

IE535: LINEAR PROGRAMMING

Fall 2025

Instructor: Dr. Andrew Liu

Course Description: This course serves as a graduate-level introduction to the subject of linear programming (also referred to as linear optimization), which has been widely used in operations research, engineering and economics. The course will start with a brief review to needed knowledge on linear algebra and matrix analysis. It will cover theories on optimality conditions, the simplex method, duality, and sensitivity analysis of linear programming problems. This course will also introduce students to modeling languages, such as AMPL and GAMS, and various off-the-shelf solvers through NEOS Online Optimization Server.

Course Goals: The major goal of this course is to build a solid foundation in both theory and algorithms of linear optimization problems so that students can apply such knowledge to their own research problems. The other goal is to familiarize the students with modern solvers of optimization problems.

Learning Objectives: At the conclusion of this course, the students should be able to:

- Understand what extreme points and basic feasible solutions are.
- Understand the optimality conditions of linear programming.
- Understand why the simplex method works and can solve small-scale LPs using the simplex method.
- Understand the weak and strong duality theorems of linear programming.
- Can perform simple sensitivity analysis on LPs.
- Can use modeling languages/tools to solve larger-scale LPs.

Prerequisite: Multivariate calculus. Linear algebra and matrix analysis at the undergraduate level. Programming skills.

Required Textbook:

- *Linear Programming and Network Flows* by Mokhtar S. Bazaraa, John J. Jarvis and Hanif D. Sherali, **4th edition**, Wiley, 2010. [BJS10] (Freely available online through Purdue's library)
- *Introduction to Linear Optimization* by Dimitris Bertsimas and John N. Tsitsiklis, Athena Scientific, 1997. [BT97]

Recommended Reference Books:

- *Linear and Nonlinear Programming* by David G. Luenberger and Yinyu Ye, 3rd edition, 2008. Springer, New York.

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- *Linear Programming: Foundations and Extensions* by Robert J. Vanderbei, 4th edition, 2014. Springer, New York.
- *Linear Programming with MATLAB* by Michael C. Ferris, Olvi L. Mangasarian and Stephen J. Wright, 2007. SIAM Philadelphia.
- *Matrix Analysis* by Roger A. Horn and Charles R. Johnson, 1st edition (1985) or 2nd edition (2012), Cambridge University Press.

Homework:

- There will be about 8 homework assignments (**submissions through Gradescope**).
- Handwritten submission is sufficient; however, you must write as neatly as possible. (No requirements on types of paper.)

Exams: There will be 3 exams (the last one is not a final exam), all in-class (open books/notes).

Project: There will be a final project. (More technical details and requirements will be provided.)

The project will involve using a **programming language** to code one of the LP algorithms and to use the coded algorithm to solve some small LPs.

Grading scale (not on a curve) –

- A [85, 100]
- B [70, 84]
- C [55, 69]
- D [45, 54]
- F < 45

Tentative Course Outline:

Block 1. Introduction

- What is linear programming
- Mathematical modeling
 - Linear programming modeling problems ([BJS10] Sec. 1.2)
 - Graphical representation and solution ([BJS10] Sec. 1.3)
- Linear algebra background ([BJS10] Sec. 2.1 – 2.3)
 - Vectors and matrices
 - Subspace, span, and basis
 - Linear system of equations

Block 2. Geometry of linear programming

- Polyhedra and convex sets ([BJS10] Sec. 2.4, 2.5)
- Extreme points, vertices and basic feasible solutions ([BJS10] Sec. 2.6)
- Representation of polyhedral sets ([BJS10] Sec. 2.7)

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Block 3. The simplex method

- Optimality conditions ([BJS10] Sec. 3.1, 3.2)
- Motivation and insights of the simplex method ([BJS10] Sec. 3.3, 3.4)
- Algebra of the simplex method ([BJS10] Sec. 3.5)
- Implementation of the simplex method
 - Termination ([BJS10] Sec. 3.7)
 - Tableau format ([BJS10] Sec. 3.8)
 - Starting the simplex method ([BJS10] Sec. 4.1 – 4.4)
 - Degeneracy and cycling ([BJS10] Sec. 4.6)

Block 4. Duality theory and sensitivity analysis

- Duality theory (primal-dual relationships) ([BJS10] Sec. 6.1, 6.2)
- Economic interpretation of the dual ([BJS10] Sec. 6.3)
- Dual simplex method ([BJS10] Sec. 6.4, 6.5)
- Sensitivity analysis and parametric programming ([BJS10] Sec. 6.7, 6.8)

Block 5. Interior-point method (depends on the available lecture time) or large-scale linear programming (decomposition methods)

Important omissions: 1. Network-flow problems (will cover if time permits); 2. Integer programming; 3. Stochastic programming