

IE690 – Optimization, Game Theory, and Uncertainty

Spring 2024, Class time: TTh 4:30 PM – 5:45 PM, GRIS 126

Instructor: Dr. Andrew L. Liu (andrewliu@purdue.edu)

Office Hours: TTh 10:45 – 11:45 AM (GRIS 388)

Course Description: This course will serve as an advanced graduate-level course to provide students with both solid theoretical foundations of optimization in general, and knowledge of the state-of-the-art development of the area of complementarity problems (CPs), (finite-dimensional) variational inequalities (VIs), mathematical problems with equilibrium constraints (MPECs) and their applications in game theory. Building upon such knowledge, the course will also provide an introduction, not a comprehensive treatment, of games under uncertainty, including recent developments in reinforcement learning games. While the course will focus on the theoretical aspects of the subjects, it will also introduce students to modeling languages, such as AMPL, GAMS and JuMP, and the complementarity-problem solver PATH, through NEOS Online Optimization Server.

Course Objectives: At the end of this course, the students are expected to be able to

- ✓ Understand the basic concepts of games and equilibria
- ✓ Formulate an equilibrium-finding problem of certain games as a CP/VI or an MPEC
- ✓ Rigorously analyze the existence and uniqueness of solutions to CPs, VIs and MPECs, and hence able to prove the existence and uniqueness of Nash equilibria to certain games
- ✓ Understand and utilize various algorithms to solve CPs, VIs and MPECs
- ✓ Understand the basic concepts of games with uncertainty

Prerequisite: Graduate-level linear algebra and/or matrix analysis, real analysis, linear programming, nonlinear programming

Textbooks: There will not be a single textbook, but a series of textbooks. The books are summarized below by subjects.

Background Materials	▪ R. A. Horn and C. R. Johnson. <i>Matrix Analysis</i>, Cambridge University Press, 1985. ▪ J. Nocedal and S. J. Wright. <i>Numerical Optimization</i>, 2nd edition, Springer, 2006. [NW06] ▪ M. S. Bazaraa, H. D. Sherali and C. M. Shetty. <i>Nonlinear Programming: Theory and Algorithms</i>, 3rd edition, Wiley, 2006. [BSS06] ▪ D. P. Bertsekas. <i>Nonlinear Programming</i>, 2nd edition, Athena Scientific, 1999. [Bertsekas99]
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VI/CP	▪ R. W. Cottle, J.-S. Pang and R. E. Stone. <i>The Linear Complementarity Problem</i>. Academic Press, 1992. [LCP] ▪ F. Facchinei and J.-S. Pang. <i>Finite-Dimensional Variational Inequalities and Complementarity Problems: Volume I and Volume II</i>. Springer, 2003. [VICP]
Hierarchical Optimization	▪ S. Dempe. <i>Foundations of Bilevel Programming</i>. Kluwer Academic Publisher, 2002. [Bilevel] ▪ Z.-Q. Luo, J.-S. Pang and D. Ralph. <i>Mathematical Programs with Equilibrium Constraints</i>. Cambridge University Press, 1996. [MPEC]
Game Theory	▪ R. Myerson. <i>Game Theory: Analysis of Conflict</i>, Harvard University Press, 1997. [Game1] ▪ D. Fudenberg and J. Tirole. <i>Game Theory</i>, The MIT Press, 1991. [Game2] ▪ R. Gibbons. <i>Game Theory for Applied Economists</i>. Princeton University Press, 1992. [Game3]
Algorithmic Game Theory, Learning in Games	▪ Noam Nisan, Tim Roughgarden, Eva Tardos, Vijay V. Vazirani. <i>Algorithmic Game Theory</i>. Cambridge University Press, 2007. [AGT] ▪ N. Cesa-Bianchi and G. Lugosi. <i>Prediction, Learning, and Games</i>. Cambridge University Press, 2006. [PLG]

Note: Books highlighted in green are available online through Purdue library.

Homework: There will be about five homework assignments; submissions through Gradescope.

Project: There will be one project to deliver in the final week of the semester, in the form of both oral presentation and a written report. The report should include the following items:

- description of the problem to be studied (motivation) and brief literature review
- mathematical formulation of the problem
- discussion of the problem's properties
- discussion of algorithms to solve the resulting problems
- solve the problem either by a suitable solver or by coding your own algorithm
- discuss the solution quality and limitations of your approach, and suggest ways that may improve the results

You are STRONGLY encouraged to talk to be instructor about the project you will be working on BEFORE you start.

Final project presentation: Thursday, May 2nd, GRIS 134, 3 – 6 PM (EDT)

Grading Policy:

- Homework: 35%
- Project: 50%
- Class attendance: 15%

Other Policies

Attendance

- Attendance of lectures is required and will account for 15% of the final grade. Sign-in sheets will be distributed at the beginning of RANDOMLY selected classes.

Class Cancellations

TBD

Grief Absence Policy for Students

“Purdue University recognizes that a time of bereavement is very difficult for a student. The University therefore provides the following rights to students facing the loss of a family member through the Grief Absence Policy for Students (GAPS). GAPS Policy: Students will be excused for funeral leave and given the opportunity to earn equivalent credit and to demonstrate evidence of meeting the learning outcomes for misses assignments or assessments in the event of the death of a member of the student's family.”

Emergencies

“In the event of a major campus emergency, course requirements, deadlines and grading percentages are subject to changes that may be necessitated by a revised semester calendar or other circumstances beyond the instructor's control. Relevant changes to this course will be posted onto the course website or can be obtained by contacting the instructors or TAs via email or phone. You are expected to read your @purdue.edu email on a frequent basis.”

Emergency Assembly Area (EAA) of Grissom Hall

In case of fire evacuation or other situations, the EAA of GRIS is the East end of the Main Aisle of Stewart Center on the First Floor (Near the doors that go outside to the Memorial Union).

Tentative Course Outline:

Block 1. Background – linear algebra, real analysis and nonlinear programming

- Linear algebra ([NW06] Appendix A.1, [Bertsekas99] Appendix A, [LCP] Sec 2.2)
 - Vectors, matrices, norms
 - Eigenvalues, eigenvectors and singular-value decomposition
 - Semidefinite and definite matrices
- Real analysis ([NW06] Appendix A.2, [Bertsekas99] Appendix A, [LCP] Sec 2.1)
 - Limits, continuity
 - Functions of several variables, differentiability
 - Contraction mappings, fixed point theorems

- Convex analysis ([NW06] Appendix B, [BSS06] Chapter 2 and 3)
 - Convex sets, convex hulls
 - Separation and support of sets
 - Convex functions, differentiability and subgradients
 - Generalizations of convex functions
- Nonlinear programming
 - Unconstrained optimization: optimality conditions, algorithms ([NO] Chapter 2 - 6)
 - Constrained optimization:
 - Tangent cones ([NW06] Sec 12.2, [BSS06] Sec 5.1)
 - Constrained qualifications ([NW06] Sec 12.2)
 - Optimality conditions ([NW06] Sec 12.3 – 12.7)
 - Lagrange multipliers and duality ([NW06] Sec 12.8 – 12.9, [BSS06] Chapter 6)
 - NLP algorithms: penalty approach ([NW06] Chapter 17), SQP ([NW06] Chapter 18), interior point methods ([NW06] Chapter 19)

Block 2. Game theory and linear complementarity problems

- Game Theory
 - Non-cooperative games: normal-form representation, the concept of Nash equilibrium ([Game2] Chapter 1)
 - Proof of Nash equilibrium through fixed-point theorem
- LCP
 - Reformulation of games to CPs
 - Solution existence of LCPs ([LCP] Chapter 3)
 - Lemke's Method ([LCP] Chapter 4)

Block 3. Nonlinear complementarity problems and variational inequalities

- Problem definitions and application areas ([VICP] Chapter 1)
- Solution analysis ([VICP] Chapter 2 and 3)
- Algorithms for CPs ([VICP] Chapter 9)
- Algorithms for VIs ([VICP] Chapter 10)

Block 4. Mathematical problems with equilibrium constraints (MPECs) ([MPEC] Chapter 1

– 5, additional readings)

- Extensive-form games ([Game1] Chapter 2) and Stackelberg games
- MPECs
 - Formulation
 - Solution existence, constraint qualifications, and optimality conditions
 - Algorithms for MPECs

Block 5. Games under uncertainty

- Incomplete information and equilibrium concepts
- Algorithmic game theory
- Reinforcement learning games

Note: This syllabus is subject to change by the instructor. Any changes will be notified to the students promptly.