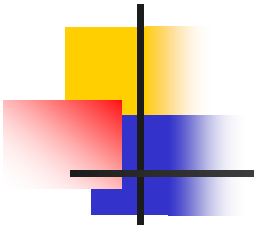


Preparatory material for FE Examination

Engineering Economics III

(Depreciation of Civil Engineering Systems)

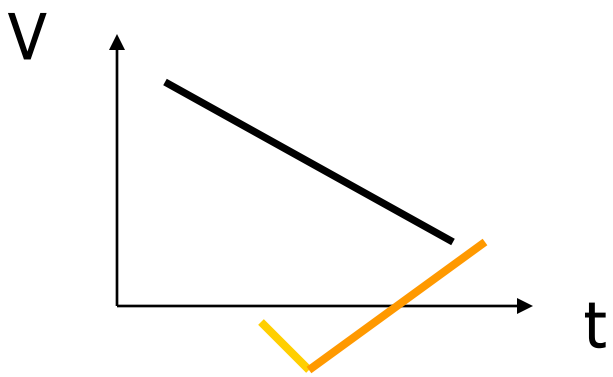
S. Labi
Purdue University

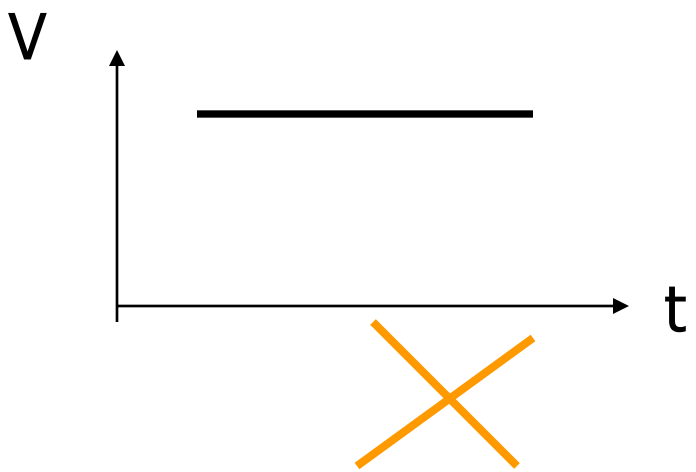
Definition of Depreciation

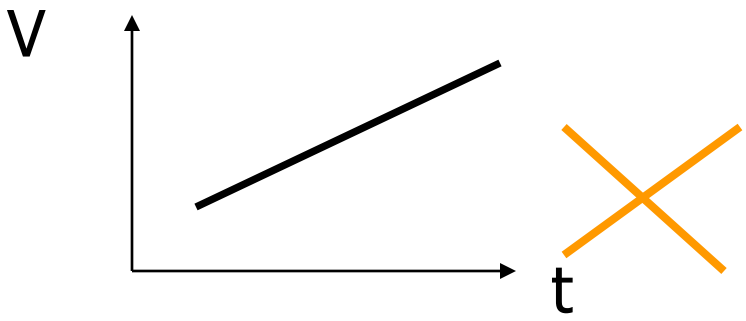
Defined as the systematic reduction in the value of an asset over a period of time.

Reduction means …

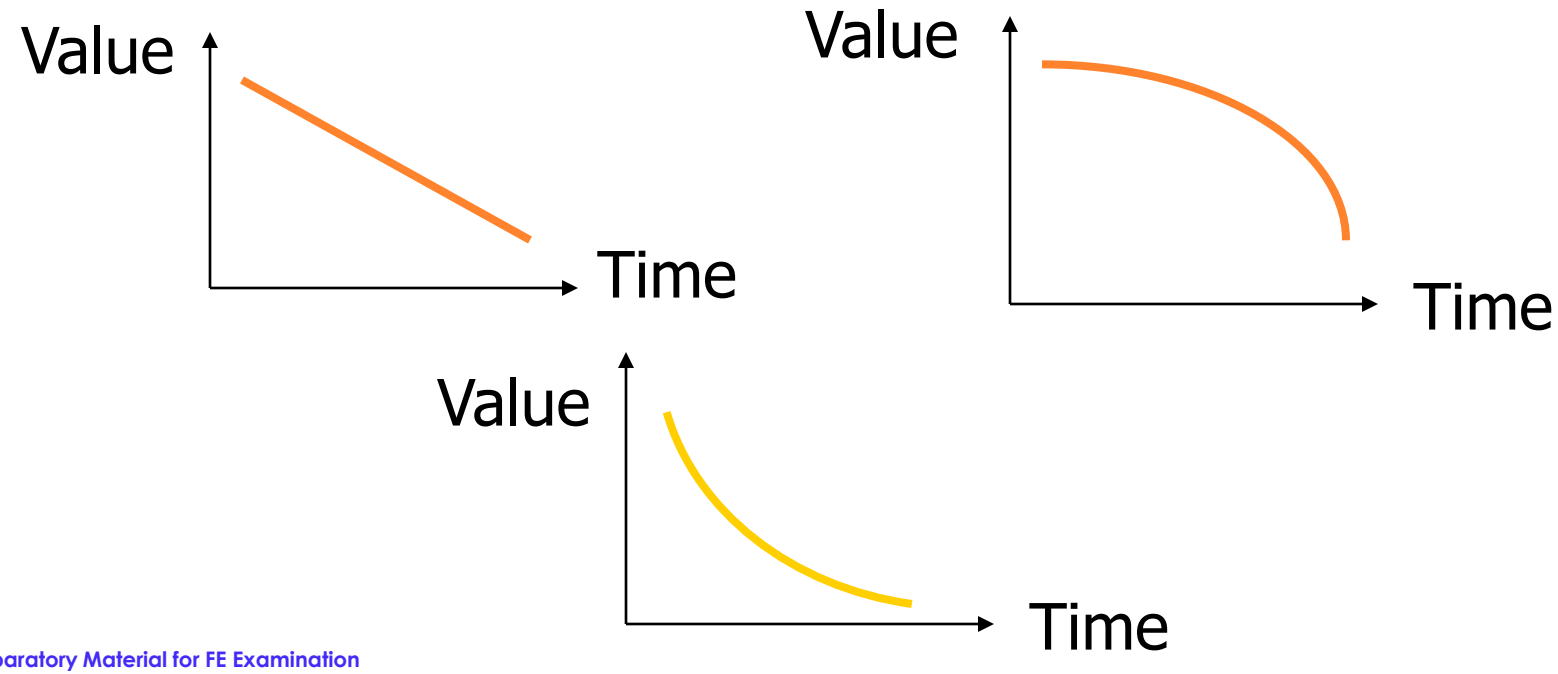
Decreasing Value



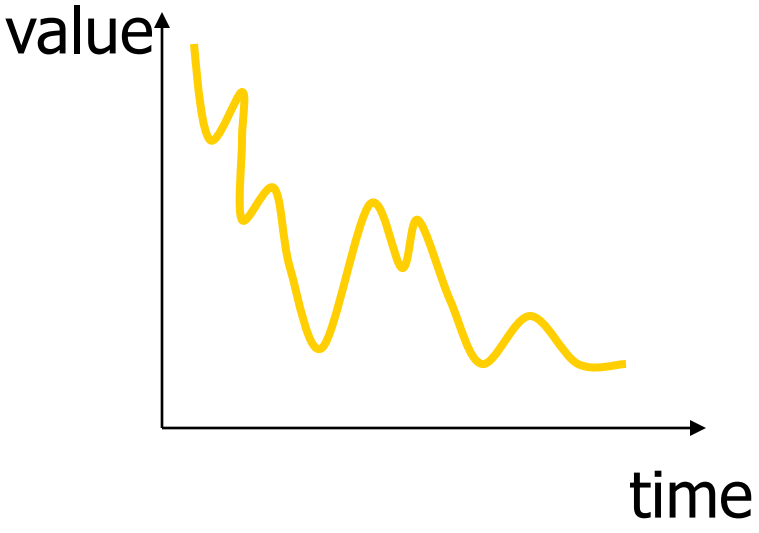


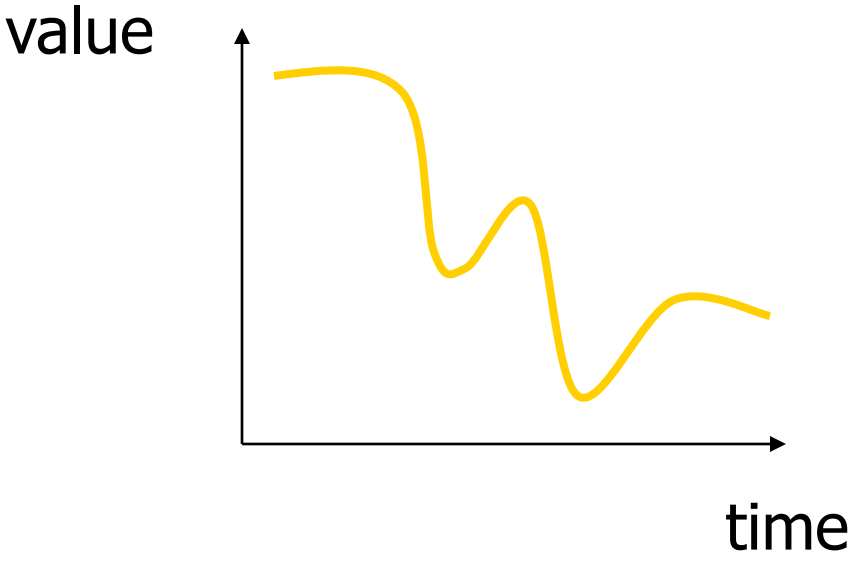


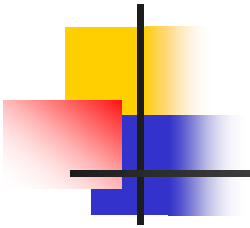
Systematic means: in a non-random fashion, like these …



.. And not like these:







Therefore an asset is said to depreciate only when …

… the value of the asset reduces over time

… the reduction is systematic

Why do assets depreciate?

- b'cos they wear out over time
  (through usage and weather effects)
  e.g., equipment, shoes

- b'cos they obsolete over time
  (e.g., computers)

Therefore it is clear that assets depreciate even when interest rate is zero (i.e., if inflation is 0 and there is no opportunity cost).

Do all assets depreciate?

NO!

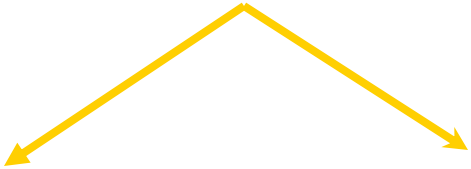
Assets that depreciate	Assets that don't depreciate
Cars	Land
Computers	Antiques
Equipment	
etc.	

Assets that don't depreciate

- Land does not depreciate because it does not wear out over time or become obsolete with time

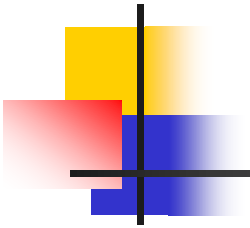
- Antiques are assets that depreciate but were manufactured so long ago that a special historic value is attached to them

ASSETS THAT DEPRECIATE



Real Assets
Buildings
Structural parts
of buildings
CE Structures

Personal Assets
All other assets other than real assets, e.g.,
Cars
Computers
Equipment, etc.



ASSETS THAT DEPRECIATE (cont'd)

Note that …

…. Both real and personal assets may be owned and used either by individuals or by businesses

Mathematical Definition of Depreciation

$$D = P - S$$

Where D = total depreciation over the payment or planning period

P = Initial value of the asset

S = Salvage value of the asset i.e., value of the asset at the end of the planning period)

Note that P and S should be expressed at the same time value!

Example: What is the depreciation of a $3000 computer bought in 1993 if its salvage value is $50 in 2003?

Initial value of asset = $3000 (1993$)

Salvage value of asset = $50 (2003$)

"Present" worth of Salvage value of asset (Worth in $1993)

= $50*PWF(5%, 10yrs) = $30.70

Depreciation = $3000 - $ 30.70 = $ 2969.30

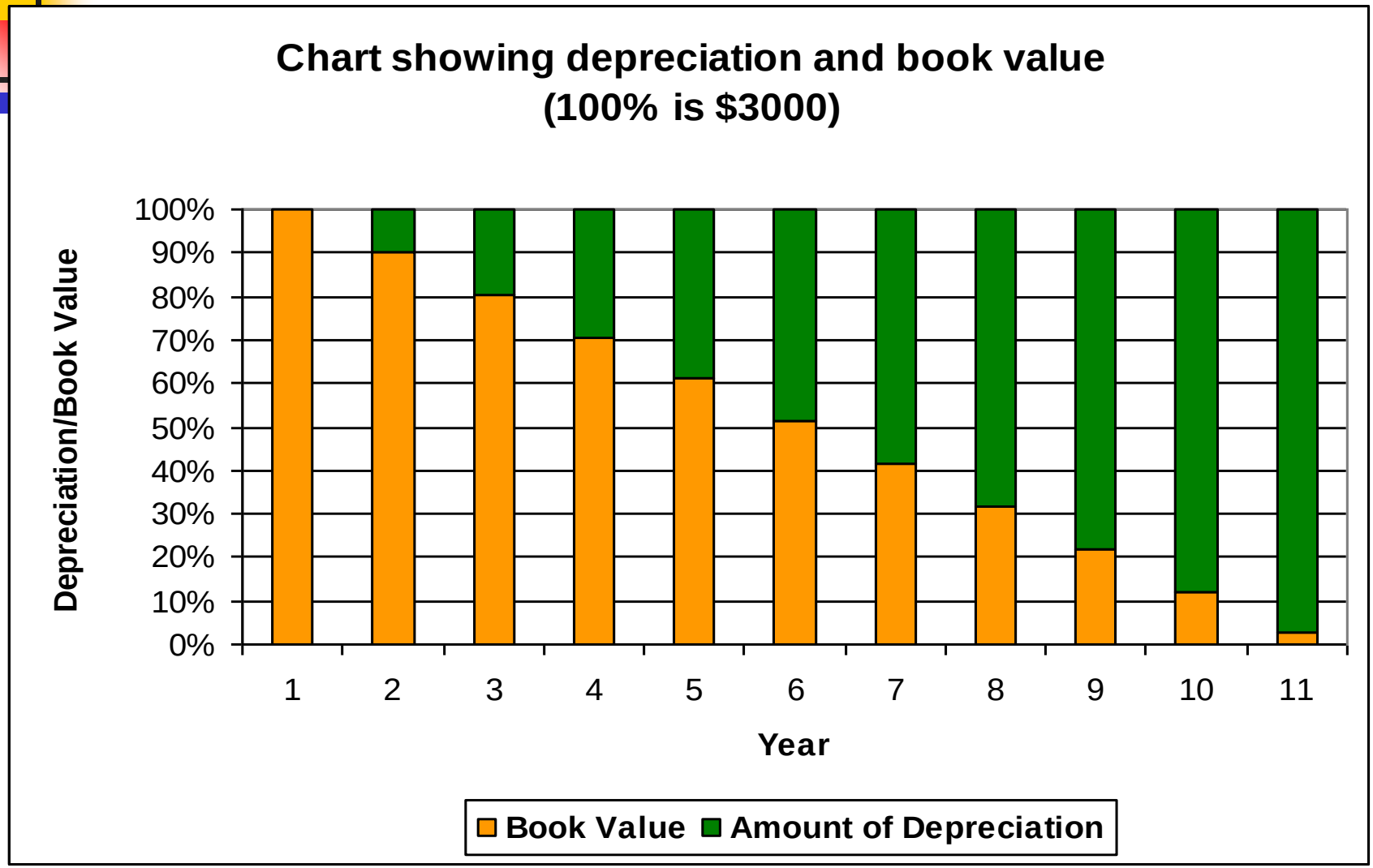
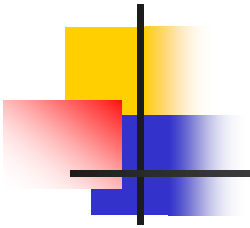


Chart showing depreciation and book value
(100% is $3000)
Depreciation/Book Value
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
1
2
3
4
5
6
7
8
9
10
11
Year
Book Value
Amount of Depreciation



Observations from Chart:

Book Value at any Year

= Initial value – Depreciation

Book Value at the end of planning period

= Salvage value

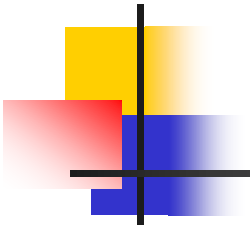
Example:

Calculate the book value of a truck that has initial value of $50,000 and has undergone depreciation of 30% its initial value.

Ans: TD = 30%*50,000 = 15,000

BV = P - TD

= 50,000- 15,000 = $35,000



Laura bought a new car in 1983 for $10,000. The salvage value of the car now (2003) is $500.

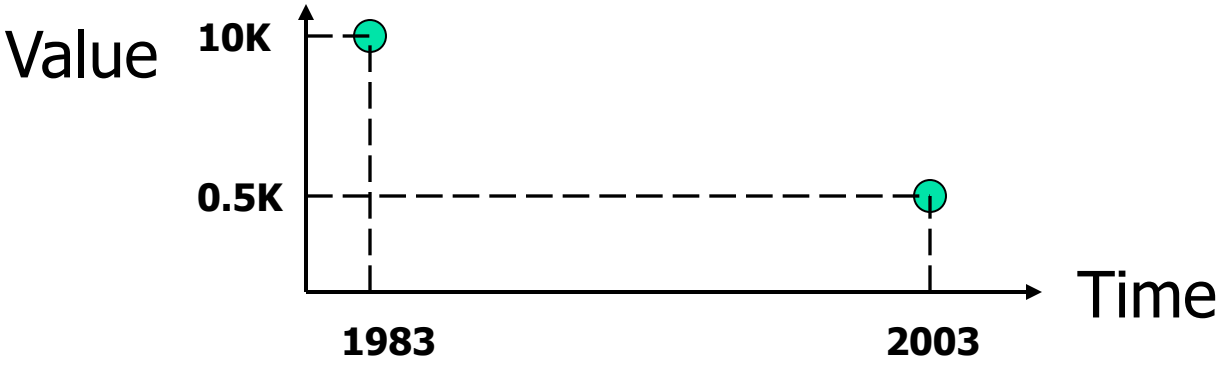
Find the following:

1) Salvage value of the car in 1983 dollars (assume 10% interest rate).

2) Total depreciation within the period

3) Annual depreciation

Assuming depreciation is LINEAR, the question can be answered as follows:



Initial Value of Car in 1983 dollars = $10,000

Salvage value of car in 2003 dollars = $500

Salvage value of car in 1983 dollars = $500*SPPWF(5%, 20 yrs) = $188.44

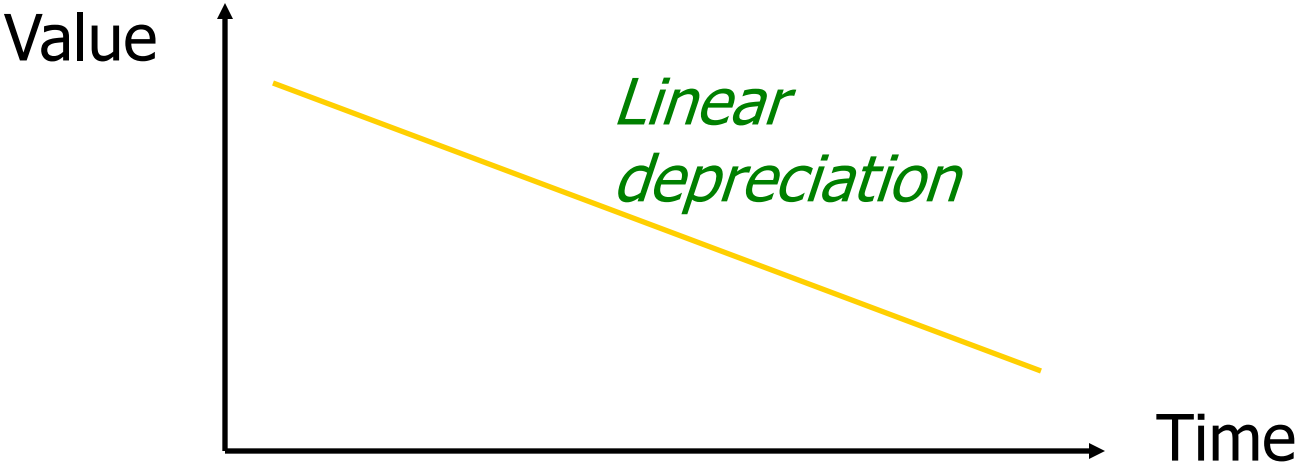
Total depreciation over the planning period = $10,000 - $188.44 = $9811.56

Annual depreciation = $9811.56/20 = $490.58

Some Patterns of Depreciation

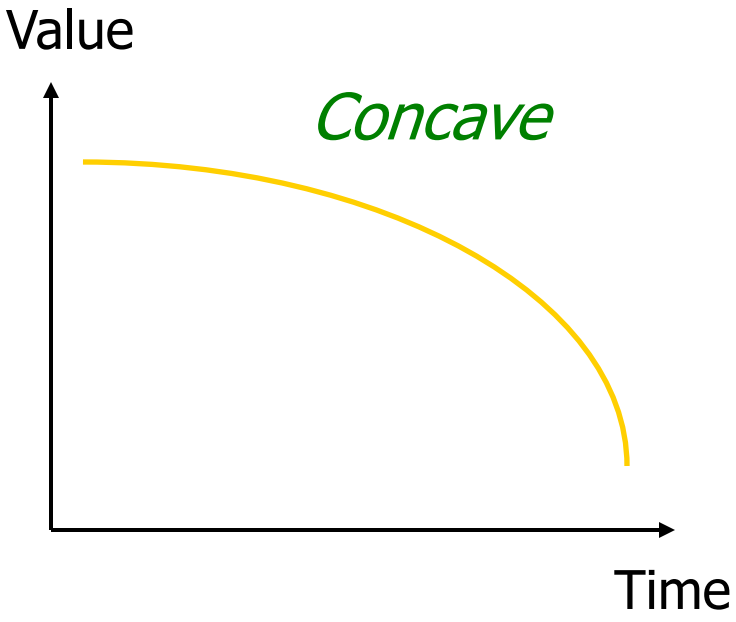
(recall that depreciation is always systematic and involves decreasing value)

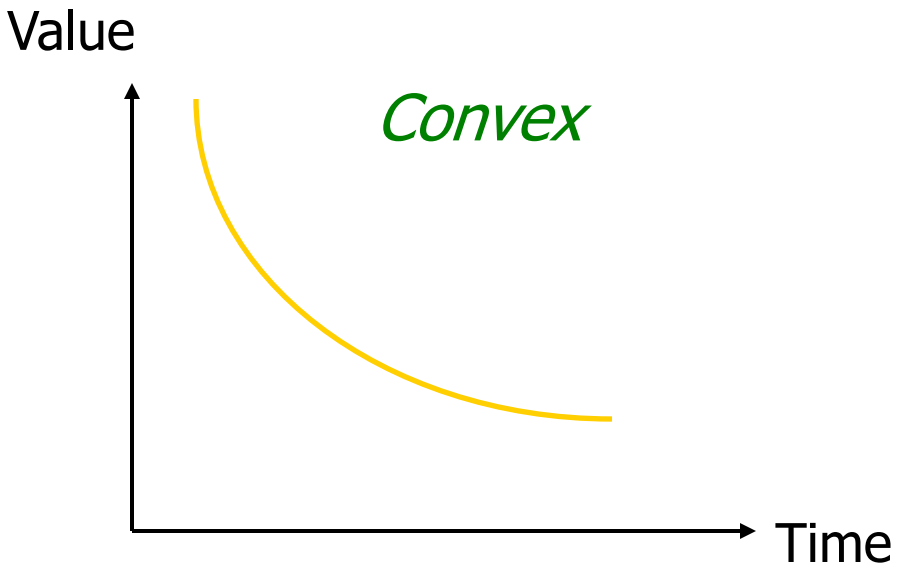
For linear, note that it is not depreciation that is decreasing. It is rather the value of the asset that is decreasing.



Some Patterns of Depreciation (cont'd)

(recall that depreciation is always systematic and involves decreasing value)





Early Stage: Slow depreciation

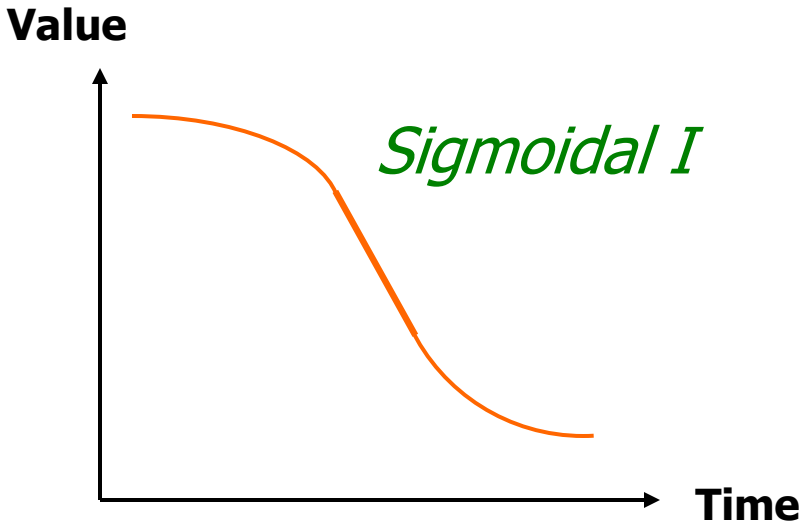
Late Stage: Fast depreciation

Early Stage: Fast depreciation

Late Stage: Slow depreciation

Some Patterns of Depreciation (cont'd)

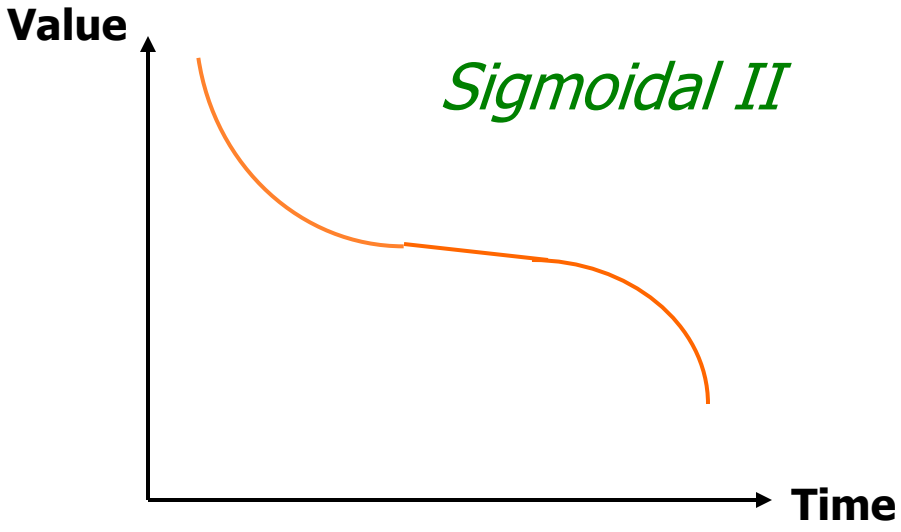
(recall that depreciation is always systematic and involves decreasing value)



Early Stage: Slow depreciation

Mid Stage: Fast depreciation

Late Stage: Slow depreciation

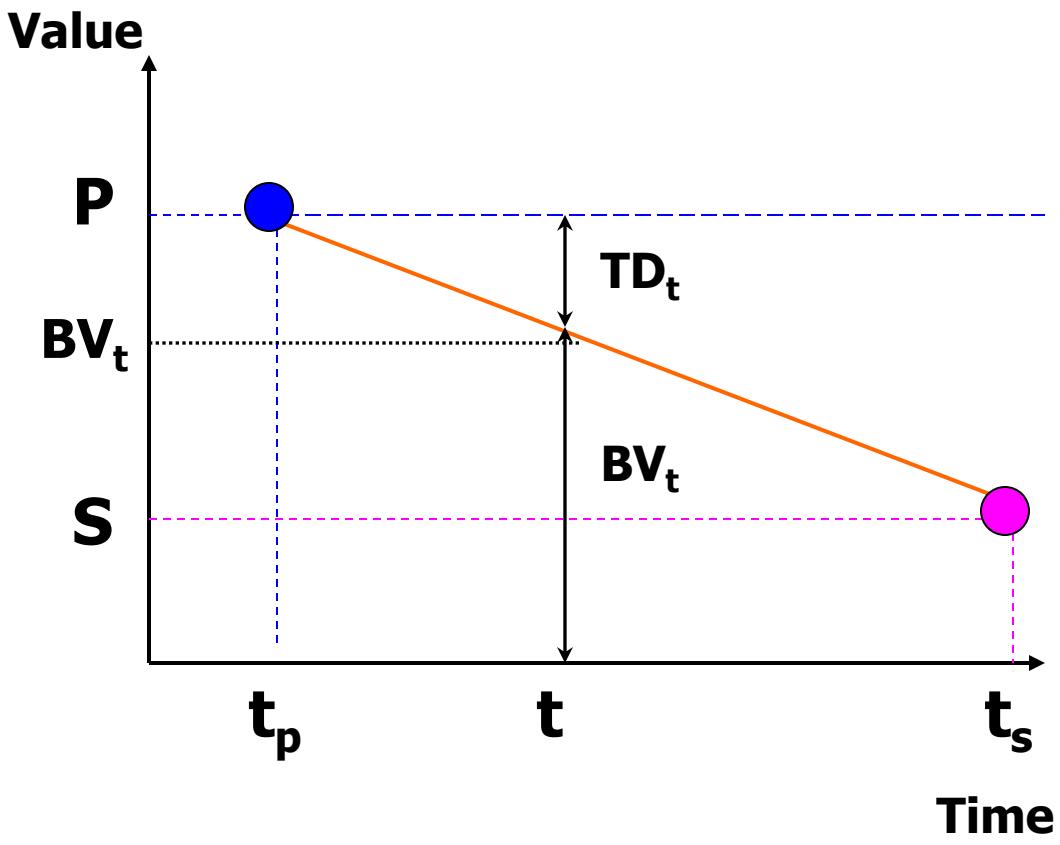


Early Stage: Fast depreciation

Mid Stage: Slow depreciation

Late Stage: Fast depreciation

Straight-Line Depreciation (SLD)



P= initial value of asset

BV_t=book value of asset at any time t

S=salvage value of asset i.e., book value at end of service life

TD_t =Total (cumulative) depreciation at any time t

t_p=time of initial value, usually taken as zero

t_s=time of salvage value,

(1) Book Value at time t (BV_t)

$$BV_t = P - \frac{P - S}{t_s - t_p}(t - t_p)$$

This formula can be used to find the book value at any time t.

P= initial value of asset

BV_t=book value of asset at any time t

S=salvage value of asset i.e., book value at end of service life

TD_t =Total (cumulative) depreciation at any time t

t_p=time of initial value, usually taken as zero

t_s=time of salvage value,

(2) Total Depreciation at any time (TD_t)

$$TD_t = P - BV_t$$

$$TD_t = \frac{P - S}{t_S - t_P}(t - t_P)$$

P= initial value of asset

BV_t=book value of asset at any time t

S=salvage value of asset i.e., book value at end of service life

TD_t =Total (cumulative) depreciation at any time t

t_p=time of initial value, usually taken as zero

t_s=time of salvage value,

Where P, S, etc. have their usual meanings

Recap- Equations for Straight Line Depreciation

Book Value at end of any year t:

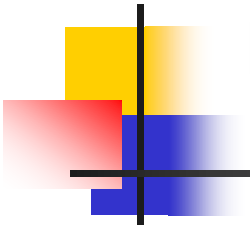
$$BV_t = P - \frac{P-S}{t_S - t_P} * (t - t_P)$$

Accumulated Depreciation at end of any year t:

$$ACD_t = \frac{P-S}{t_S - t_P} * (t - t_P)$$

Depreciation in any year t:

$$D_t = \frac{P-S}{t_S - t_P}$$



Example:

In May 1990, Johnson Doe bought a truck for $26,000. In May 2000, he sold it for $5,000.

Using Straight Line Depreciation, determine depreciation and book value schedules over the planning period. Take i as 5%. Draw depreciation and book value curves for this question.

Answer

P=$26,000 in 1990 dollars

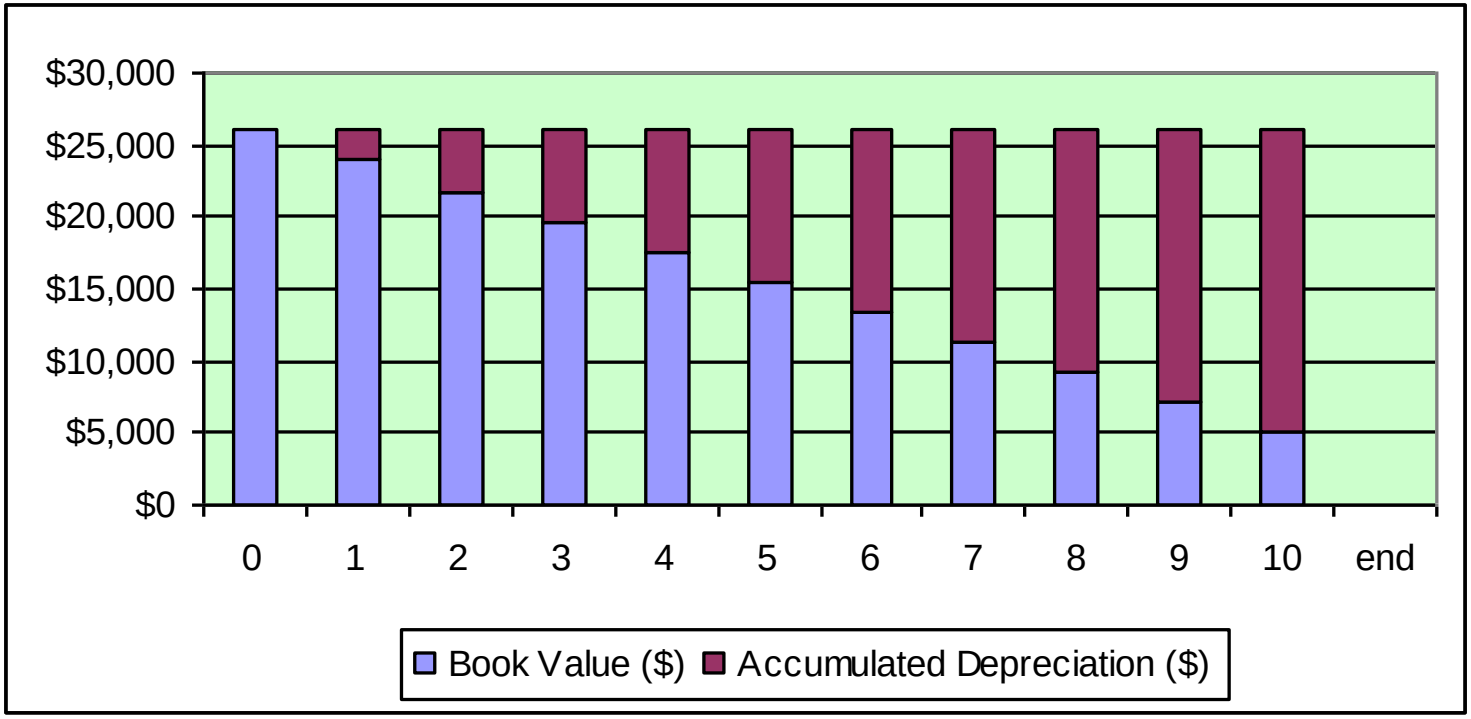
S = $5,000 in 2000 dollars

= $5,000*SPPWF(5%, 10 yrs)=$3,070

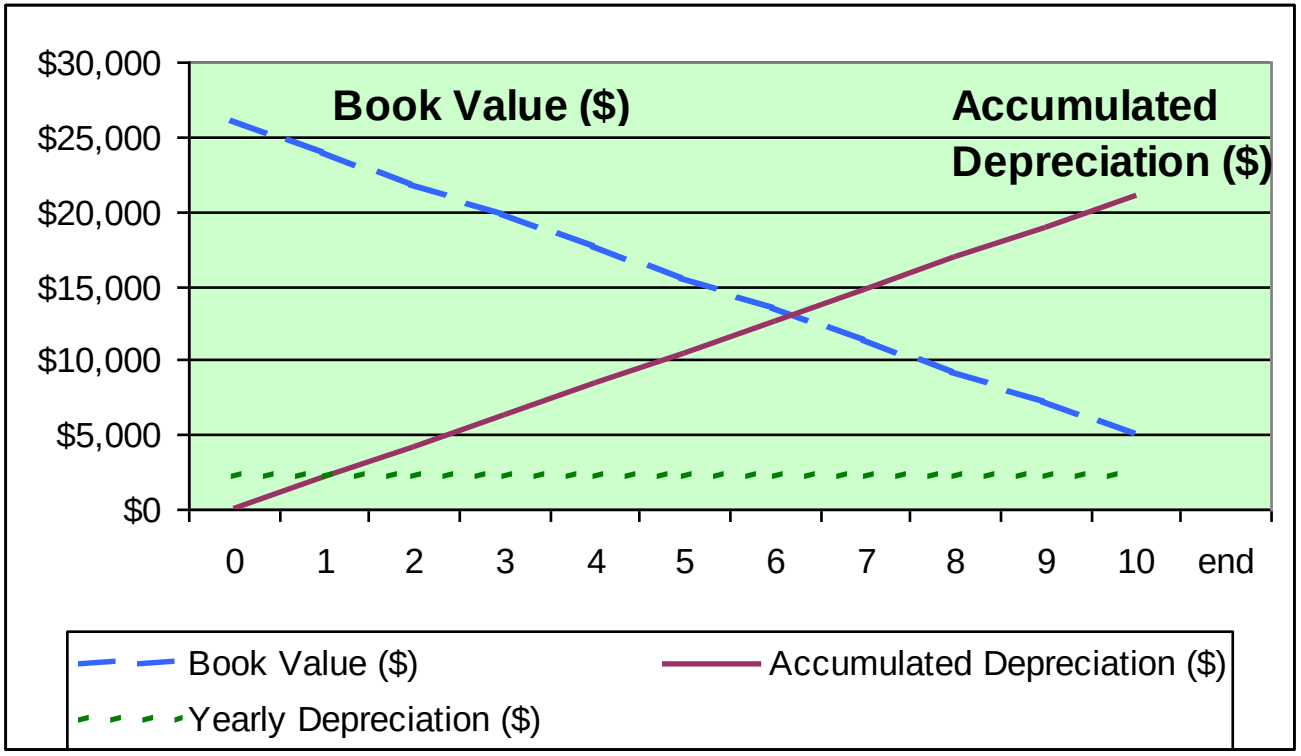
t_P=1990, t_S=2000, Thus t_S-t_P-=10 years

See Excel spreadsheet for Depreciation Schedule, Chart and Curves

Straight Line Depreciation – Example Chart



Straight Line Depreciation – Example Curve



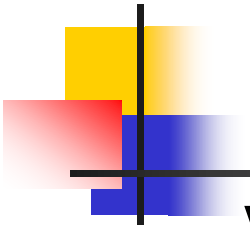
Other Common Patterns of Depreciation

- Sum-of-the-Years-Digits Depreciation (SOYD)
- Declining Balance Depreciation (DBD)
- Double Declining Balance Depreciation (DDBD)
- Modified Accelerated Cost Recovery Depreciation (MACRD)

Typical Depreciation Questions

In most questions involving depreciation, you'll be asked to find any of the following:

- Depreciation in any given year, D_t
- Accumulated depreciation by end of a given year (from initial year), ACD_t
- Book value at end of any given year, BV_t

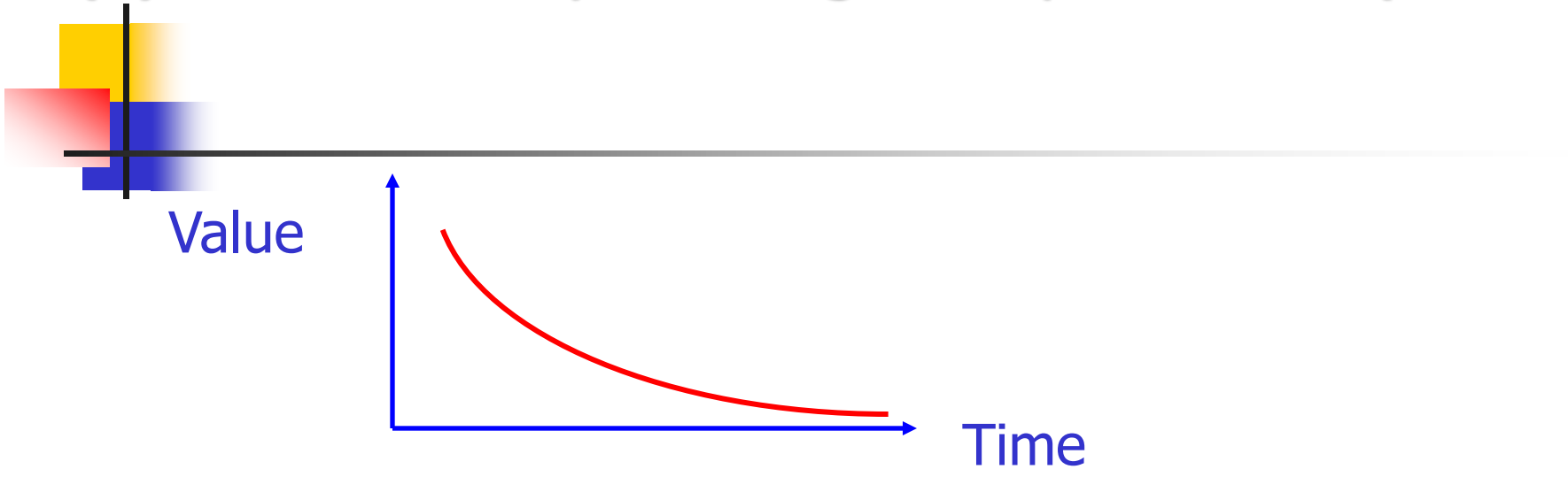


Very often, questions may also involve the drawing of depreciation and book value schedules (tables), charts or curves.

Schedules, charts and curves are useful for the determination of book value, accumulated depreciation, and any year's depreciation, given the year (t).

Conversely, such tools may be used to determine the year at which BV, D, or ACD reach a certain value.

(2) Sum-of-the-years-Digits Depreciation (SOYD)



-Large annual depreciation when asset is young

-Smaller annual depreciation when asset is old

Advantage of SOYD: Because SOYD involves larger depreciation in earlier periods and smaller depreciation at later periods, it permits asset-owning companies to claim large tax deductions (for depreciation) at earlier periods and smaller deductions at later periods. Large deductions made at earlier periods can be invested to earn interest.

Sum-of-the-years-Digits Depreciation (SOYD), continued

SOYD depreciation in any year t, is given by:

$$D_t = \frac{N - t + 1}{(N/2)(N+1)} * (P - S)$$

Where N-t+1 = useful remaining life at beginning of year t

N = planning period or service life

t = given year

P = Initial amount, S = Salvage or terminal value

Sum-of-the-years-Digits Depreciation (SOYD), continued

The accumulated depreciation (from the initial year) at the end of any year, is …

$$ACD_t = D_1 + D_2 + \ldots + D_t$$

The book value at the end of any year, is …

$$BV_t = \text{Initial Amount} - ACD_t$$

Sum-of-the-years-Digits Depreciation (SOYD), continued

Example:

The present value of a certain piece of construction machinery is $3,000,000. After its 30-year service life, the terminal value will be $200,000 in terms of present dollars.

Using SOYD depreciation method, find:

a) total depreciation at the end of three years

b) the book value at the end of three years

c) Plot the book value/depreciation curves

d) use you curve to determine the year in which total depreciation exceeds $1,000,000.

Sum-of-the-years-Digits Depreciation (SOYD), continued

P = $3,000,000 in today's dollars

S = $200,000 in today's dollars

N = 30 years

$$D_1 = \frac{30-1+1}{(30/2)*(30+1)} * (3000000 - 200000)$$

$$D_2 = \frac{30-2+1}{(30/2)*(30+1)} * (3000000 - 200000)$$

$$D_3 = \frac{30-3+1}{(30/2)*(30+1)} * (3000000 - 200000)$$

Sum-of-the-years-Digits Depreciation (SOYD), continued

D_1= $180,645

D_2= $174,623

D_3= $ 168,602

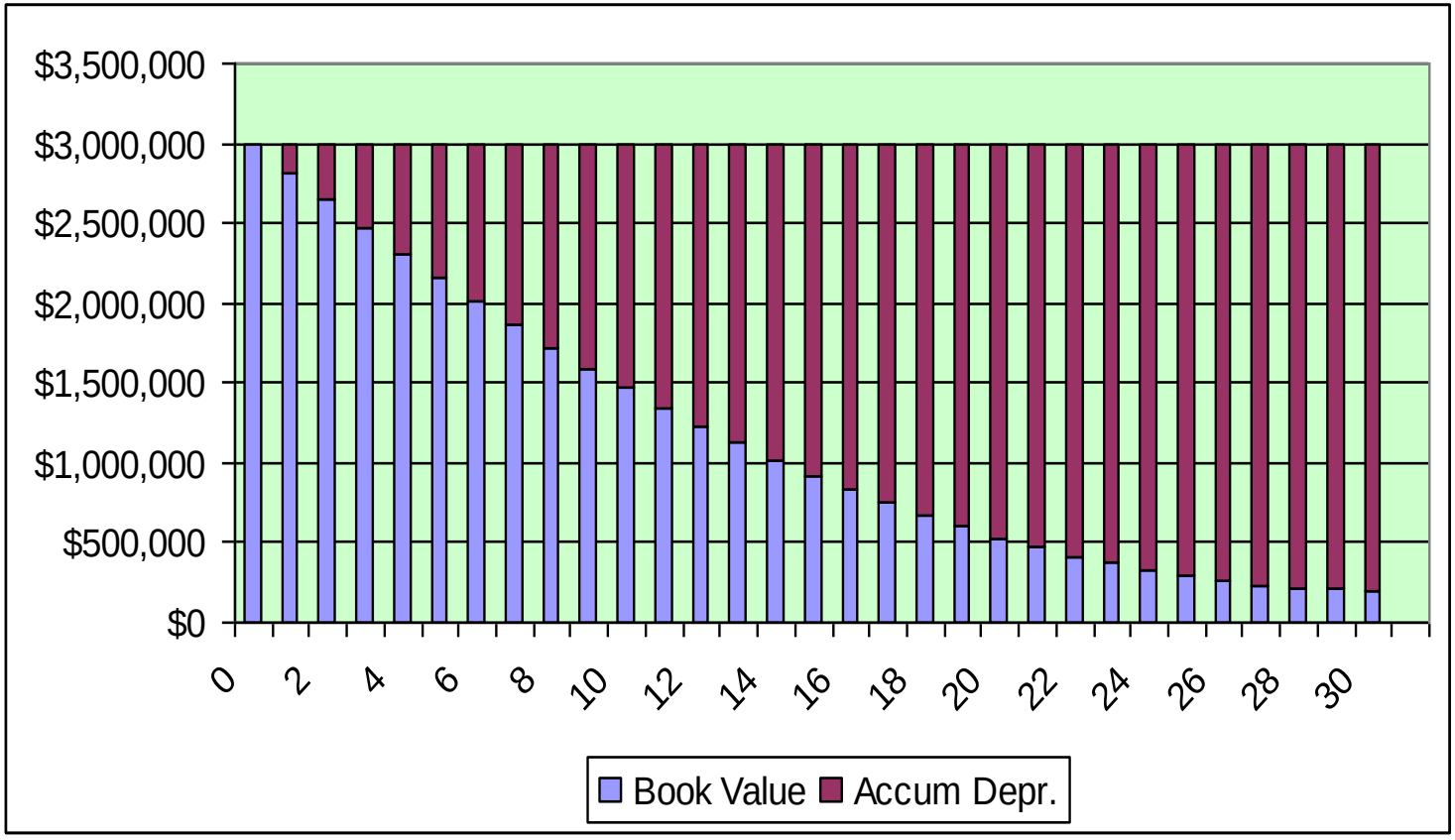
Total depreciation at end of Year 3 = $523, 870

Book value at end of Year 3 = $3m - $523,870

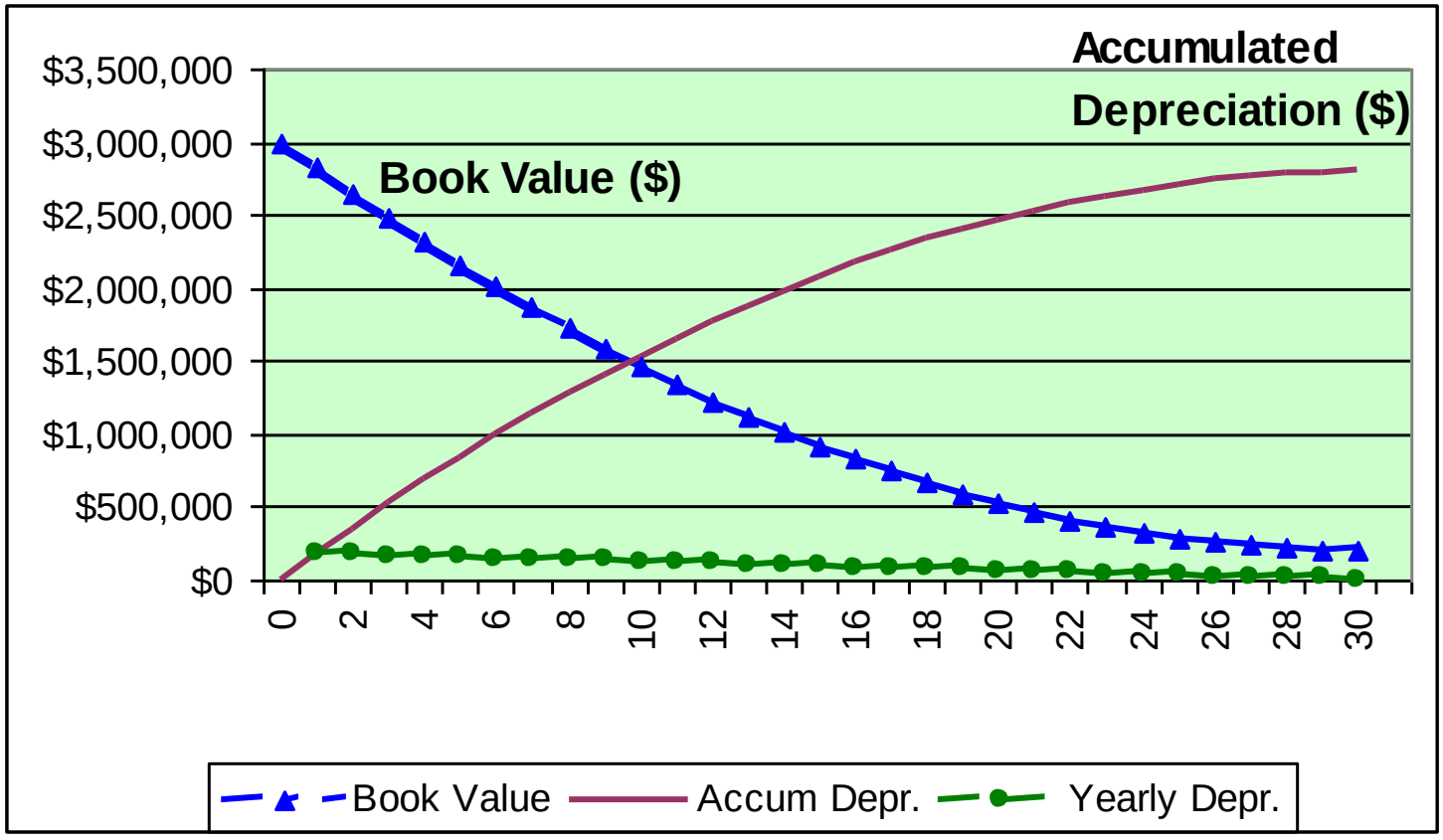
= $2,476,130

For depreciation curves, see Excel Sheet #2

Sum-of-the-years-Digits Depreciation (SOYD) – Example Chart



Sum-of-the-years-Digits Depreciation (SOYD) – Example Curve



(3) Declining Balance Depreciation (DBD)

In DBD, depreciation in each year is calculated as a constant fraction (x%) of the End of Previous Year (EOPY) book value.

Depreciation in any year t, D_t = X% of BV_{t-1}

Accumulated depreciation at end of any year t, ACD_t

= $D_1 + D_2 + \ldots + D_t$

OR = $ACD_{t-1} + D_t$

Book Value at end of any year t, BV_t

= Initial Amount -ACD_t

Declining Balance Depreciation (continued)
General Schedule:

YEAR (t)	EOPY BV (BV_{t-1})	Yearly Depr. (D_t)	Accum. Depr. (ACD_t)	EOCY BV (BV_t)
0	-	-	-	$BV_0=P$
1	BV_0	D_1=X% of BV_0	$ACD_1= D_1$	$BV_1 =P- ACD_1$
2	BV_1	D_2=X% of BV_1	$ACD_2= ACD_1+ D_2$	$BV_2 =P- ACD_2$
. .				
N	BV_{N-1}	D_N=X% of BV_{N-1}	$ACD_N= ACD_{N-1}+ D_N$	$BV_N =P- ACD_N$

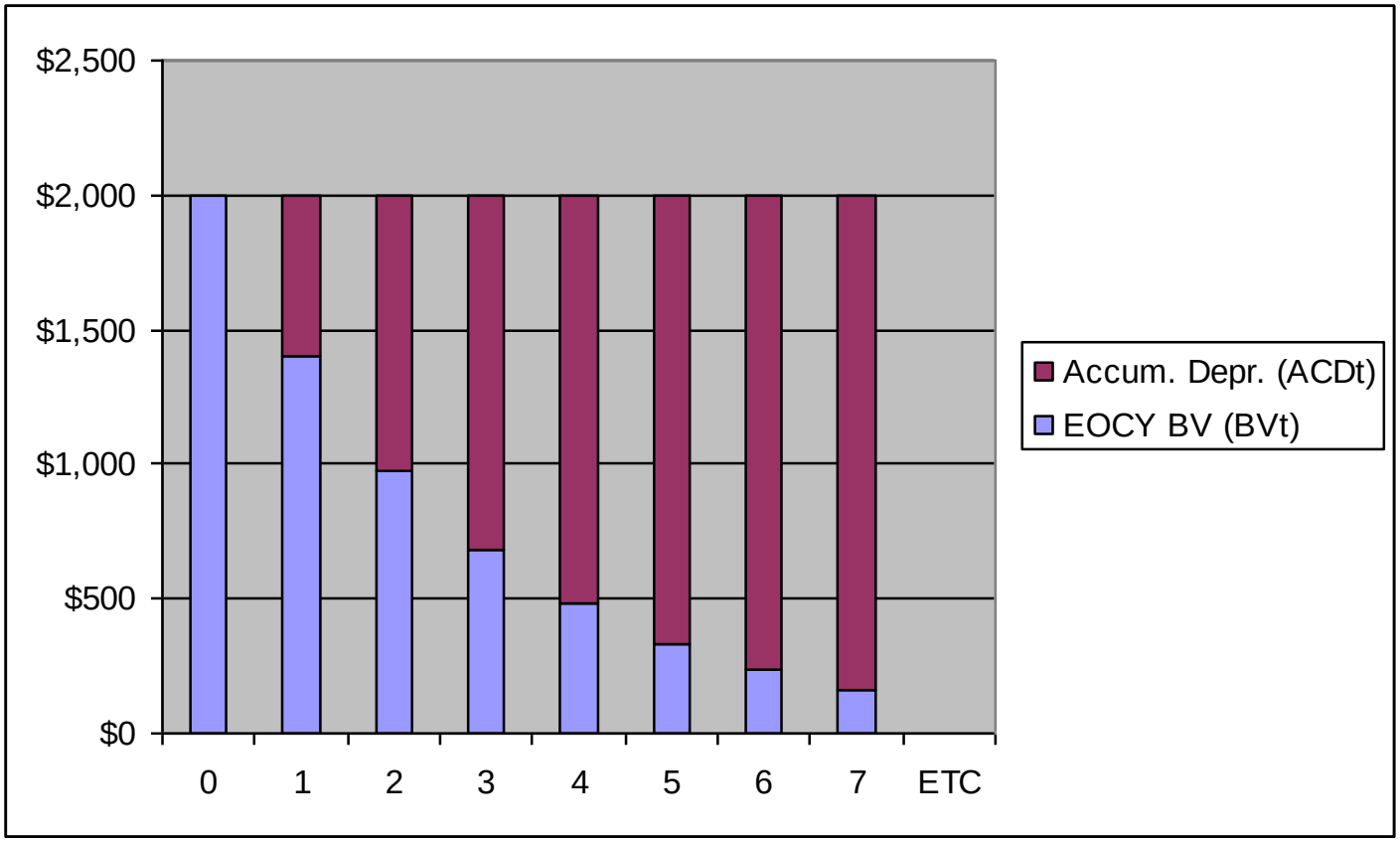
Declining Balance Depreciation (continued)

Note: EOPY = End of Previous Year, EOCY= End of Current Year

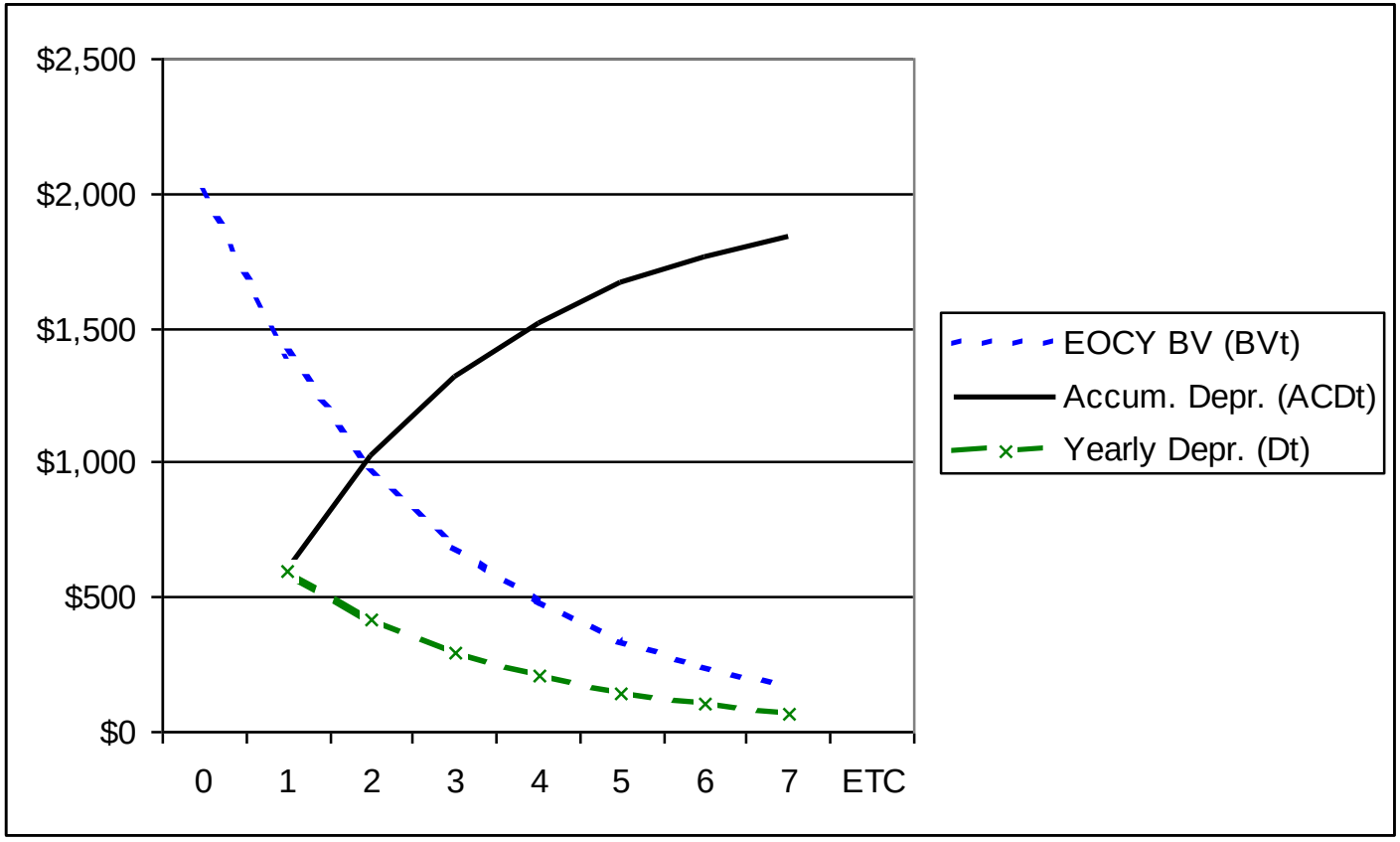
EXAMPLE: ***Asset is worth $2000 now. Declining balance is 30%. 4-year schedule is shown below. See Excel sheet 6-3 for curves.***

YEAR (t)	EOPY BV (BV_{t-1})	Yearly Depr. (D_t)	Accum. Depr. (ACD_t)	EOCY BV (BV_t)
0	-	-	-	$2000
1	$2000	30% of 2000 =$600	$600	$1400
2	$1400	30% of 1400 =$420	$1020	$980
3	$980	30% of 980 =$294	$1314	$686
4	$686	30% of 686 =$205	$1519	$481

Declining Balance Depreciation -Example Chart



Declining Balance Depreciation – Example Curve



(4) Double Declining Balance Depreciation (DDBD)

DDBD is a special case of DBD: Here, depreciation in each year is also calculated as a constant fraction of the End of Previous Year (EOPY) book value. The value of the constant fraction is 2/N, where N is planning period.

Depreciation in any year t, $D_t = 2/N * BV_{t-1}$

Accumulated depreciation at end of any year t, ACD_t

$= D_1 + D_2 + \ldots + D_t$

OR $= ACD_{t-1} + D_t$

Book Value at end of any year t, BV_t

$=$ Initial Amount $- ACD_t$

(4) Double Declining Balance Depreciation (DDBD)

GENERAL SCHEDULE:

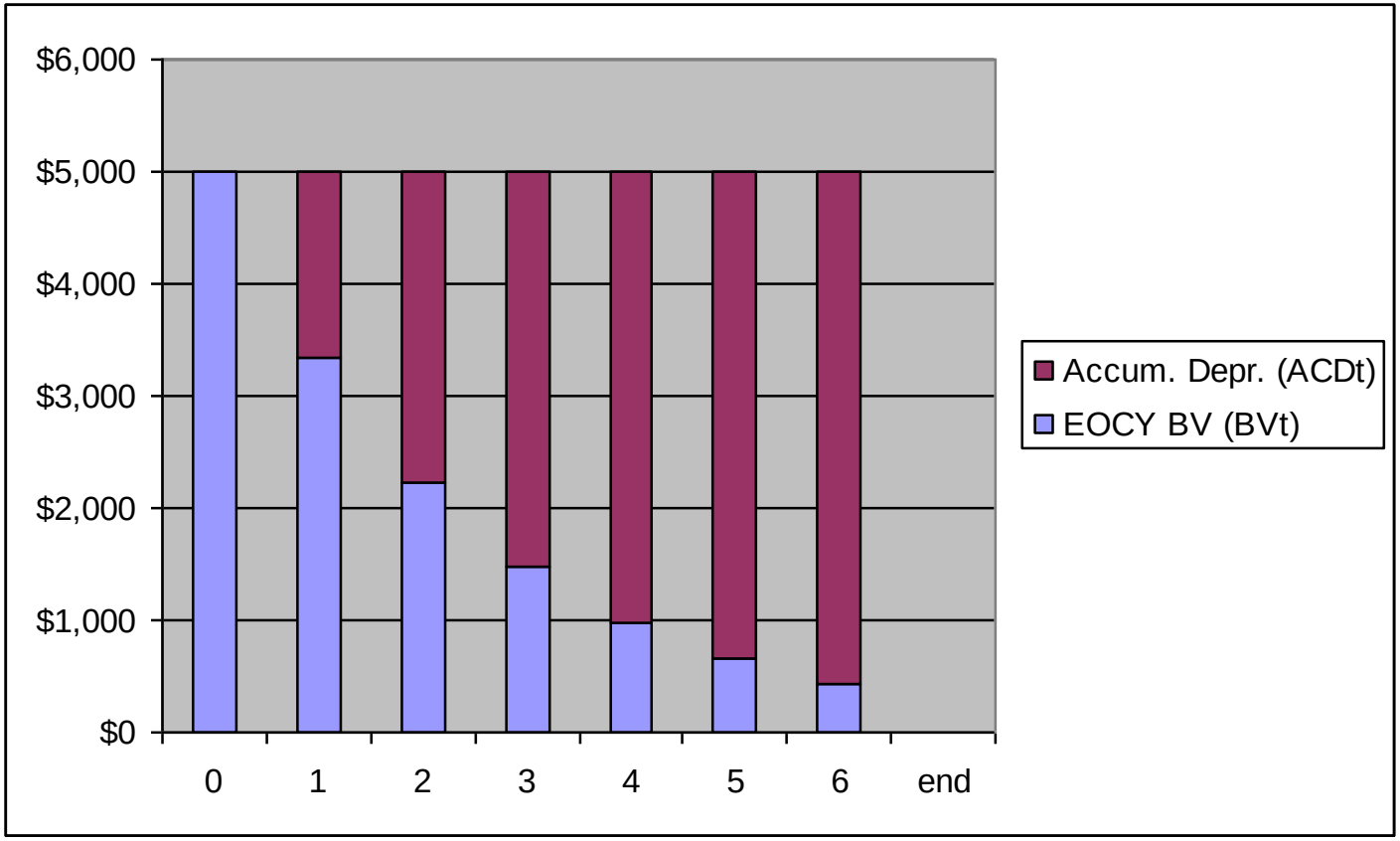
YEAR (t)	EOPY BV (BV_{t-1})	Yearly Depr. (D_t)	Accum. Depr. (ACD_t)	EOCY BV (BV_t)
0	-	-	-	$BV_0=P$
1	BV_0	$D_1=2/N* BV_0$	$ACD_1= D_1$	$BV_1 =P- ACD_1$
2	BV_1	$D_2= 2/N* BV_1$	$ACD_2= ACD_1+ D_2$	$BV_2 =P- ACD_2$
. .				
N	BV_{N-1}	$D_N= 2/N* BV_{N-1}$	$ACD_N= ACD_{N-1}+ D_N$	$BV_N =P- ACD_N$

(4) Double Declining Balance Depreciation (DDBD)

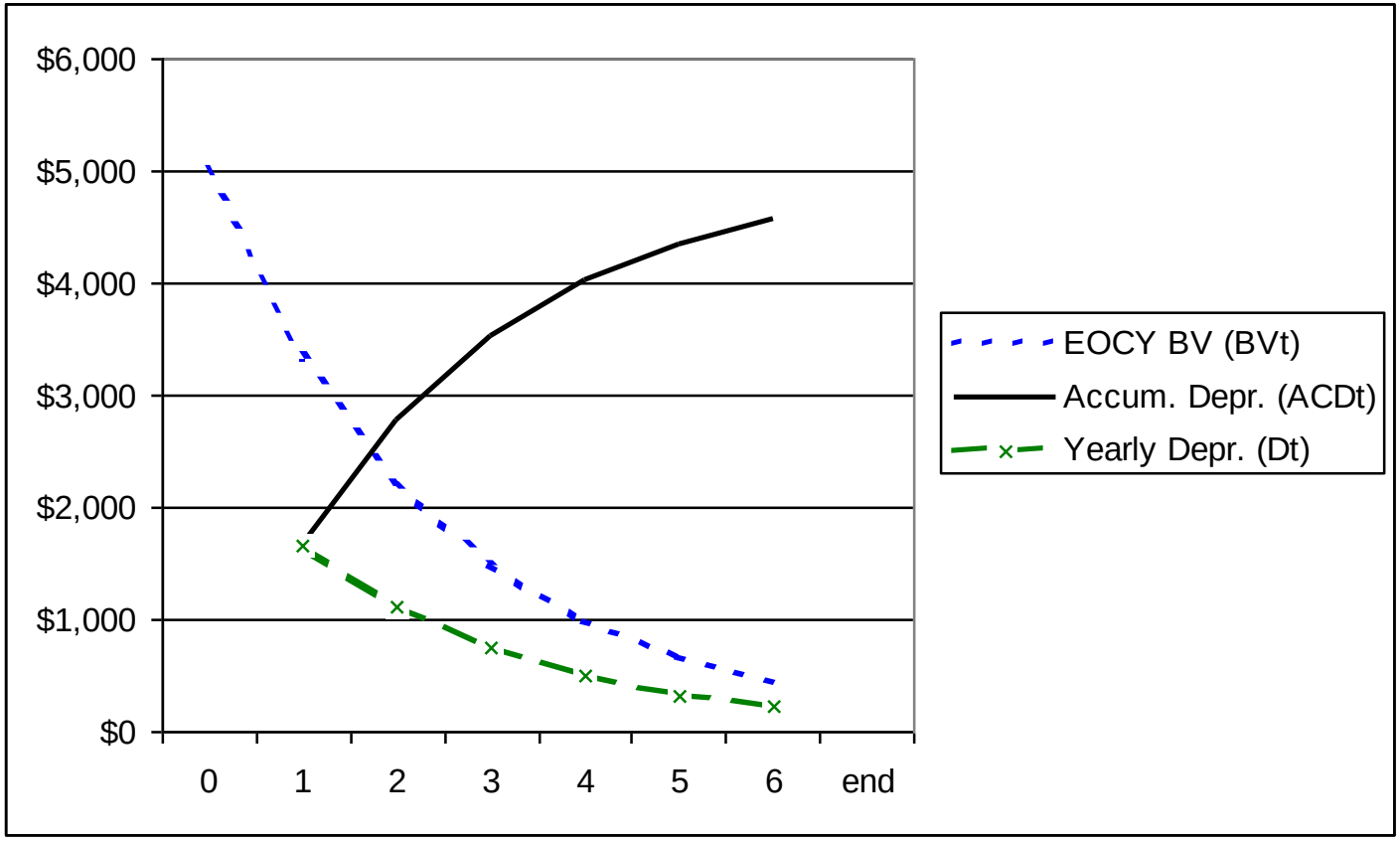
EXAMPLE: ***Asset is worth $5000 now. Use Double declining balance. 4-year schedule is shown below. See Excel sheet 6-4 for curves.***

YEAR (t)	EOPY BV (BV_{t-1})	Yearly Depr. (D_t)	Accum. Depr. (ACD_t)	EOCY BV (BV_t)
0	-	-	-	$5000
1	$5000	2/4* 5000 =$2500	$2500	$2500
2	$2500	2/4 * 2500 =$1250	$3750	$1250
3	$1250	2/4 * 1250 =$625	$4375	$625
4	$625	2/4 * 625 =$312.5	$4687.5	$312.5

Double Declining Balance Depreciation – Example Chart



Double Declining Balance Depreciation Chart



General steps in typical depreciation problems:

Determine the Expression for depreciation in any year t

↓

Calculate the depreciation in each year, up to a given year

↓

Find the accumulated depreciation at given year

= sum of all depreciation of the asset up to the given year

↓

Determine the book value at the given year= (initial value – total depreciation)