Preparatory material for FE Examinations

Engineering Economics III

(Other basic topics:

Rule of 72; Special Interest Equations; Cost Capitalization; Loan Amortization)

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This Lecture

a) Rule of 72
b) Arithmetic and Geometric Gradient Series
c) Cost Capitalization
d) Loan Amortization

The Rule of 72

States that:

Annual Rate (%) = Number of Years taken by a Sum of Money (e.g., a debt) to Double / 72

APR * YTD = 72

Note: In this formula, APR is expressed as a number, e.g., 12% APR is 12, not 0.12

The Rule of 72 (cont'd)

APR * YTD = 72

The Rule of 72 is useful to find …

- the APR at which a borrowed sum of money is expected to double within a given period (years).
- the period (years) that a sum of money takes to double, given the APR.

The Rule of 72 (cont'd)

Example 1:

At what APR will a borrowed sum of money double in only 3 years?

ANSWER: APR = 72/3 = 24%

The Rule of 72 (cont'd)

Example 2:

How long will it take for a sum of money to double at 12% APR?

ANSWER: YTD = 72/12= 6 years

The Rule of 72 (cont'd)

Example 3:

Sally owes $800 on her credit card. She has lost her job and therefore cannot make any payments. How long will it take for her debt to double? The APR is 18%

ANSWER: YTD = 72/18= 4 years

The Rule of 72 (cont'd)

Example 3 (continued):

Sally decides to transfer her credit card balance to a card with a lower APR (5%). With this new card, how long does it take for her debt to double?

ANSWER: YTD = 72/5= 14.4 years

Special Cases of Annual Payments

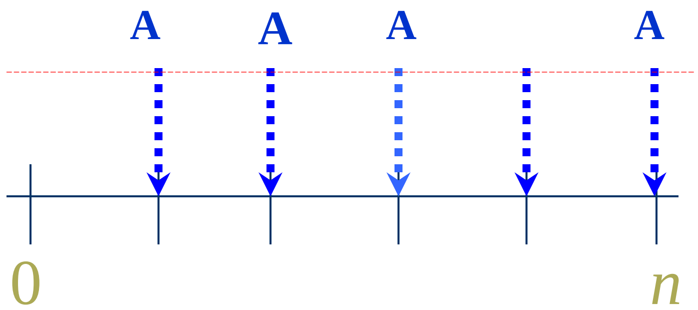
Simple Case: Where annual payments are uniform

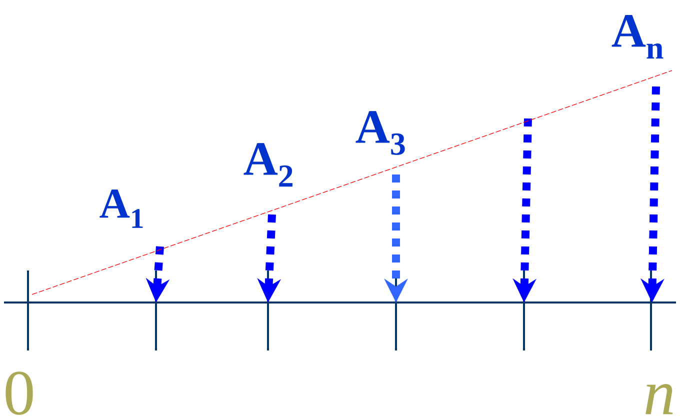
Special Cases: Non-Uniform…

- Arithmetic Gradient
- Geometric Gradient

Special Cases of Annual Payments

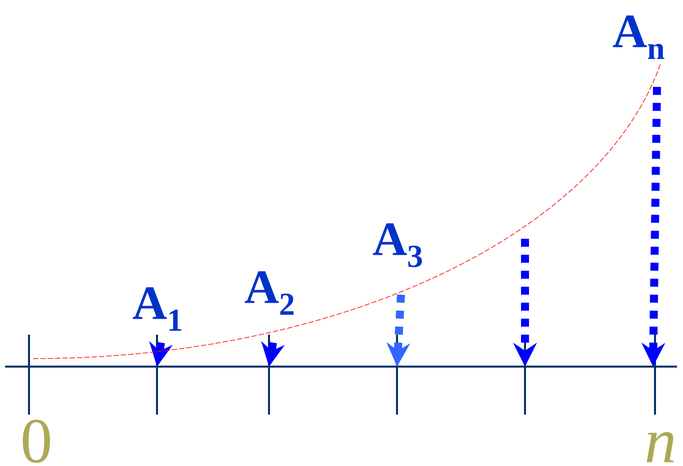
Uniform





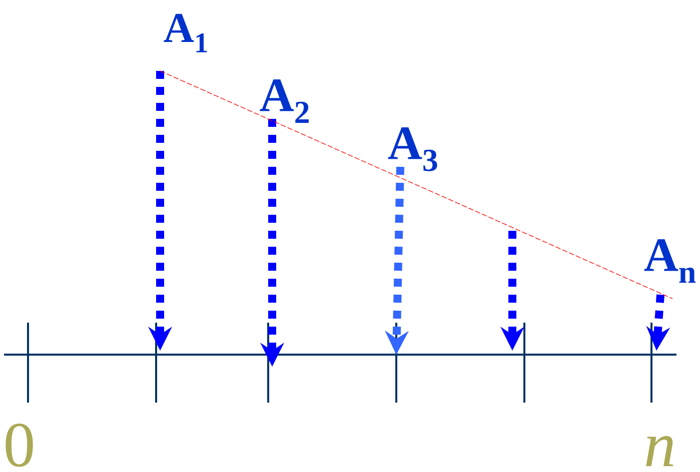
Linearly Increasing

(Arithmetic Gradient Series)



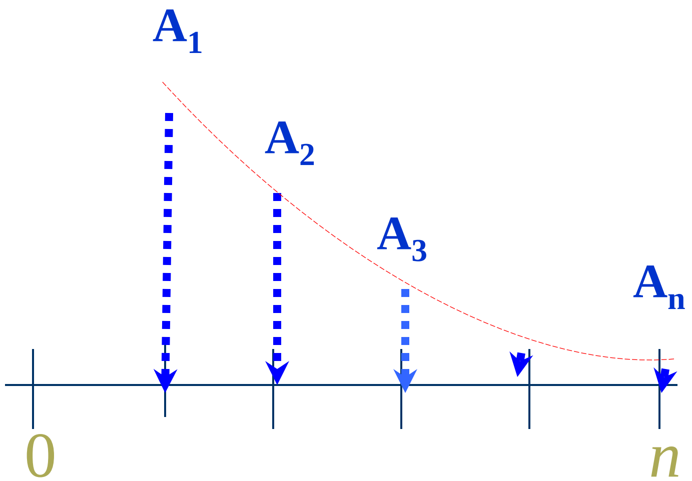
Geometrically Increasing

Special Cases of Annual Payments



Linearly Decreasing

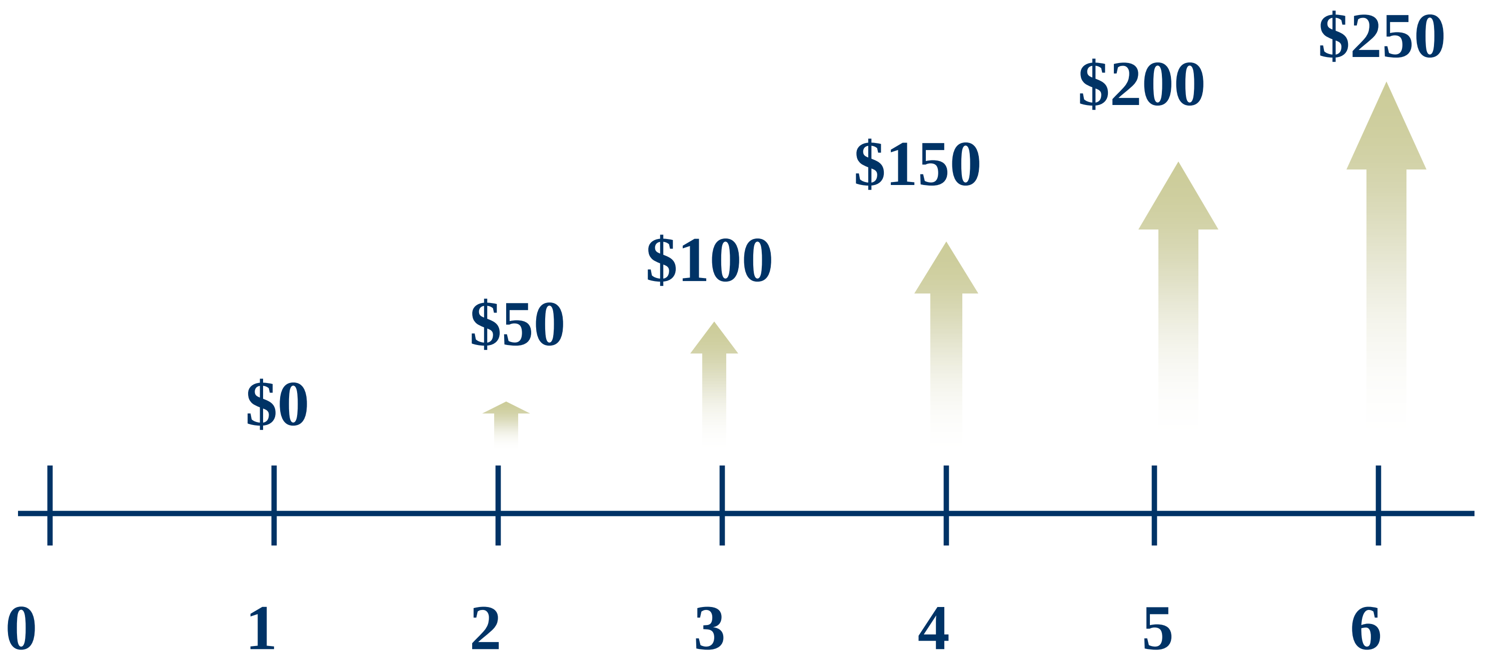
(Arithmetic Gradient Series)



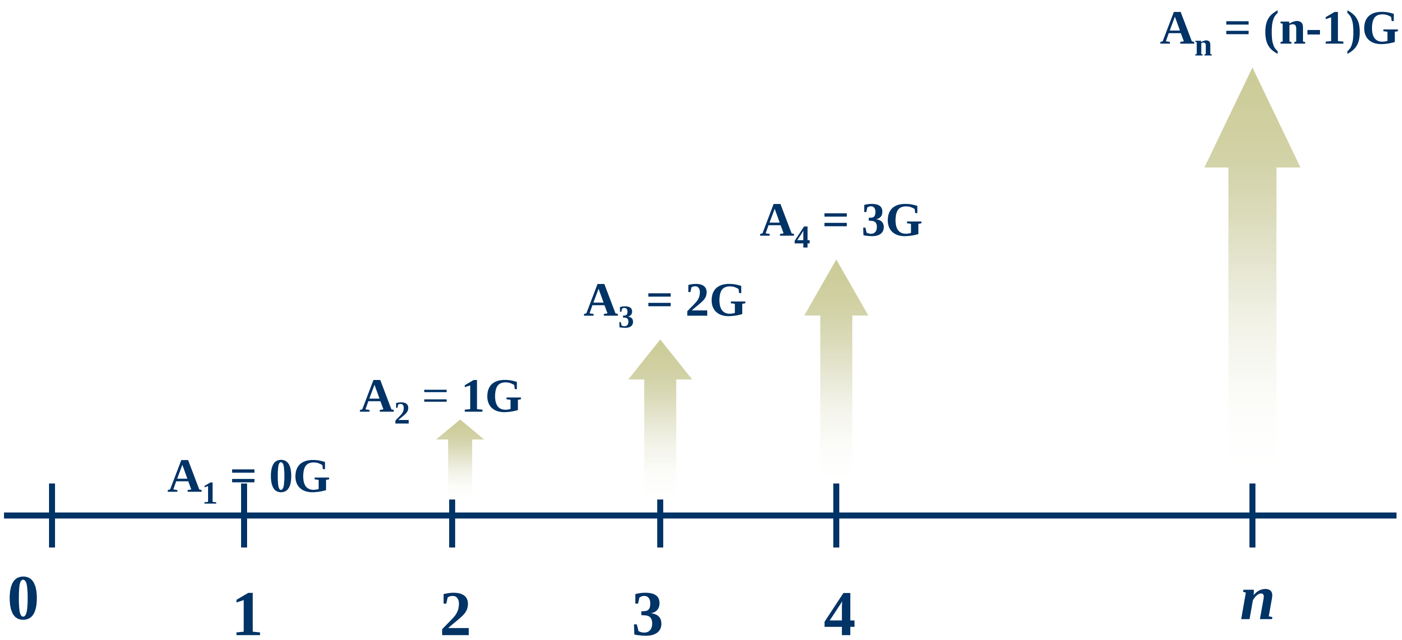
Geometrically Decreasing

Special Cases of Annual Payments (cont'd)

The Arithmetic Gradient Series



Special Cases of Annual Payments, Arithmetic Gradient Series (cont'd)



Special Cases of Annual Payments, Arithmetic Series (cont'd)

To find Future Worth, F

$$F = \frac{G}{i} \times [\frac{(1+i)^n - 1}{i} - n]$$

This equivalency equation can be used to find F, given G, or to find G given F.

Special Cases of Annual Payments, Arithmetic Series (cont'd)

To find Present Worth, P

$$P = G \times \frac{1}{i}\left[\frac{(1+i)^n - 1}{i(1+i)^n} - \frac{n}{(1+i)^n}\right]$$

This equivalency equation can be used to find P, given G, or to find G given P.

Special Cases of Annual Payments, Arithmetic Series (cont'd)

To find Uniform Annual Series that is equivalent to the Arithmetic Gradient Series

$$A = G \times [\frac{1}{i} - \frac{n}{(1+i)^n - 1}]$$

This equivalency equation can be used to find A, given G, or to find G given A.

The Case of Perpetual Payments

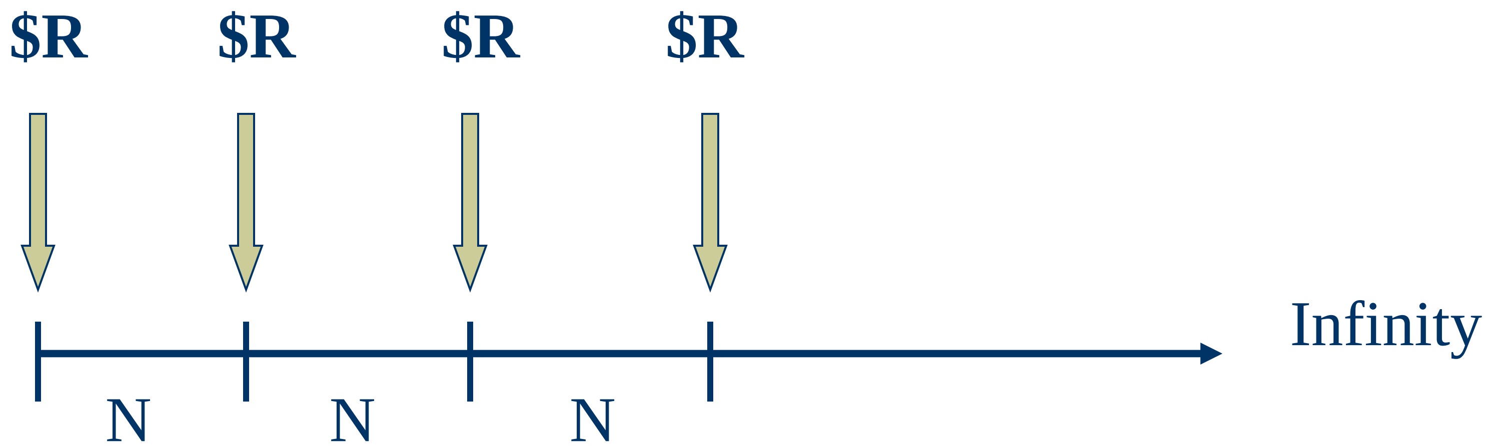
Perpetual means… recurring forever.

Most Civil Engineering Infrastructure involve…

- **Periodic Payments** for rehabilitation, as they do not last forever

- **Perpetuity** of such periodic payments, as they are always needed by society

The Case of Perpetual Payments (cont'd)



R = Reconstruction or rehabilitation costs

N= Interval of rehab/reconstruction (15 years for roads, 40 years for dams, 20 years for bridges, etc.)

The Case of Perpetual Payments (cont'd)

Present Worth of periodic perpetual payments is given as:

$$P = R \times \left[\frac{1}{(1+i)^N - 1}\right]$$

Where R = Amount of periodic payments (rehabilitation/reconstruction costs)

i = interest rate,

N = Interval of reconstruction/rehab, (i.e. service life of the facility)

The Case of Perpetual Payments (cont'd)

What does CAPITALIZATION mean?

means finding the initial capital needed to build and maintain the infrastructure.

How to capitalize?

Bring all expected costs to their present worth using the present worth factor (if it is a perpetual investment, use equation on previous slide).

The Case of Perpetual Payments (cont'd)

Example

A dam costs $50million to construct and requires $20 million every 20 years to rehabilitate. What is the capitalized cost in perpetuity? Assume interest rate = 10%.

Loan Amortization (Capital Recovery)

What is loan amortization?

Is the payment of a borrowed loan means helping the lender (bank) to recover its capital.

For easy monitoring of the loan, a **loan amortization schedule** involving certain key statistics need to be prepared by either the lender or borrower or both.

Loan Amortization (Capital Recovery)

Typical parameters used in Amortization are:

- **Initial amount borrowed**
- **Payment Period**
- **Interest (APR)**
- **Payment Serial Number**
- **Periodic Payment Amount**

Loan Amortization (Capital Recovery)

Typical Amortization Parameters (cont'd)

- Part of periodic payment towards interest
- Part of periodic payment towards principal
- Total payment to date
- Total payment to date towards interest
- Total payment to date towards principal
- Outstanding balance

Loan Amortization (Capital Recovery)

- **Example**: John took a loan of $5000 to buy a motorbike on December 31st of 2001 at an APR of 12% to be paid over 24 months.

  Assuming he makes all payments regularly, what are his amortization statistics on January 31st of 2002.

Loan Amortization (Capital Recovery)

- Initial amount borrowed = $5000
- Payment period = 24 months
- Annual percentage rate (APR) = 12%

- Monthly interest rate = 0.12/12= 0.01
- Periodic payment = monthly payment (*m*)

m= [5000*0.01(1.01)^{24}]/ [(1.01)^{24}-1]= $235.37

Loan Amortization (Capital Recovery)

- Part of monthly payment towards interest

  = monthly interest rate * amount owed at the beginning of the month

  = 1% * $5000 = $50

- Part of monthly payment towards principal

  = total monthly payment- part of monthly payment towards interest

  = $235.37-$50 = $185.37

Loan Amortization (Capital Recovery)

- Total payment made to date
  = sum of all monthly payments up to January 31st 2001
  = $235.37

- Total payments towards interest
  = sum of all monthly payments made towards interest up to January 31st 2001
  = $50

Loan Amortization (Capital Recovery)

- Total payments towards principal
  = sum of all monthly payments made towards principal up to January 31st 2001
  = $185.37

- Outstanding balance as of Jan 31st 2001
  = total loan amount- total payment made towards <u>principal</u> to date
  = $5000- $185.37= $4814.63

Loan Amortization (Capital Recovery)

A spreadsheet can be used to compute the loam amortization statistics for each payment date.

Such amortization schedules are useful for…

1) monitoring cash flows in the construction, operation, maintenance and finance of large civil engineering projects

2) Your personal finance (car loans, credit cards, etc.)