Second Order Linear Equations

$y'' - 5y' + 6y = 0 \quad y = y(x)$

$\rightarrow$ search soln $y(x) = e^{rx}$

$y' = re^{rx}$

$y'' = r^2 e^{rx}$

Plug in

$r^2 e^{rx} - 5re^{rx} + 6e^{rx} = 0$

$r^2 - 5r + 6 = 0$

$r = 2, r = 3 \rightarrow e^{2x}, e^{3x}$

$y = c_1 e^{2x} + c_2 e^{3x}$

real #'s determined by initial conditions

In general $y'' - ay' + by = 0 \rightarrow y = e^{rx}$

$r^2 + ay + b = 0$

with soln r_1, r_2

① if $r_1 \neq r_2 \in \mathbb{R} \Rightarrow y = c_1 e^{r_1 x} + c_2 e^{r_2 x}$

② if $r_1 = r_2 = r \Rightarrow y = c_1 e^{rx} + xe^{rx} c_2$

③ if $r_{12} = \alpha \pm i\beta$ (complex number) $\rightarrow y = c_1 e^{\alpha x} \cos \beta x + c_2 e^{\alpha x} \sin \beta x)$

example of this

$y'' + 2y' + 2y = 0$

$r^2 + 2r + 2 = 0$

$(r+1)^2 = -1$

$r + 1 = \pm i$

$r = -1 \pm i$

$\alpha = -1, \beta = 1$

$e^{-x} \cos x, e^{-x} \sin x$

Euler Eqn

a b

find general soln: $x^2y'' + 5xy' + 8y = 0$

trial x^n

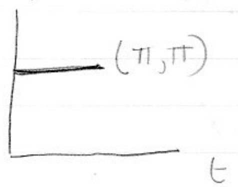
$n^2 + (5-1)n + 8$

$= n^2 + 4n + 8 = 0$

$r = \frac{-4 \pm \sqrt{16-32}}{3} = -2 \pm 2j$

$e^{\alpha + i\beta} = e^{\alpha}(\cos\beta + \sin\beta)$

Laplace Transforms



$F(s) = \mathcal{L}(f) = \int_0^\infty e^{-st} f(t)\,dt$

convolution

$y' = 2\int_0^x e^{t-x} y(t)\,dt \quad y(0) = 3$

$(f * g)(x) = \int_a^b f(t-x)\,g(t)\,dt$

$y' = 2e^{-x} * yx$