

IE538: Nonlinear Optimization

Spring 2025, MW 06:30 PM - 07:45 PM, Grissom Hall 133

Instructor: Dr. Andrew Liu (andrewliu@purdue.edu).

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Office Hours: 2 – 3 PM, MW at GRIS 388

TA:

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- Office hour: GRIS 157C, Tuesdays 10:30-11:29am

Course Description: This course serves as a graduate-level introduction to the subject of nonlinear optimization, which has been widely used in operations research, engineering, and economics. The course will be self-contained, starting with an introduction to basic real analysis and linear algebra needed for this course. It will cover theories on optimality conditions and various algorithms for both unconstrained and constrained optimization problems. This course will also introduce students to modeling languages, such as AMPL and GAMS, and various off-the-shelf solvers through NEOS Online Optimization Server.

Course Objectives: The aim of this course is to build a solid foundation in both theory and computation of nonlinear optimization problems. It will emphasize on convex analysis, KKT conditions (aka optimality conditions), duality, and convergence analysis of various algorithms (especially of penalty methods, SQP and interior-point methods).

Prerequisite: Multivariate calculus. Linear algebra and real analysis at the undergraduate level. Basic programming skills.

Required Textbook: (both books are freely available through Purdue Library)

1. *Nonlinear Programming: Theory and Algorithms* by M. S. Bazaraa, H. D. Sherali and C. M. Shetty, **3rd edition**, Wiley, 2006. [BSS06]
2. *Numerical Optimization* by Jorge Nocedal and Stephen J. Wright, **2nd edition**, Springer, 2006. [NW06]

Recommended Reference Textbooks:

- *Nonlinear Programming* by D. P. Bertsekas, 2nd edition, Athena Scientific, 1999. [B99]
- *Convex Analysis and Optimization* by D. P. Bertsekas, Athena Scientific, 2010 [B03]

Reference Books for Mathematical Background

- *Matrix Analysis* by Roger A. Horn and Charles R. Johnson, 1st edition (1985) or 2nd edition (2012), Cambridge University Press.

- *Real Analysis and Applications: Theory in Practice* by K. R. Davidson and A. P. Donsig, Springer Science & Business Media, 2009. (Available online through Purdue Library)

Attendance:

- Attendance is required and will be checked through iClicker Cloud.
- Each student will be given three free passes; no need to email me if you cannot attend a lecture. Only email me if you have used up all the free passes and anticipate missing more lectures.

Homework: There will be homework assignments every one or two weeks, and the assignments will be due in class.

Exams: There will be three exams, weighted equally

Project: There will be one programming/coding project to deliver in the end of this semester. You are expected to know at least one programming language, preferably in Matlab, Python, Julia, C++ or equivalent. Detailed requirements of the project will be provided before the Spring Break.

Grading Policy:

- Attendance: 8%
- Homework: 18%
- Exam 1: 18% -- 02/24/2025 (Monday, 90-minute, in class)
- Exam 2: 18% -- 04/02/2025 (Wednesday, 90-minute, in class)
- Exam 3: 18% -- 05/05/2025 (Monday): 07:00p - 08:30p GRIS 134 (not 133)
- Final Project: 20%, Due on 05/07 (Wednesday), 11:59 PM (EDT)

Academic Dishonesty:

○ General policies

Purdue prohibits "dishonesty in connection with any University activity. Cheating, plagiarism, or knowingly furnishing false information to the University are examples of dishonesty." [Part 5, Section III-B-2-a, [University Regulations](#)] Furthermore, the University Senate has stipulated that "the commitment of acts of cheating, lying, and deceit in any of their diverse forms (such as the use of substitutes for taking examinations, the use of illegal cribs, plagiarism, and copying during examinations) is dishonest and must not be tolerated. Moreover, knowingly to aid and abet, directly or indirectly, other parties in committing dishonest acts is in itself dishonest." [University Senate Document 72-18, December 15, 1972]

○ Specific policies:

- You can discuss homework with your peers, but you must do it on your own!
- The exams will be in person (if you are in quarantine or need special accommodations for the exams, please contact the instructor as early as possible).
- If you have concrete evidence that someone in the class is cheating, I strongly encourage you to submit the evidence to me. Any cheating behavior, with concrete

- evidence, will result in punitive grading – if there is evidence of exam cheating, then the corresponding exam will automatically be graded as 0; if there is evidence of plagiarism of the final project, the project will be graded as 0.
- In addition to the punitive grading, all cheating incidents will be reported to the Office of Student Rights and Responsibilities, a subdivision of the Office of the Dean of Students.

Emergencies: I will contact you through your Purdue emails in case of emergencies.

In the event of a major campus emergency, course requirements, deadlines and grading percentages are subject to changes that may be necessitated by a revised semester calendar or other circumstances beyond the instructor's control. Relevant changes to this course will be posted onto the course website or can be obtained by contacting the instructor via email or phone. You are expected to read your @purdue.edu email on a frequent basis.

EMERGENCY NOTIFICATION PROCEDURES are based on a simple concept – if you hear a fire alarm inside, proceed outside. If you hear a siren outside, proceed inside.

- Indoor Fire Alarms mean to stop class or research and immediately evacuate the building – Proceed to your Emergency Assembly Area (EAA) (Grissom's EAA is the area outside between Grissom and Heavilon Hall) away from building doors. Remain outside until police, fire, or other emergency response personnel provide additional guidance or tell you it is safe to leave.

- All Hazards Outdoor Emergency Warning Sirens mean to immediately seek shelter (Shelter in Place) in a safe location within the closest building. – “Shelter in place” means seeking immediate shelter inside a building or University residence. This course of action may need to be taken during a tornado, a civil disturbance including a shooting or release of hazardous materials in the outside air. Once safely inside, find out more details about the emergency*. Remain in place until police, fire, or other emergency response personnel provide additional guidance or tell you it is safe to leave.

*In both cases, you should seek additional clarifying information by all means possible...Purdue Home page, email alert, TV, radio, etc...review the Purdue Emergency Warning Notification System multi-communication layers at http://www.purdue.edu/ehps/emergency_preparedness/warning-system.html

EMERGENCY RESPONSE PROCEDURES: Review the Emergency Procedures Guidelines https://www.purdue.edu/emergency_preparedness/flipchart/index.html

EMERGENCY PREPAREDNESS AWARENESS VIDEOS

- "Shots Fired on Campus: When Lightning Strikes," is a 20-minute active shooter awareness video that illustrates what to look for and how to prepare and react to this type of incident. See: https://www.youtube.com/watch?v=5mzI_5aj4Vs

MORE INFORMATION

Reference the Emergency Preparedness web site for additional information:
http://www.purdue.edu/emergency_preparedness

Tentative Course Outline:

Block 1. Background – real analysis and linear algebra

- Linear algebra ([NW06] Appendix A.1)
 - Vectors, matrices, norms
 - Subspaces
 - Eigenvalues, eigenvectors and singular-value decomposition
 - Matrix factorization – Cholesky, LU, QR (skip)
 - Semidefinite and definite matrices
- Real analysis ([NW06] Appendix A.2)
 - Sequence and convergence, rate of convergence (postpone)
 - Topology of the Euclidean Space $\mathbb{R}^n$
 - Continuity and limits
 - Differentiability
 - Mean value theorem, implicit function theorem
 - Weierstrass's Theorem

Block 2. Convex analysis

- Convex sets ([BSS06] Chapter 2)
 - Convex sets and their properties
 - Convex hulls
 - Separation and support of sets
- Convex functions ([BSS06] Chapter 3)
 - Definitions and properties
 - Differentiability and subgradients
 - Generalizations of convex functions

Block 3. Unconstrained optimization

- Optimality conditions ([NW06] Chapter 2)
- Algorithms
 - Line-search methods ([NW06] Chapter 3)
 - Trust-region methods ([NW06] Chapter 4)
 - Conjugate gradient methods ([NW06] Chapter 5)
 - Quasi-Newton methods ([NW06] Chapter 6)

Block 4. Constrained optimization

- Theory
 - Tangent cones ([NW06] Sec 12.2, [BSS06] Sec 5.1)
 - Constrained qualifications ([NW06] Sec 12.2)
 - Optimality conditions ([NW06] Sec 12.3 – 12.7)

- Lagrange multipliers and duality ([NW06] Sec 12.8 – 12.9, [BSS] Chapter 6)
- Algorithms
 - Penalty and augmented Lagrangian methods ([NW06] Chapter 17)
 - Sequential quadratic programming ([NW06] Chapter 18)
 - Interior-point methods ([NW06] Chapter 19)