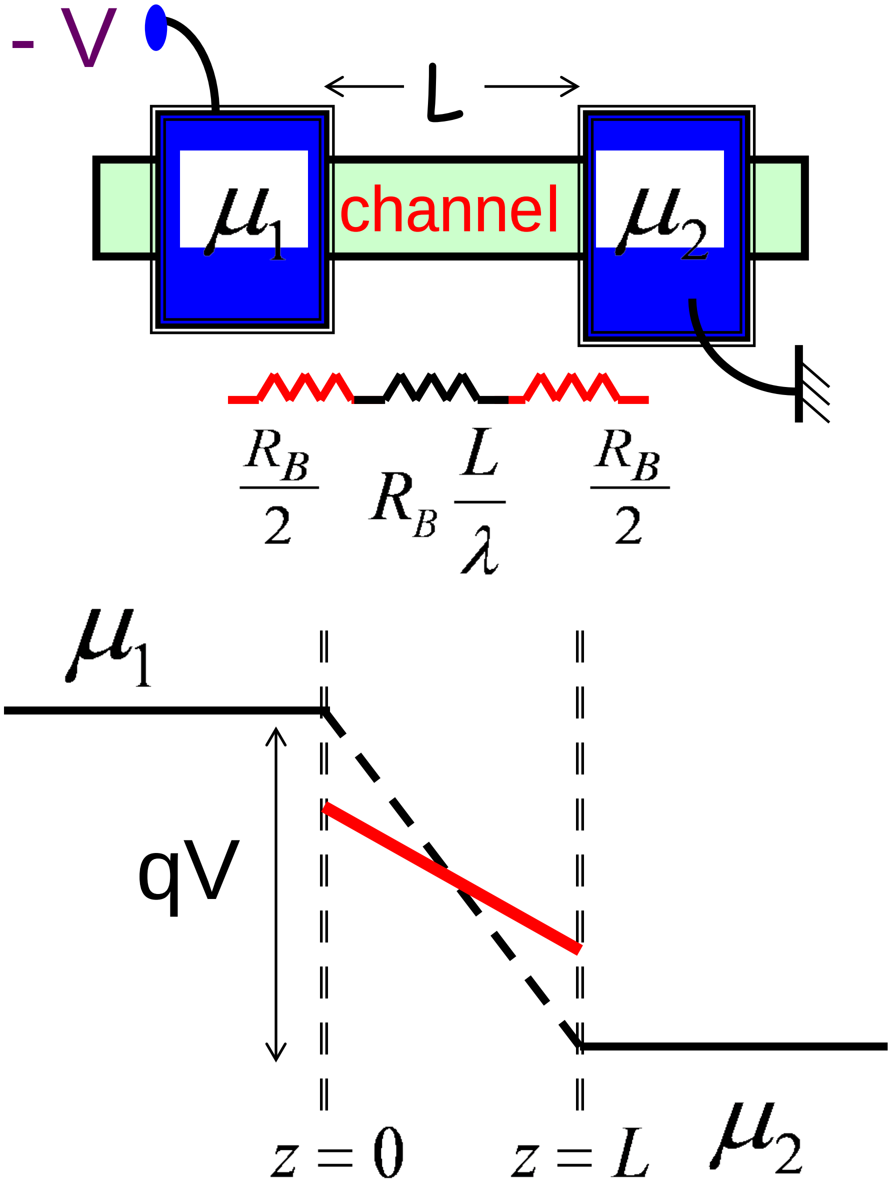

$$I = -\frac{\sigma_0 A}{q}\frac{d\mu}{dz}\ ,\quad \frac{dI}{dz} = 0$$

$$= \frac{1}{q}\frac{\sigma_0 A}{L}(\mu_1-\mu_2)$$

*Instead of*

$$I = \frac{1}{q}\frac{\sigma_0 A}{L+\lambda}(\mu_1-\mu_2)$$

# 3.2b A New Boundary Condition




*Equation is fine !*

$$I = -\frac{\sigma_0 A}{q}\frac{d\mu}{dz}, \quad \frac{dI}{dz} = 0$$

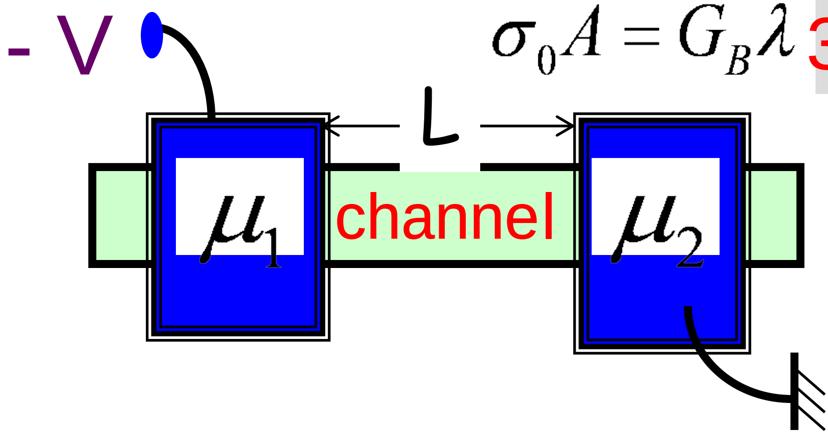
*Boundary condition is wrong !!*

$$\mu(z=0) = \mu_1$$

$$\mu(z=L) = \mu_2$$

# 3.2c A New Boundary Condition

$$\sigma_0 A = G_B \lambda$$




$$I = -\frac{\sigma_0 A}{q}\frac{d\mu}{dz}$$

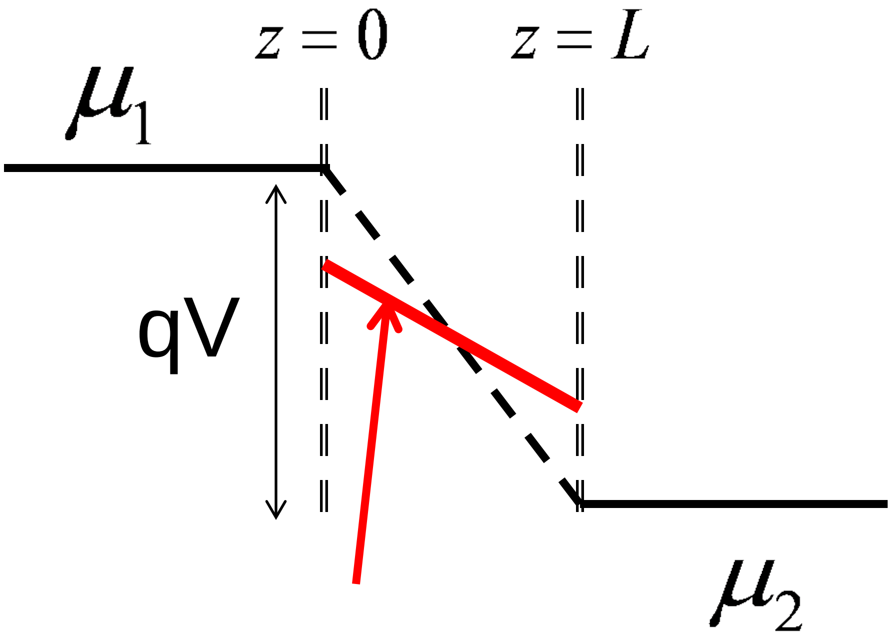
$$= \frac{1}{q}\frac{\sigma_0 A}{L}\left(\mu_1 - \mu_2 - qIR_B\right)$$

$$I\left(1 + \frac{\overset{\lambda}{\cancel{\sigma_0 A R_B}}}{L}\right) = \frac{1}{q}\frac{\sigma_0 A}{L}\left(\mu_1 - \mu_2\right)$$

$$\boxed{\rightarrow \quad I = \frac{1}{q}\frac{\sigma_0 A}{L+\lambda}\left(\mu_1 - \mu_2\right)}$$

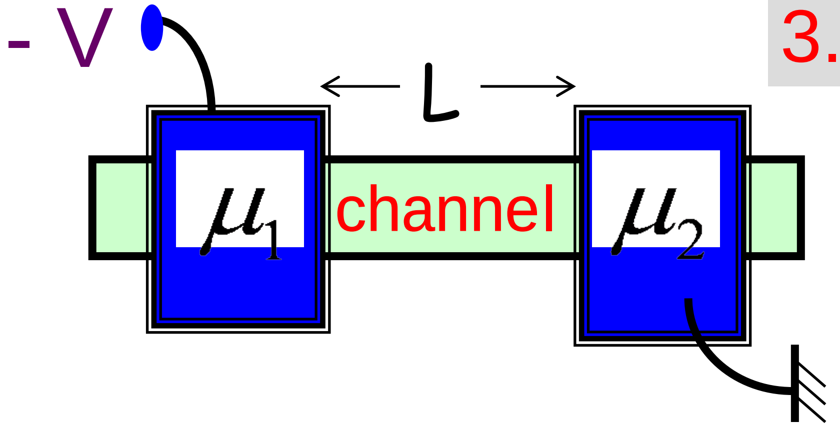
$$\mu(z=0) = \mu_1 - \frac{qIR_B}{2}$$

$$\mu(z=L) = \mu_2 + \frac{qIR_B}{2}$$



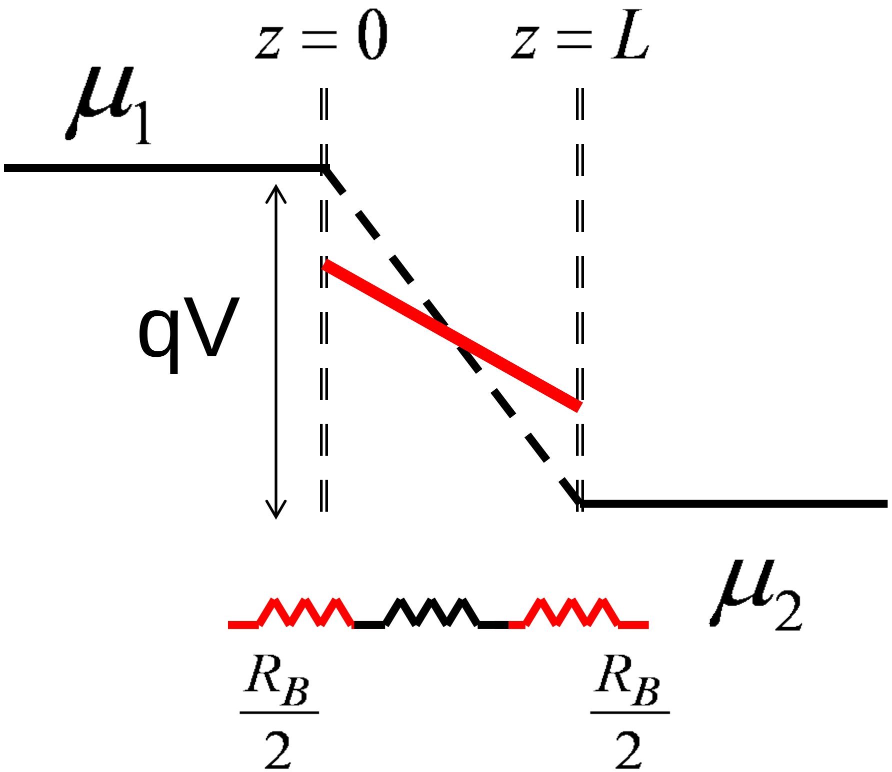

$$-\frac{d\mu}{dz} = \frac{\mu_1 - \mu_2}{L} - qIR_B$$

# 3.2d A New Boundary Condition




$$I = -\frac{\sigma_0 A}{q}\frac{d\mu}{dz}$$

*Same equation !*



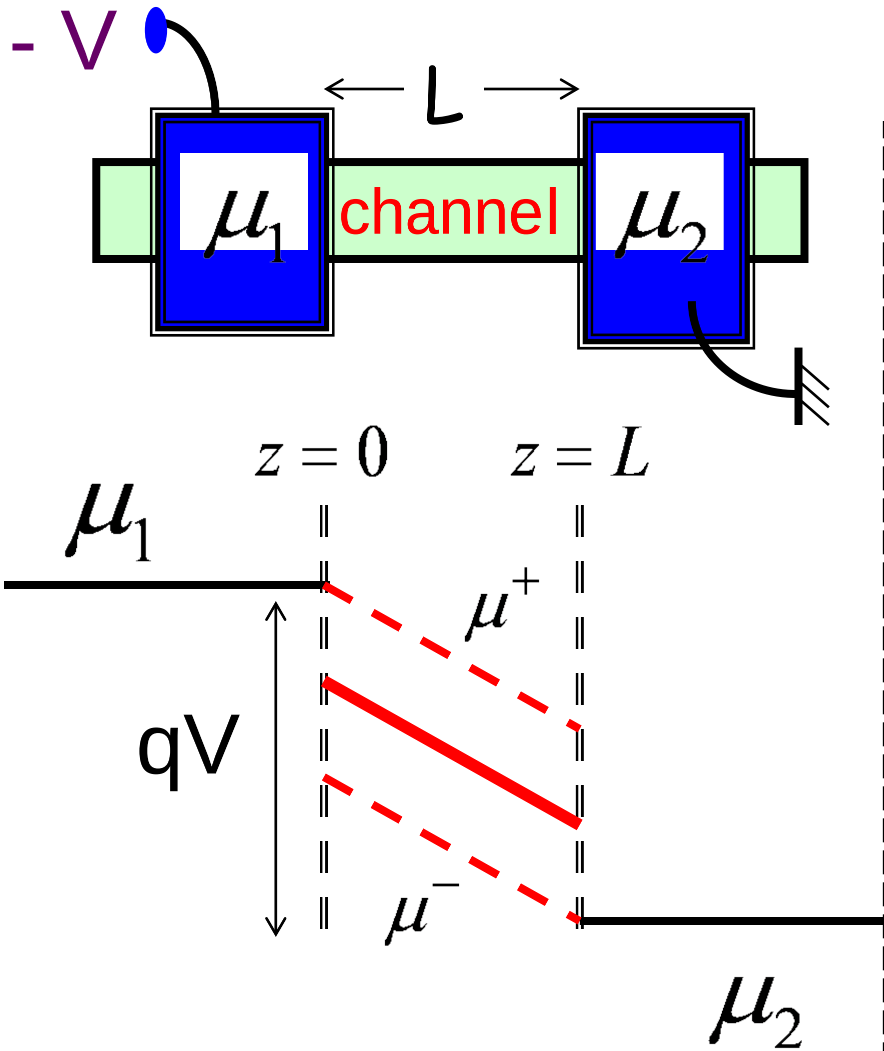

*New boundary condition !!*

$$\rightarrow \quad I = \frac{1}{q}\frac{\sigma_0 A}{L+\lambda}(\mu_1 - \mu_2)$$

$$\mu(z=0) = \mu_1 - \frac{q I R_B}{2}$$

$$\mu(z=L) = \mu_2 + \frac{q I R_B}{2}$$

# Coming up next ..



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*Spring 2015*

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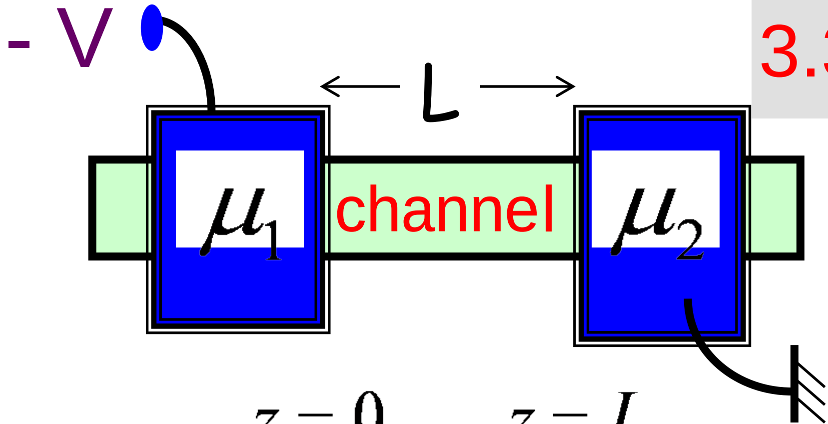
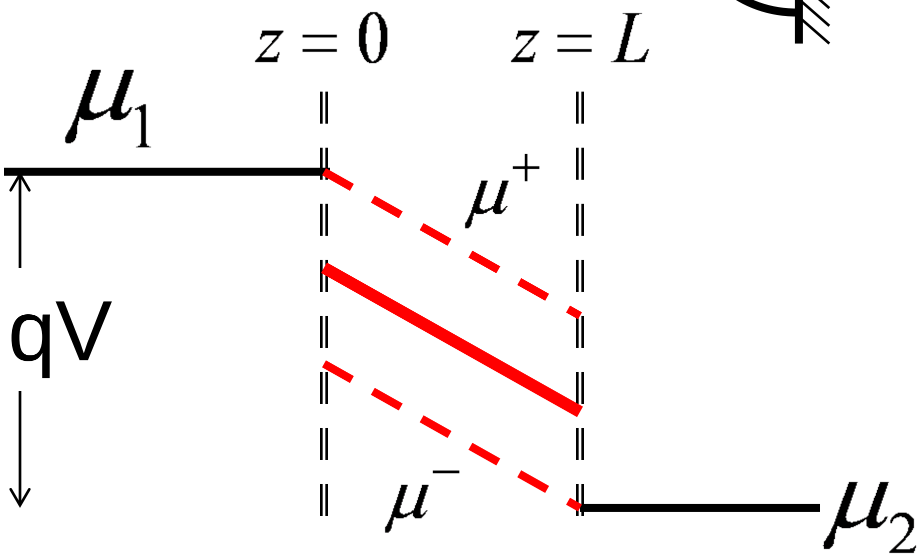
# 3.3a Quasi-Fermi Levels (QFL's)



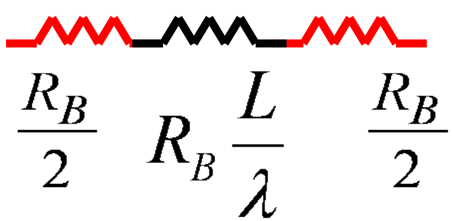




$$\mu^+(z=0) = \quad \mu_1$$
$$\mu^-(z=L) = \quad \mu_2$$

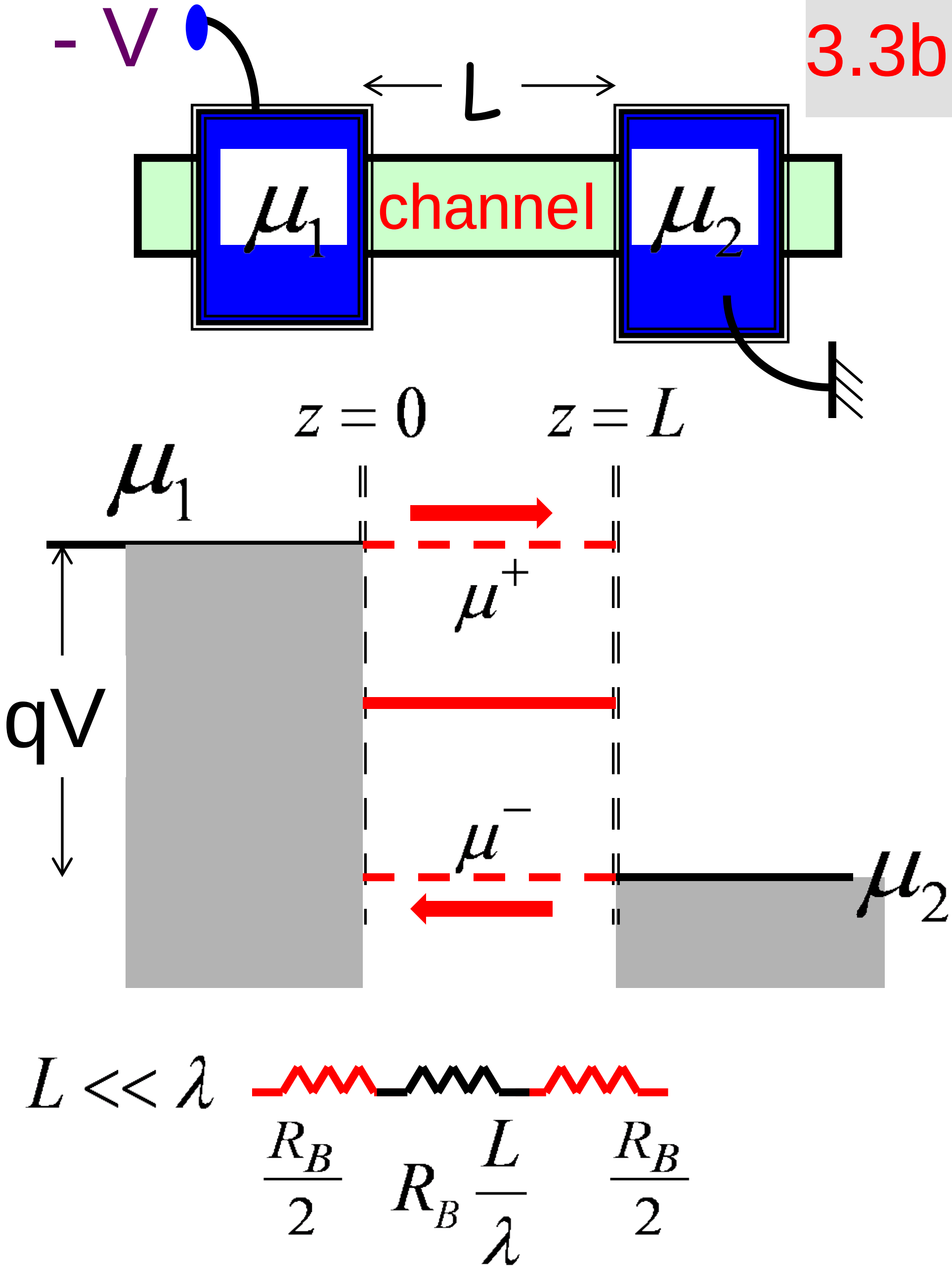
Lecture 4.4

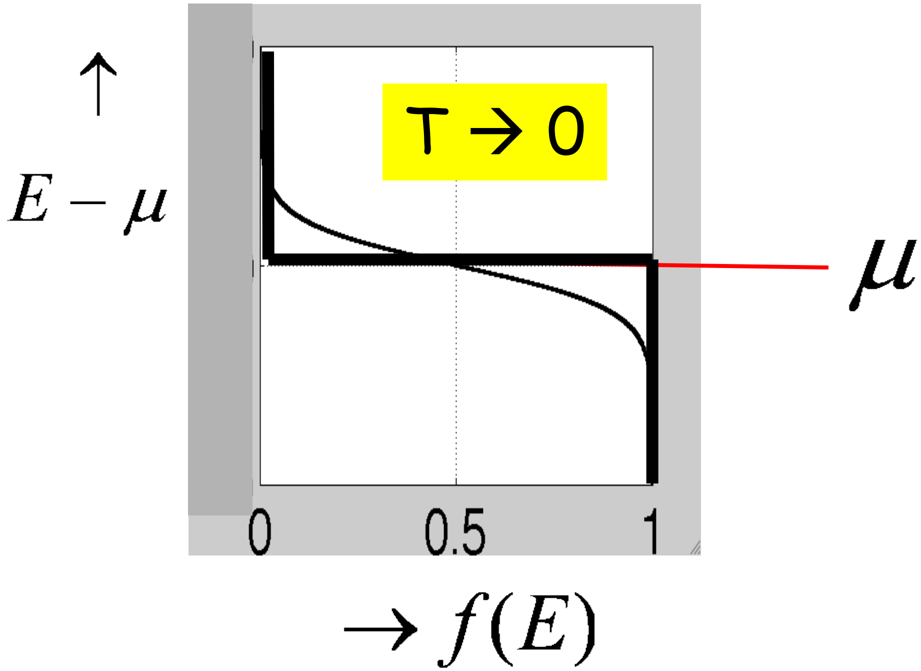
$$\mu(z=0) = \quad \mu_1 \quad - \quad \frac{q\,I\,R_B}{2}$$
$$\mu(z=L) = \quad \mu_2 \quad + \quad \frac{q\,I\,R_B}{2}$$

$L << \lambda$

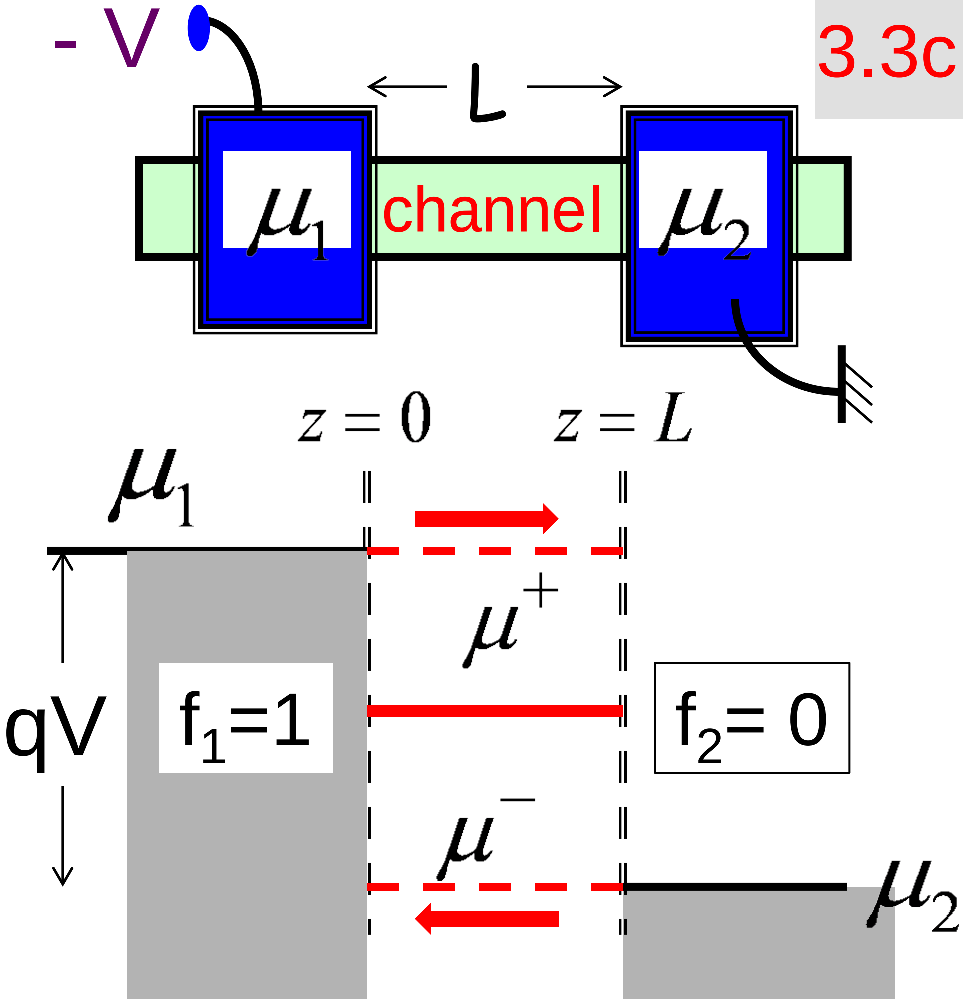


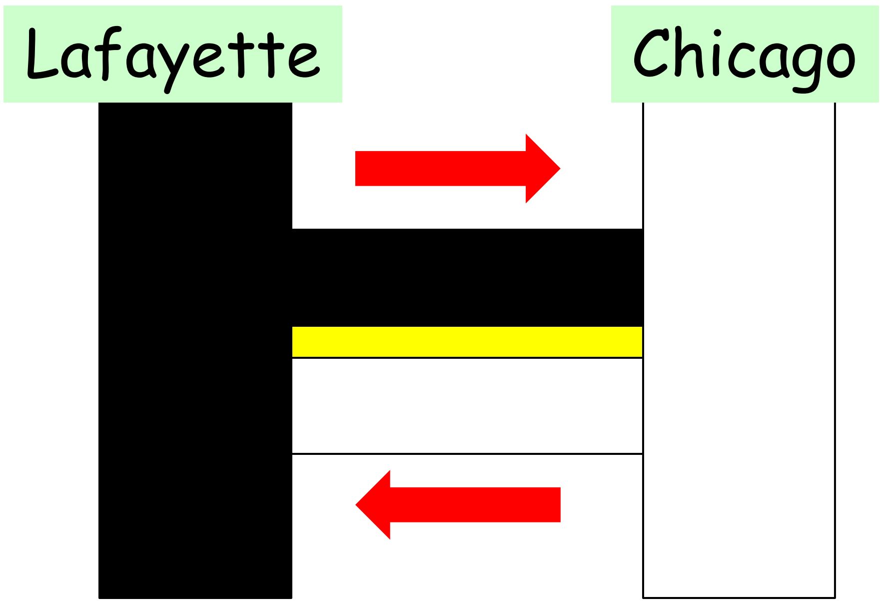
# 3.3b Quasi-Fermi Levels (QFL's)



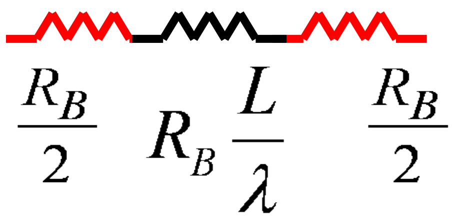



# 3.3c Quasi-Fermi Levels (QFL's)

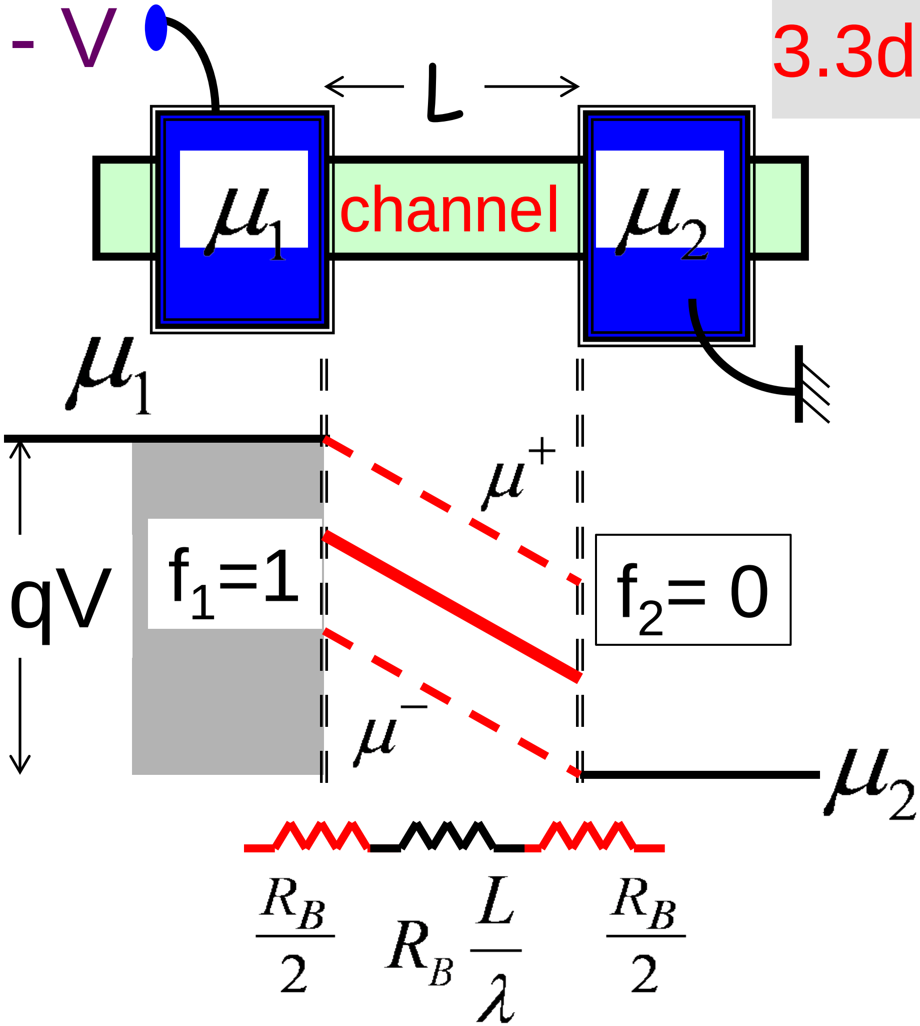



$L << \lambda$

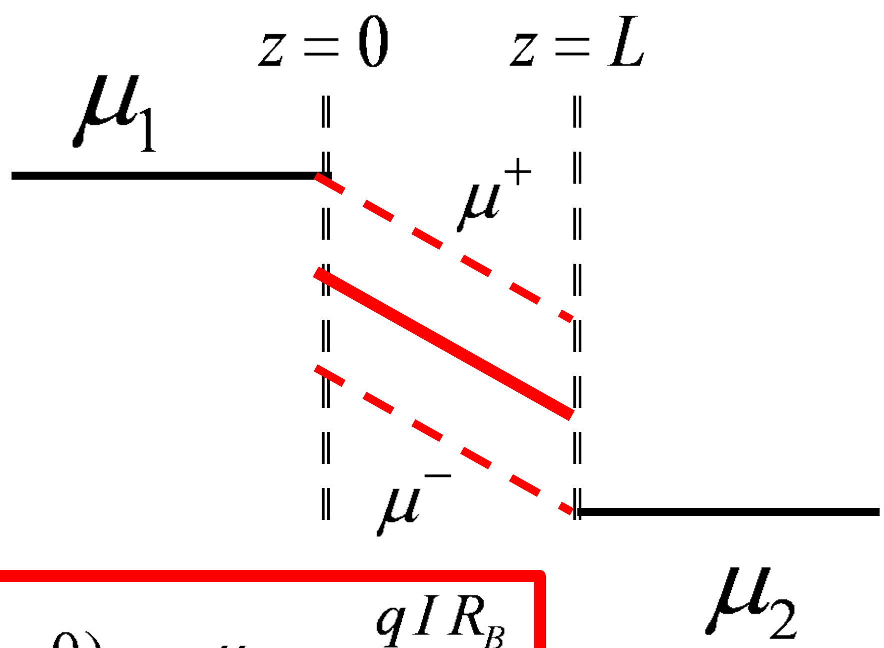


# 3.3d Quasi-Fermi Levels (QFL's)




$$\mu^+(z=0) = \mu_1$$
$$\mu^-(z=L) = \mu_2$$

## Coming up next ..




$$\mu(z=0) = \mu_1 - \frac{q I R_B}{2}$$

$$\mu(z=L) = \mu_2 + \frac{q I R_B}{2}$$

$$\mu^+(z=0) = \mu_1$$

$$\mu^-(z=L) = \mu_2$$

*QFL Boundary Conditions*

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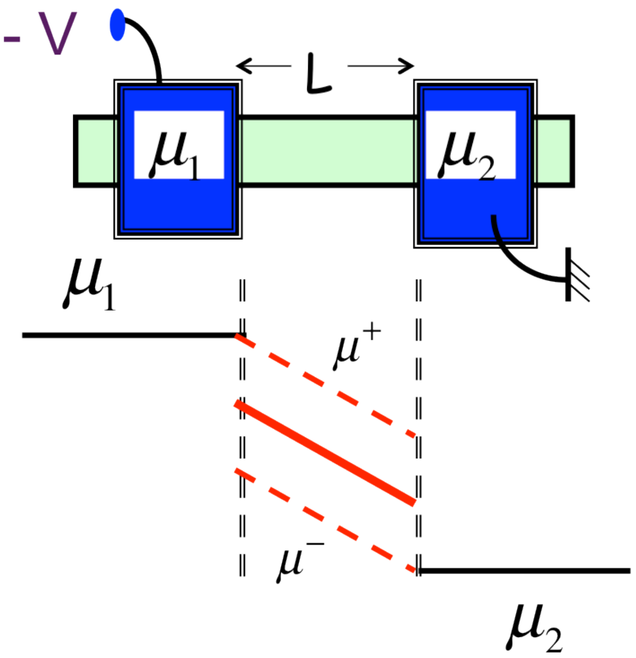
# 3.4a Current from QFL's




$$\mu = \frac{\mu^+ + \mu^-}{2}$$

$$= \mu^+ - \frac{\mu^+ - \mu^-}{2} = \mu^+ - \frac{qIR_B}{2}$$

$$= \mu^- + \frac{\mu^+ - \mu^-}{2}$$

$$= \mu^- + \frac{qIR_B}{2}$$

z = 0

z = L

$$I = \frac{G_B}{q}\left(\mu^+ - \mu^-\right)$$

$$\rightarrow \quad \mu^+ - \mu^- = qIR_B$$

QFL Boundary Conditions

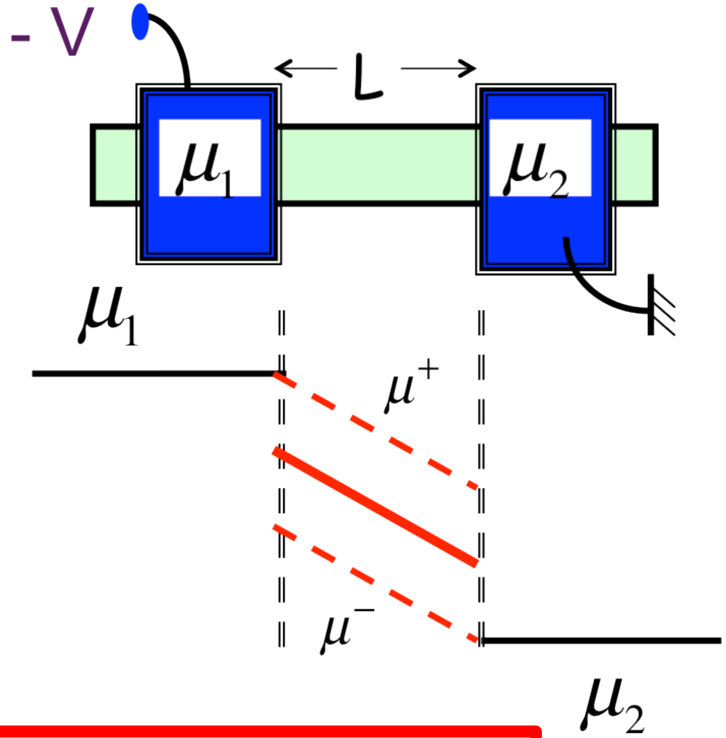
$$\mu^+(z=0) = \mu_1$$

$$\mu^-(z=L) = \mu_2$$

$$\mu(z=0) = \mu_1 - \frac{qIR_B}{2}$$

$$\mu(z=L) = \mu_2 + \frac{qIR_B}{2}$$

# 3.4b Current from QFL's




$$\boxed{I = \frac{G_B}{q}\left(\mu^+ - \mu^-\right)}$$

$$I^+ = \frac{q}{h} M(E) * f^+(E)\,dE$$

$$I^+ = q * \frac{D(E)\bar{u}}{2L} * f^+(E)\,dE$$

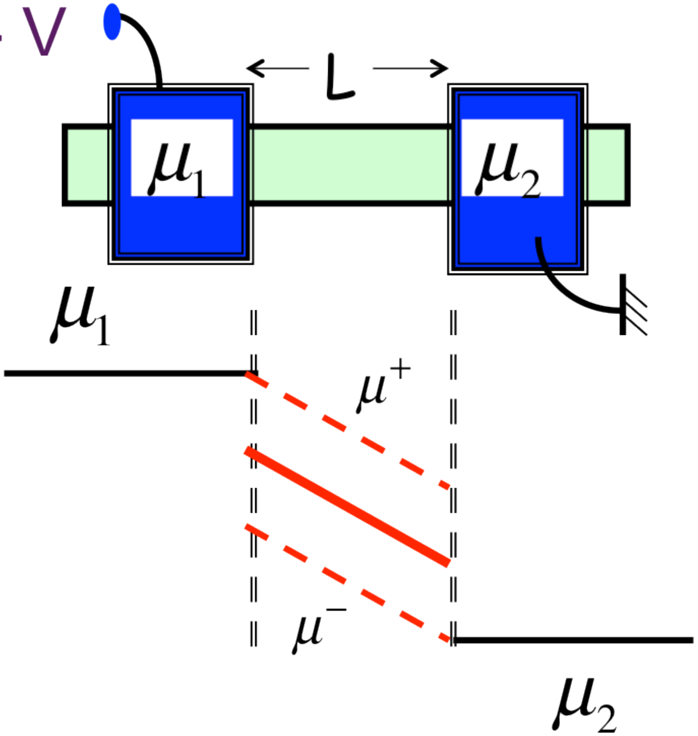
$$I^+ = q * \underbrace{\frac{D(E)\,dE}{2} * f^+(E)} * \underbrace{\frac{1}{L/\bar{u}}}$$

*Number of right-moving electrons*

*Time spent*

$$\frac{Graduates}{year} = \#\,of\ students * \frac{1}{Years\ spent}$$

# 3.4c Current from QFL's




$$I^+ = \frac{q}{h} M(E) * f^+(E)\, dE$$

$$I = I^+ - I^-$$

$$= \frac{q}{h} M(E) * \left( f^+(E) - f^-(E) \right) dE$$

$$\approx \frac{q}{h} M(E) * \left( -\frac{\partial f_0}{\partial E} \right) dE \left( \mu^+ - \mu^- \right)$$

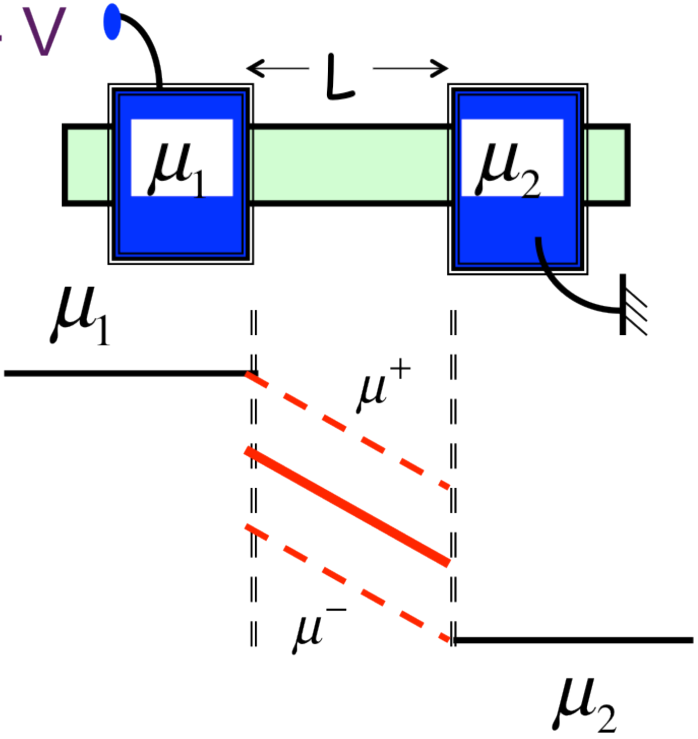
$$\boxed{I = \frac{G_B}{q} \left( \mu^+ - \mu^- \right)}$$

$$I = \frac{q}{h} M_0 \left( \mu^+ - \mu^- \right)$$

$$G_B = \frac{q^2}{h} M_0$$

$$M_0 = \int M(E) * \left( -\frac{\partial f_0}{\partial E} \right) dE$$

# 3.4d Current from QFL's




$$\mu^+ - \mu^- = (\mu_1 - \mu_2)\frac{\lambda}{L+\lambda}$$

$$\mu^+ - \mu^- = (\mu_1 - \mu_2)\frac{G_0}{G_B}$$

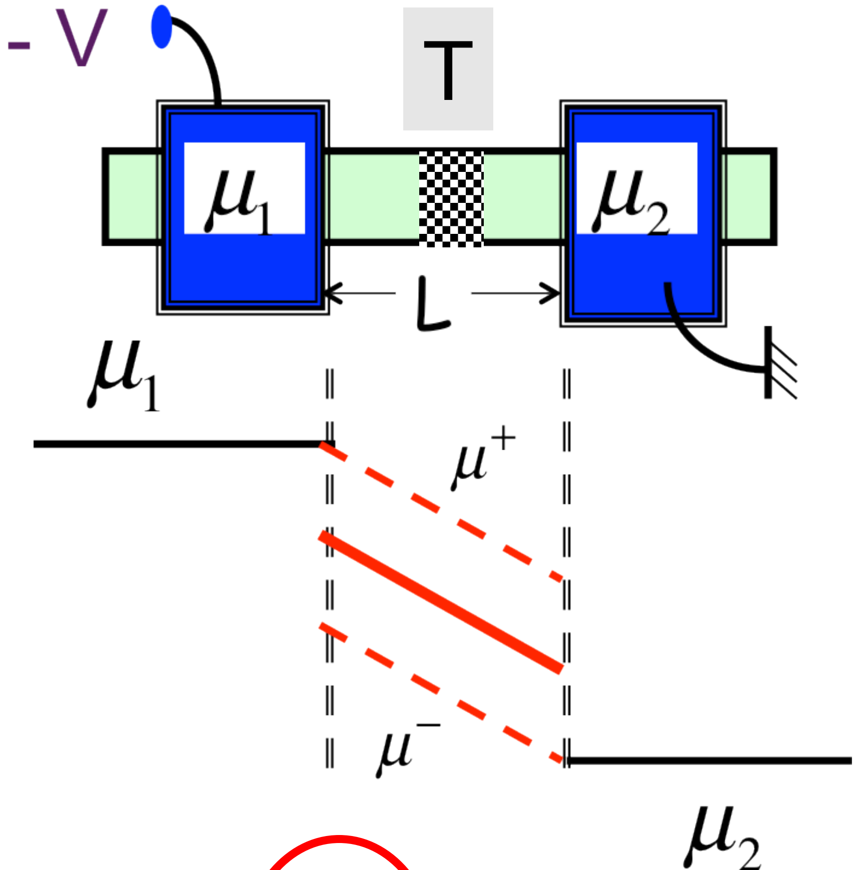
$$\boxed{I = \frac{G_B}{q}\left(\mu^+ - \mu^-\right)}$$

$$G_B = \frac{q^2}{h}M_0$$

$$I = \frac{G_0}{q}\left(\mu_1 - \mu_2\right)$$

$$G_0 = G_B\frac{\lambda}{L+\lambda}$$

## *Coming up next ..*




$$G_0 = G_B \left( \frac{\lambda}{L+\lambda} \right)$$

T

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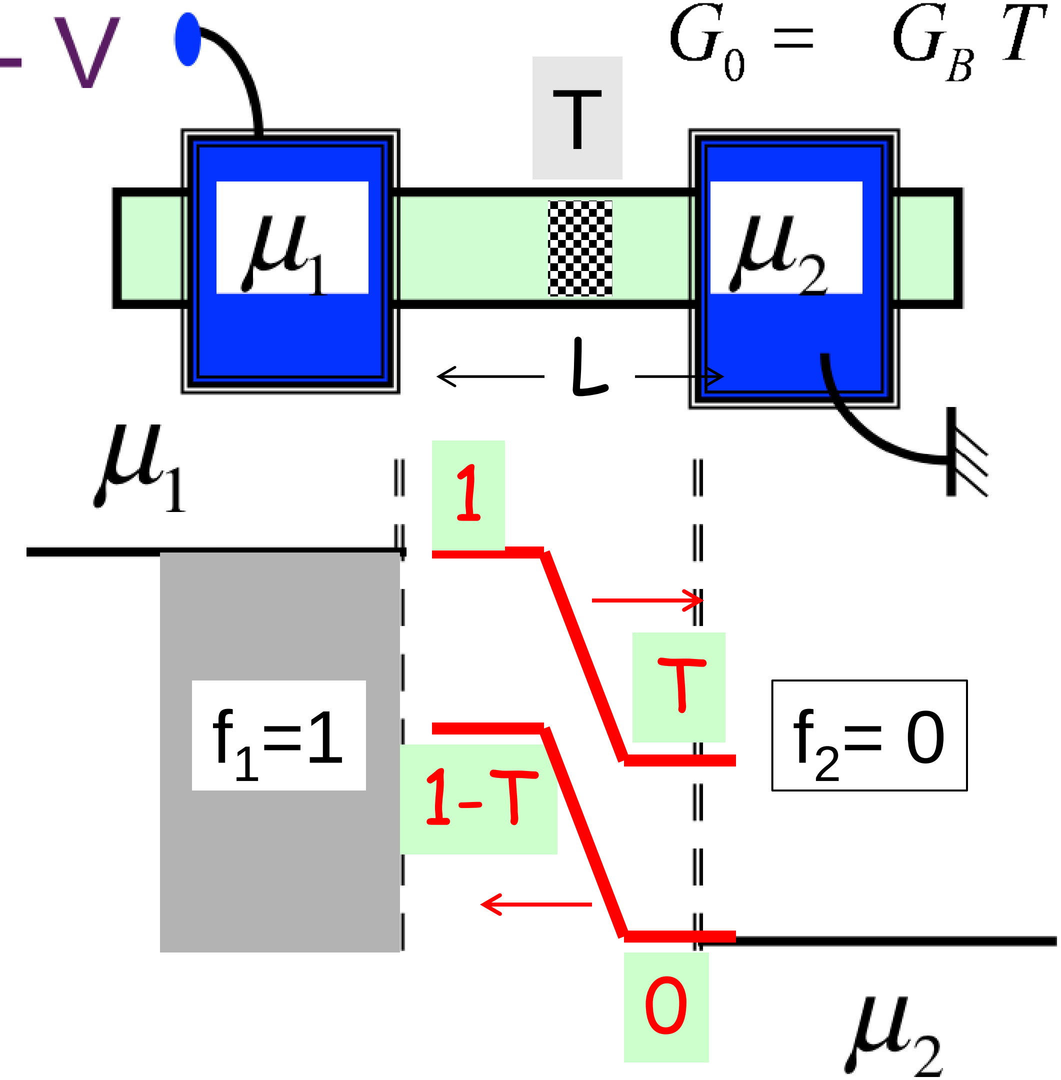
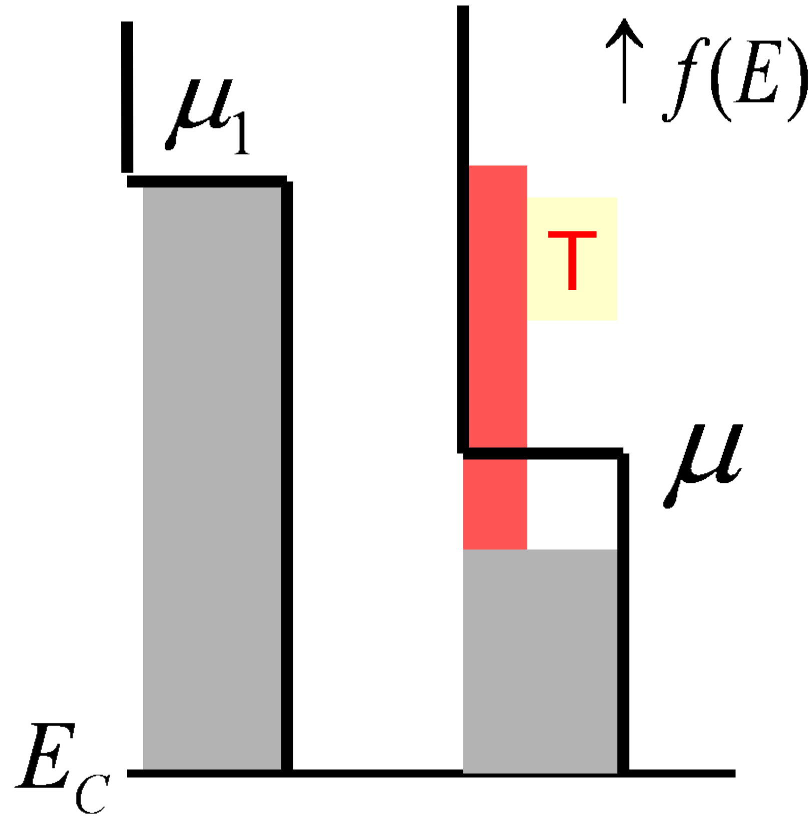
# 3.5a Landauer Formulas

$$G_0 = G_B\, T$$




$$G_0 = G_B \left(\frac{\lambda}{L+\lambda}\right)$$

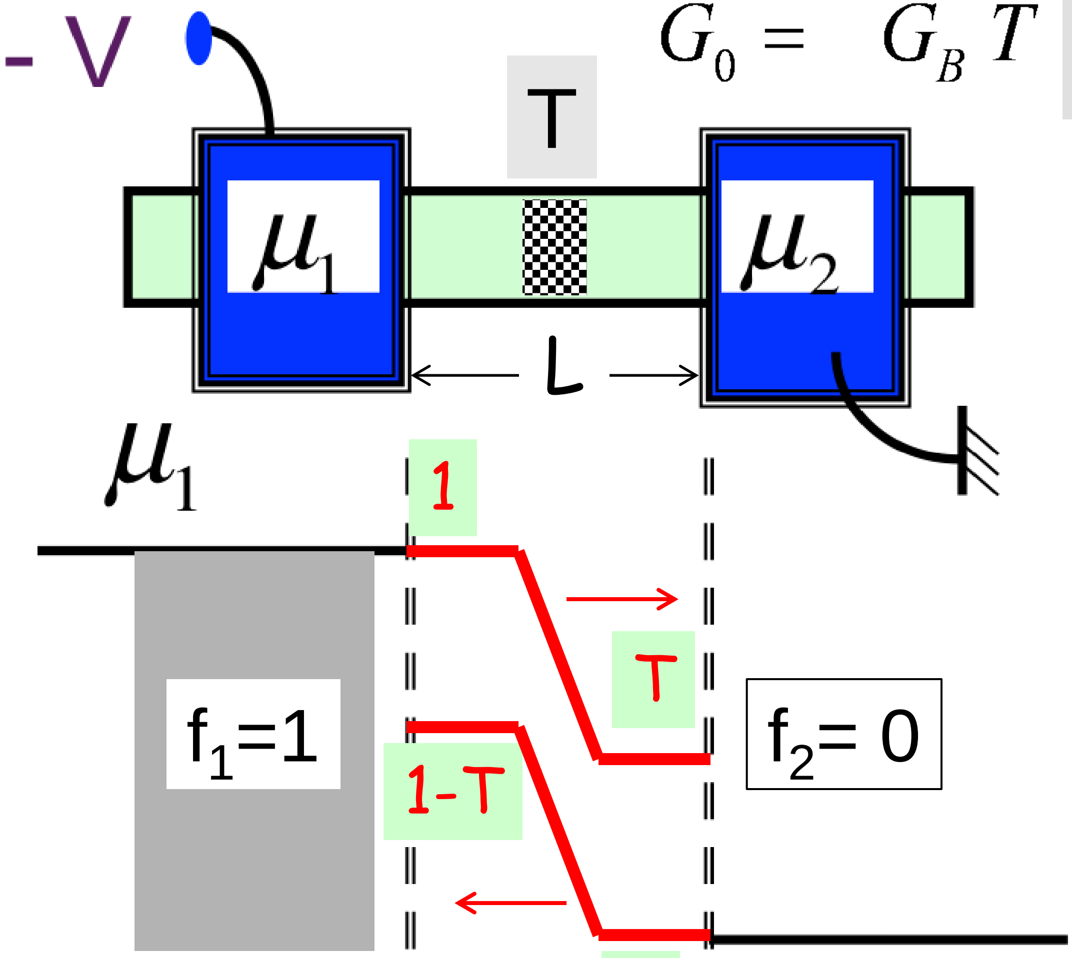
T




$$\boxed{\mu - \mu_2 \equiv qV\, f}$$

# 3.5b Landauer Formulas

$$G_0 = G_B\,T$$




$$\mu - \mu_2 \equiv qV\,f$$

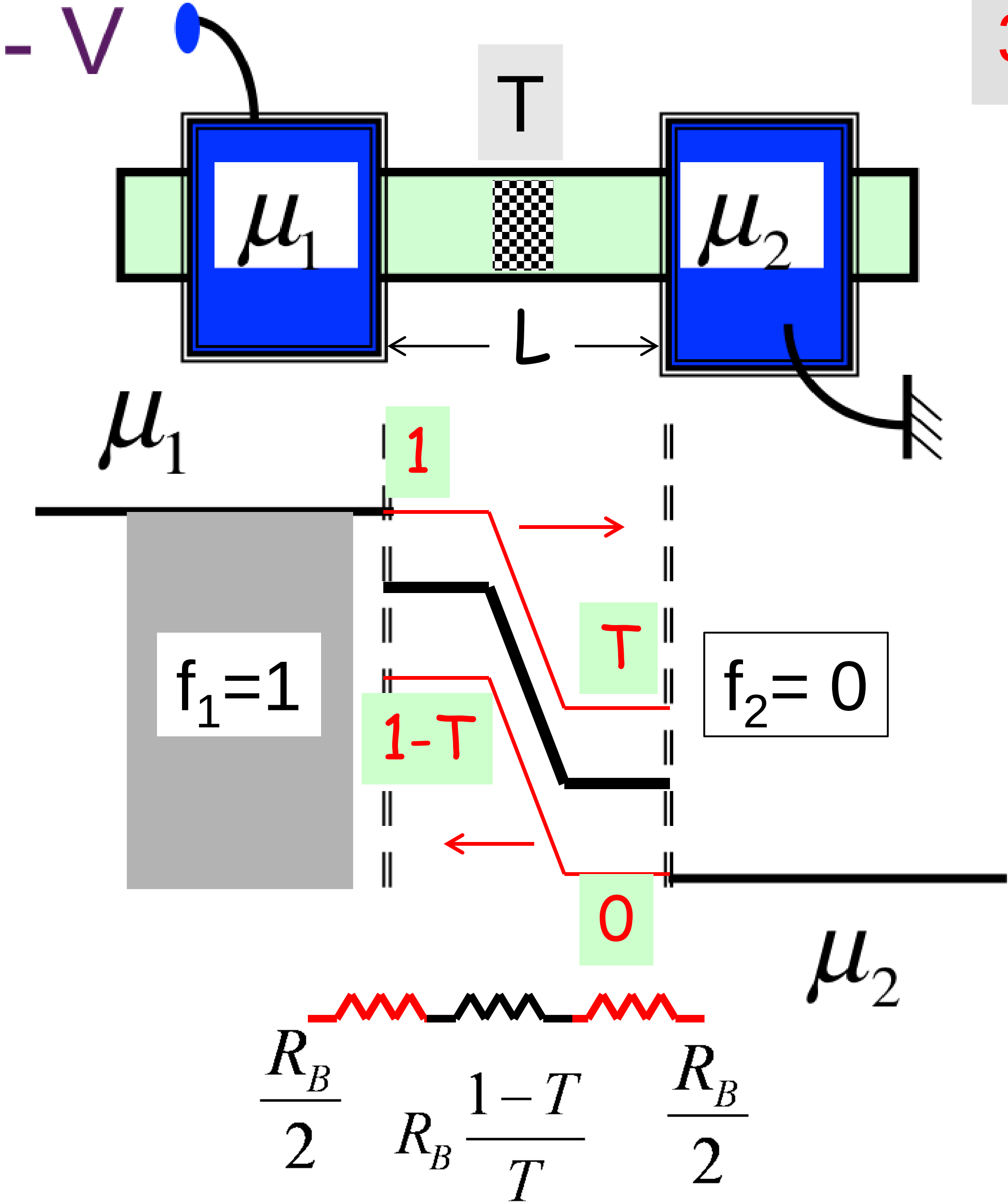
$$\delta V = V\,(1-T)$$

$$I = \frac{G_B}{q}\underbrace{\left(\mu^+ - \mu^-\right)}_{qV\,T}$$

$$\frac{I}{V} = G_B\,T \quad\rightarrow\quad R = R_B\,\frac{1}{T}$$

$$\frac{I}{\delta V} = G_B\,\frac{T}{1-T} \quad\rightarrow\quad R_S = R_B\,\frac{1-T}{T}$$

# 3.5c Landauer Formulas




$$\mu - \mu_2 \equiv \quad qV\,f$$

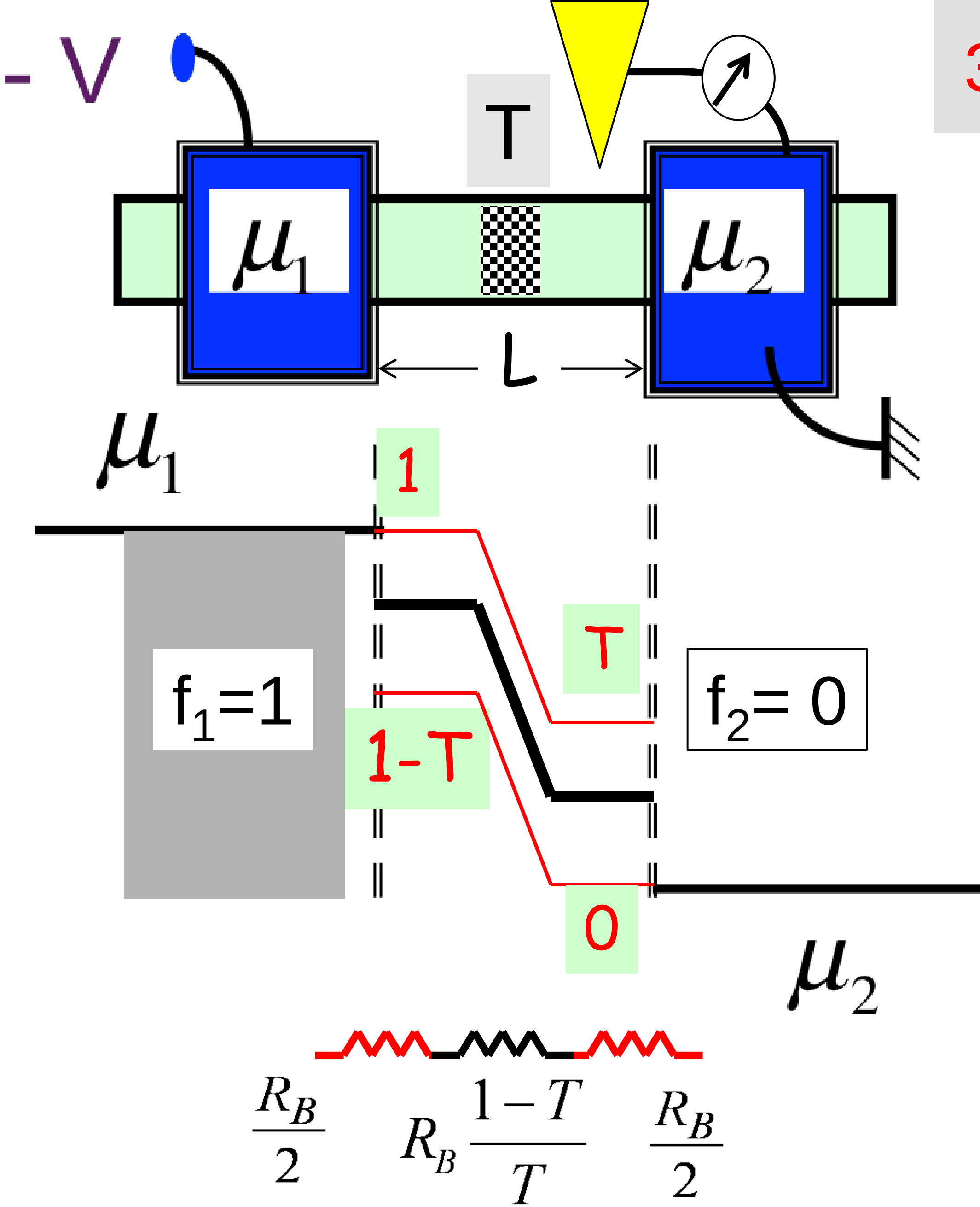
$$\delta V = \quad V\,(1-T)$$

$$R = \quad R_B \frac{1}{T}$$

$$R_S = \quad R_B \frac{1-T}{T}$$

$$R - R_S = \quad R_B$$

# 3.5d Landauer Formulas

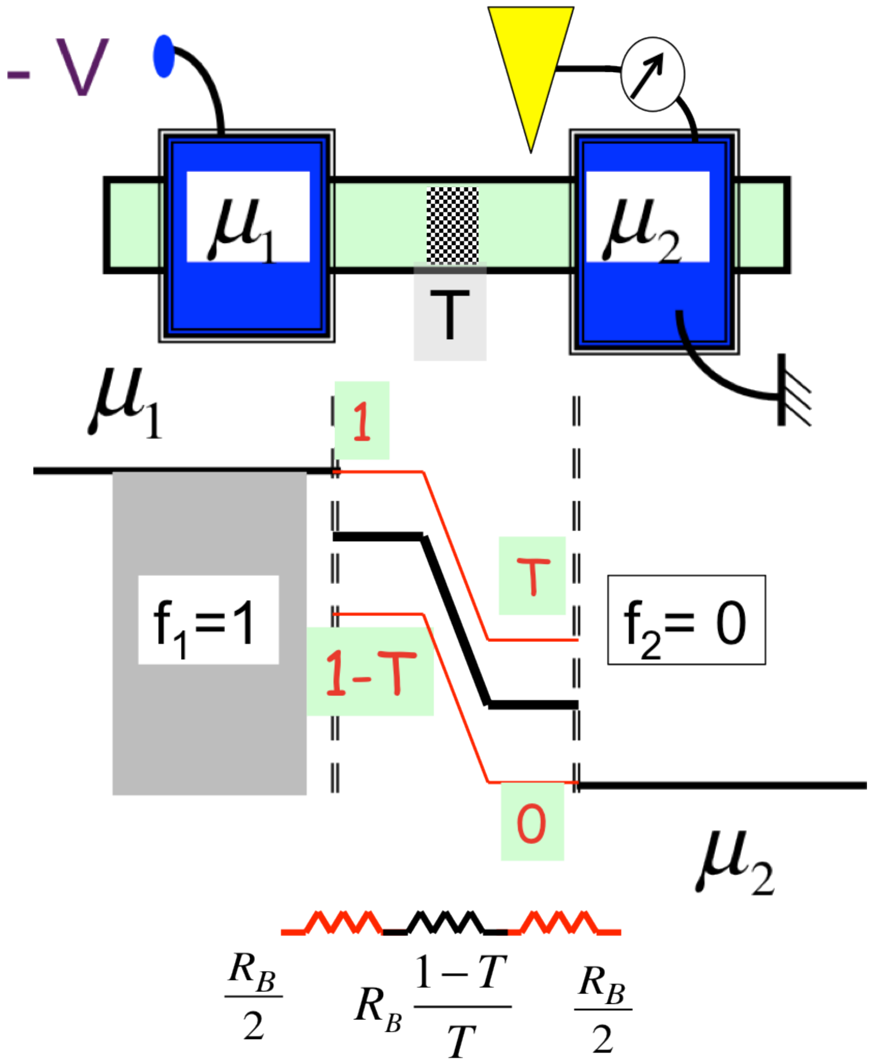



*Two approaches to making it “concrete”*

4.6. Measure with a probe

4.7. Electrostatic potential

# Coming up next ..

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