Role of Control-Structure Interaction in Protective System Design

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Abstract

Most of the current research in the field of structural control for mitigation of responses due to environmental loads neglects the effects of control-structure interaction in the analysis and design. The importance of including control-structure interaction when modeling a control system is discussed herein. A specific model for hydraulic actuators typical of those used in many protective systems is developed, and experimental verification of this model is given. Examples are provided which employ seismically excited structures configured with active bracing, active tendon, and active mass driver systems. These examples show that accounting for control-structure interaction and actuator dynamics can significantly improve the performance and robustness of a protective system.

Introduction

The concept of structural control for civil engineering applications originated in the early 70’s (Yao, 1972). In the two decades since, much progress has been made toward exploiting the potential benefits offered by control for protection of structures against environmental loads such as strong earthquakes and high winds (e.g., Soong, 1990; Housner and Masri, 1990, 1993; ATC-17, 1993).

Nearly all of the current literature on control of civil engineering structures does not directly account for control-structure interaction and actuator/sensor dynamics in the analysis and design of protective systems. Unmodeled control-structure interaction (CSI) effects can severely limit both the performance and robustness of protective systems. This is true for both active and semi-active systems. To study effectively the control-structure interaction problem, one must have good models for the dynamics of the associated actuators.

This paper presents a general framework within which one can study the effect of control-structure interaction. Specific models are developed for hydraulic actuators typical of those used in many active structural control situations. A natural velocity feedback link is shown to exist, which tightly couples the dynamic characteristics of a hydraulic actuator to the dynamics of the structure to which it is attached. Neglecting this feedback interaction can produce poor, or perhaps catastrophic, performance of the controlled system due to the unmodeled or mismodeled dynamics of the actuator-structure interaction. In addition, the time lag in generation of control forces is accommodated through appropriate modeling of the actuator and the associated CSI. Experimental verification of the main concepts is presented. The implications on protective system design are illustrated through examples of seismically excited structures. Active bracing, active tendon and active mass driver (AMD) systems are considered.

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Problem Formulation

Figure 1 provides a schematic diagram of a general active/semi-active structural control problem. The controller receives measurements from the sensors and forms a command input vector \( \mathbf{u} \) to the control actuator. The control actuator then applies a force vector \( \mathbf{f} \) to the structure. However, when mechanical actuators (e.g., hydraulic actuators) are used to control structures, there is generally a dynamic coupling between the actuator and the structure, as represented by the dotted arrow in Fig. 1. This coupling indicates that it is not possible to separate dynamical representations of the structure and the actuator, and model them as independent systems connected in series.

To understand the implications of the dynamical coupling, examine more closely the actuator and structural portions of the control system (i.e., the region in Fig. 1 enclosed in the dashed box). Consider the case in which the system has one actuator, with a single command input \( \mathbf{u} \) generating a single output force \( \mathbf{f} \). Fig. 2 provides a block diagram description of this case, in which the interaction can be modelled from the output. Here \( G_a \) is the transfer function of the actuator, and \( G_{yf} \) is the transfer function from the force applied by the actuator to the structural responses. Because the dashed line in Fig. 1, representing the feedback interaction between the structure and the actuator, often has associated dynamics, these dynamics are
represented by the transfer function \( H_i \) in Fig. 2. Thus, the overall transfer function from the control input \( u \) to the structural response \( y \) is given by

\[
G_{yu} = \frac{G_y G_a}{1 + G_y G_a H_i}.
\]  

(1)

If these dynamic systems are represented by numerator and denominator polynomials in \( s \), the representation of the system becomes

\[
G_{yu} = \frac{n_{yu}}{d_{yu}} = \frac{n_{yu} n_{a} d_{i}}{d_{yu} d_{a} d_{i} + n_{yu} n_{a} n_{i}}.
\]  

(2)

The transfer function from the command input \( u \) to the force \( f \) applied to the structure is given by

\[
G_{fu} = \frac{G_a}{1 + G_y G_a H_i}.
\]  

(3)

From Eq. (3), it is clear that the dynamics of the transmission from \( u \) to \( f \) are not simply the actuator dynamics \( G_a \), but contain dynamics due to the structure and the actuator. More insight can be gained by rewriting Eq. (3) in terms of the associated numerator and denominator polynomials, i.e.,

\[
G_{fu} = \frac{n_{fu}}{d_{fu}} = \frac{d_{fu} n_{a} d_{i}}{d_{fu} d_{a} d_{i} + n_{fu} n_{a} n_{i}}.
\]  

(4)

Comparing Eqs. (2) and (4), notice that unless pole/zero cancellation occurs, the transfer functions \( G_{yu} \) and \( G_{fu} \) have the same poles, and that the poles of the structure (i.e., the poles of \( G_y \)) are zeros of \( G_{fu} \). Cancellation is most unlikely, and therefore this possibility is disregarded in the remainder of the paper. Because the poles of the structure are zeros of \( G_{fu} \), actuators attached to lightly damped structures will have a greatly limited ability to apply forces at the structure’s natural frequencies; and if the structure is undamped, the actuator will not be able to apply a force at its natural frequencies. Also note that poles of the structure do not appear as poles in \( G_{yu} \), because they are cancelled by the zeros in \( G_{fu} \). These results occur regardless of how fast the dynamics of the actuator are (including the case when \( G_a \) is a constant gain).

Before closing the section, the effect of neglecting the interaction between the control actuator and the structure is considered. Using the definition of \( G_{fu} \) in Eq. (4), an alternative block diagram to that in Fig. 2 can be determined as shown in Fig. 3. Because the dynamics of an actuator, as given in \( G_a \), may often be fast relative to the structure, one might argue that the block \( G_{fu} \) may be reasonably represented as a constant both in phase and magnitude. However, as shown previously, neither the phase nor the magnitude of \( G_{fu} \) will be constant in general. Moreover, the phase and magnitude characteristics of \( G_{fu} \) will vary depending on the structure. Neglecting phase differences between the command input \( u \) and the resulting

\[\text{Figure 3. Equivalent block diagram model of the actuator/structure.}\]
force \( f \), i.e., neglecting the CSI, will result in an apparent time delay associated in the literature with generation of the control forces.

The next section presents a simple model which shows that a feedback interaction path is always present in hydraulic actuators. Experimental verification of the model is also provided.

**Hydraulic Actuator Modeling**

In the case of hydraulically actuated systems, a feedback path exists between the velocity of the actuator and the command input to the actuator. From DeSilva (1989), the equations describing the fluid flow rate in an actuator can be linearized about the origin to obtain

\[
\text{Valve: } q = k_q c - k_c f, \\
\text{Hydraulic Actuator: } q = A \dot{x} + \frac{V}{2\beta A} \dot{f},
\]

where \( q \) is the flow rate, \( c \) is the valve input, \( f \) is the force generated by the actuator, \( A \) is the cross-sectional area of the actuator, \( \beta \) is the bulk modulus of the fluid, \( V \) is the characteristic hydraulic fluid volume for the actuator, \( x \) is the actuator displacement, and \( k_q, k_c \) are system constants. Equating Eqs. (5) and (6) and rearranging yields

\[
\dot{f} = \frac{2\beta}{V} \left( A k_q c - k_c f - A^2 \dot{x} \right),
\]

which shows that the dynamics of the force applied by the actuator are dependent on the velocity response of the actuator, i.e., the feedback interaction path is intrinsic to the dynamical response of a hydraulic actuator.

Figure 4 is a block diagram representation of the hydraulic actuator model given in Eq. (7) attached to a structure. Here, \( G_{xf} \) denotes the transfer function from the force generated by the actuator to the displacement of the point on the structure where the actuator is attached, and \( f_L \) is the external load on the structure. Notice the presence of the “natural” velocity feedback in the open-loop system. Through this “natural” velocity feedback, the dynamics of the structure directly affect the characteristics of the control actuator.

![Figure 4. Block diagram of open-loop servovalve/actuator model.](image-url)
The portion of the system in Fig. 4 identified as $G_h$ has the following transfer function:

$$G_h = \frac{A}{V} \frac{1}{s + k_c} = \frac{A}{\tau_h s + 1}, \quad (8)$$

where $\tau_h = V/2\beta k_c$ is the time constant of the actuator. Thus, the transfer function from the valve input $c$ to the force $f$ is given by

$$G_{fc} = \frac{k_q G_h}{1 + sA G_h G_{xf}}, \quad (9)$$

and the transfer function from the valve input $c$ to the actuator displacement $x$ is given by

$$G_{xc} = \frac{k_q G_h G_{xf}}{1 + sA G_h G_{xf}}. \quad (10)$$

Representing these transfer functions in terms of their respective numerator and denominator polynomials gives

$$G_{fc} = \frac{n_{fc}}{d_{fc}} = \frac{k_q n_h d_{xf}}{d_h d_{xf} + sA n_h n_{xf}}, \quad (11)$$

$$G_{xc} = \frac{n_{xc}}{d_{xc}} = \frac{k_q n_h n_{xf}}{d_h d_{xf} + sA n_h n_{xf}}. \quad (12)$$

As discussed in the previous section, Eqs. (11) and (12) show that these two transfer functions have the same poles and that the poles of the structure (i.e., the poles of $G_{xf}$) will be zeros of the transfer function from the input to the applied force.

Because the open-loop system identified in Fig. 4 is typically unstable, position, velocity and/or force feedback may be used to stabilize the system. Here, a unity-gain position feedback loop, i.e., position control, is considered. Figure 5 is the block diagram for the hydraulic actuator with the position feedback included. This configuration is typical of those found in active structural control systems (see, for example, Chung, et al., 1988, 1989).

The transfer function from the command $u$ to the actuator force $f$ and to the displacement $x$ for the system in Fig. 5, including the unity-gain position feedback loop, are given respectively by

$$G_{fu} = \frac{\gamma G_{fc}}{1 + \gamma G_{fc} G_{xf}} = \frac{\gamma k_q G_h}{1 + (\gamma k_q + sA) G_h G_{xf}}, \quad (13)$$

$$G_{xu} = \frac{\gamma G_{fc} G_{xf}}{1 + \gamma G_{fc} G_{xf}} = \frac{\gamma k_q G_h G_{xf}}{1 + (\gamma k_q + sA) G_h G_{xf}}, \quad (14)$$

where $\gamma$ is the proportional feedback gain stabilizing the actuator. Rewriting the Eqs. (13) and (14) in terms of their numerator and denominator polynomials yields
Again, from Eqs. (15) and (16), the poles of the structure, \( G_{xf} \), become zeros of the transfer function from the command \( u \) to the actuator force \( f \), and are then cancelled in the transfer function from command to actuator position.

As an alternative to using position feedback to stabilize the hydraulic actuator, a velocity and/or force feedback loop can be added. Considering the general case in which a combination of all three measurements is used, the resulting transfer functions are

\[
G_{fu} = \frac{n_{fu}}{d_{fu}} = \frac{\gamma k_q n_h d_{xf}}{d_h d_{xf} + (\gamma k_q + sA) n_h n_{xf}}, \tag{15}
\]

\[
G_{xu} = \frac{n_{xu}}{d_{xu}} = \frac{\gamma k_q n_h n_{xf}}{d_h d_{xf} + (\gamma k_q + sA) n_h n_{xf}}. \tag{16}
\]

Figure 5. Block diagram of closed-loop servovalve/actuator model.

Using position feedback to stabilize the hydraulic actuator, a velocity and/or force feedback loop can be added. Considering the general case in which a combination of all three measurements is used, the resulting transfer functions are

\[
G_{fu} = \frac{\gamma k_q G_h}{1 + sA G_h G_{xf} + \delta \gamma k_q G_h + (\alpha + \eta s) \gamma k_q G_h G_{xf}}, \tag{17}
\]

\[
G_{xu} = \frac{\gamma k_q G_h G_{xf}}{1 + sA G_h G_{xf} + \delta \gamma k_q G_h + (\alpha + \eta s) \gamma k_q G_h G_{xf}}, \tag{18}
\]

where \( \alpha \) is the position feedback gain, \( \eta \) is the velocity feedback gain, and \( \delta \) is the force feedback gain. These transfer functions can be written in terms of their respective numerator and denominator polynomials as
Similarly to the case of position control, the poles of $G_{fu}$ are those of the overall transfer function $G_{xu}$, and the poles of the structure (i.e., the poles of $G_{xf}$) are the zeros of the transfer function from the actuator command $u$ to the applied actuator force $f$.

**Experimental Verification**

To demonstrate the validity of this model of a hydraulic actuator, the above results were compared to experimental data obtained at the Earthquake Engineering/Structural Dynamics and Control Laboratory at the University of Notre Dame. A scale model of the prototype building discussed in Chung, et al. (1988, 1989) was the test structure, shown in Fig. 6. The total mass of the floors of the model is 1,100 kg (500 lb), distributed evenly between the three levels, and the structure is 157.5 cm (62 in.) tall. The time scale was decreased by a factor of five, making the natural frequency of the structure five times that of the prototype. Cross-braces can be attached to the top two floors, causing the structure to respond primarily as a SDOF structure (see Fig. 6). A hydraulic control actuator with a ±5.08 cm (±2 in.) stroke was placed at the first floor of the building and attached to the seismic simulator table via a rigid frame. For this system, the actuator displacement is equivalent to the displacement of the first floor. A position sensor was therefore used to measure the displacement of the first floor and to provide feedback for the control actuator. The force

\[
G_{fu} = \frac{n_{fu}}{d_{fu}} = \frac{\gamma k_q n_h d_{xf}}{d_h d_{xf} + s A n_h n_{xf} + \delta \gamma k_q n_h d_{xf} + (\alpha + \eta s) \gamma k_q G_h G_{xf}}, \tag{19}
\]

\[
G_{xu} = \frac{n_{xu}}{d_{xu}} = \frac{\gamma k_q n_h n_{xf}}{d_h d_{xf} + s A n_h n_{xf} + \delta \gamma k_q n_h d_{xf} + (\alpha + \eta s) \gamma k_q G_h G_{xf}}. \tag{20}
\]
transmitted to the building by the control actuator was measured with a piezoelectric force ring manufactured by PCB Piezotronics, Inc.

Experimental transfer functions were found for the test structure and actuator using a Tektronix 2630 spectrum analyzer. Figure 7 shows the magnitude of the building and actuator transfer functions for the SDOF case (i.e., with the cross-braces attached). The fundamental frequency of the structure in the SDOF configuration is 7 Hz. Examining the transfer function from the input command to the applied force, $G_{fu}$, it is clear that significant modeling error would be incurred if one took this transfer function to be constant (i.e., neglected the actuator dynamics and the CSI). Notice that the zeros of $G_{fu}$ coincide with the poles of the structure. Also, in the transfer function from the command input $u$ to the actuator displacement $x$, $G_{xu}$,
the poles and zeros cancel and a new complex pole pair appears at a higher frequency. This behavior is exactly what is predicted by the model presented in this section.

Figure 8 shows the magnitude of the building and actuator transfer functions when the braces are removed and the model is in the three degree-of-freedom configuration. The actuator transfer function $G_{fu}$ in this case is significantly different than it is with the SDOF structure. Notice, as in the SDOF case, that the poles of the structure are cancelled by the zeros of the actuator in the transfer function from the control input $u$ to the structural displacement $x$, $G_{sxu}$, and new poles appear at poles of the actuator transfer function.

To verify the actuator model, the transfer function $G_{hu}$ was determined from the experimental data in the SDOF case. The proportional feedback gain stabilizing the actuator, $\gamma$, was set at 2.5. This user-defined constant can be changed through adjustment of the potentiometer of the servo-valve amplifier. By using Eq. (9) and the experimental data for $G_{fu}$ and $G_{hu}$, $A/k_q$ was determined as a function of frequency. Although $k_q$ is a nonlinear parameter dependent on the operating point and the response amplitude of the hydraulic actuator (DeSilva, 1989), it can reasonably be assumed constant over the operating and frequency range of interest. The value of $A/k_q$ which best fits the experimental data below 40 Hz was determined to be 0.15. Knowing $A/k_q$, ratios of the various parameters in Eq. (8) were determined to fit $G_{hu}$. The values of the ratios which define $k_q G_h$ are $k_q A/k_c = 25$ and $V/2\beta k_c = 0.015$, which determine the actuator transfer function as

$$k_q G_h = \frac{25}{0.015s + 1}. \quad (21)$$

Substituting Eq. (21) into Eq. (9) and using the experimentally obtained transfer function for $G_{sf}$, the transfer function $G_{fc}$ can be obtained. A comparison between these results and those obtained directly from the experimental data is given in Fig. 9. Notice the excellent agreement between the experimental and calculated results.

Because of the excellent agreement between the model and the experiment, the model of the hydraulic actuator in Eq. (7) is concluded to be valid.

![Figure 9. Experimental and calculated transfer functions $G_{fc}$ for SDOF model.](image_url)
Illustrative Examples

This section provides examples that demonstrate the importance of accounting for the interaction between the control actuator and the structure in protective system design. An active bracing, active tendon and active mass driver system are considered. Several controllers are designed for each of the example systems. Three control models are employed in the design of the various controllers. The first model, designated Model 1, uses the full equations of motion. In the second model the fluid in the hydraulic actuator was assumed to be incompressible (i.e., $\dot{f} = 0$ in equation Eq. (7)). In this pseudo static model, designated Model 2, the hydraulic stiffness and damping terms are still included. In most studies of the control of civil engineering structures, $G_{fu}$ is considered to be constant in magnitude and phase. This assumption is employed for the third model considered (Model 3). In each example, the value of $K_0$, the constant magnitude of $G_{fu}$ used for control design, was found by determining the DC value of $G_{fu}$ for the complete model which considers actuator dynamics and CSI (Model 1). Controllers were designed based on each of these models.

Several performance objectives were also considered. The objective for the type A controller was to minimize the relative displacements of each floor by equally weighting the relative displacement measurements in the performance function. For the type B controller, the performance objective was to minimize the absolute accelerations of each floor by equally weighting the respective absolute acceleration measurements. The identifications and descriptions for the various controllers are given in Table 1.

Because the measured output vector $\mathbf{y}$ is not the full state vector, the controllers are observer based and are designed using $H_2$/Linear Quadratic Gaussian (LQG) design methods (Spencer, et. al., 1991, 1994; Suhardjo, et. al. 1992). Both low authority (Case 1) and high authority (Case 2) controllers were considered. To allow for direct comparison of each type of controller, the respective weightings on the displacements and accelerations are determined such that, when the ground acceleration is taken to be a given broadband excitation, the RMS control force for each case has the same magnitude. The RMS responses are determined through solution of the associated Lyapunov equation (Soong and Grigoriu, 1991). For all control studies, the model for which the responses are calculated is Model 1, incorporating both actuator dynamics and CSI.

### Table 1: Controller Design Descriptions.

<table>
<thead>
<tr>
<th>Model used for Controller Design</th>
<th>Displacement Weighting (Type A)</th>
<th>Acceleration Weighting (Type B)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Including CSI (Model 1)</td>
<td>1A</td>
<td>1B</td>
</tr>
<tr>
<td>Neglecting compressibility (Model 2)</td>
<td>2A</td>
<td>2B</td>
</tr>
<tr>
<td>Constant $G_{fu}$ (Model 3)</td>
<td>3A</td>
<td>3B</td>
</tr>
</tbody>
</table>

### Example 1: Active Bracing

Consider the three-story, single-bay building subjected to a one-dimensional earthquake excitation $\ddot{x}_g$ with active bracing as depicted in Fig. 6. The equations of motion are

$$
\begin{bmatrix}
  m_1 & 0 & 0 \\
  0 & m_2 & 0 \\
  0 & 0 & m_3
\end{bmatrix}
\begin{bmatrix}
  \ddot{x}_1 \\
  \ddot{x}_2 \\
  \ddot{x}_3
\end{bmatrix}
= 
\begin{bmatrix}
  c_{11} & c_{12} & c_{13} \\
  c_{12} & c_{22} & c_{23} \\
  c_{13} & c_{23} & c_{33}
\end{bmatrix}
\begin{bmatrix}
  \dot{x}_1 \\
  \dot{x}_2 \\
  \dot{x}_3
\end{bmatrix}
+ 
\begin{bmatrix}
  k_{11} & k_{12} & k_{13} \\
  k_{12} & k_{22} & k_{23} \\
  k_{13} & k_{23} & k_{33}
\end{bmatrix}
\begin{bmatrix}
  x_1 \\
  x_2 \\
  x_3
\end{bmatrix}
- 
\begin{bmatrix}
  1 \\
  0 \\
  0
\end{bmatrix}
0
- 
\begin{bmatrix}
  0 & 0 & m_3
\end{bmatrix}
\begin{bmatrix}
  \ddot{x}_g
\end{bmatrix},
$$

(22)
where $x_i$ and $m_i$ are the displacement relative to the ground and the mass of the $i$th floor of the building, $c_{ij}$ and $k_{ij}$ are the damping and stiffness coefficients, respectively, and $f$ is the control force applied by the hydraulic actuator. Strictly speaking, $m_1$ includes the weight of the actuator rod and piston. However, this additional mass is usually negligible in comparison with the first floor mass. Equation (22) can be written in matrix form as

$$
M_s \ddot{x} + C_s \dot{x} + K_s x = B_s f - M_s \Gamma_s \ddot{x}_g.
$$

Incorporating unity gain displacement feedback into the hydraulic actuator model given in Eq. (7) yields

$$
\dot{f} = 2\frac{B}{V} \left(Ak_q \gamma (u - x_1) - k_c f - A^2 \dot{x}_1 \right),
$$

where $u$ is the control command. Defining the state vector of the system as $z_1 = [x' \ x' \ f]'$, the state equation is

$$
\dot{z}_1 = \begin{bmatrix}
0 & I & 0 \\
-M_s^{-1}K_s & -M_s^{-1}C_s & M_s^{-1}B_s \\
2\beta Ak_q \gamma & 0 & -2\beta A^2 \\
\frac{-2\beta A^2}{V} & 0 & -2\beta k_c \\
\frac{-2\beta A^2}{V} & 0 & -2\beta A^2 \\
\frac{-2\beta A^2}{V} & 0 & -2\beta A^2 \\
\end{bmatrix} z_1 + \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
\end{bmatrix} u + \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
\end{bmatrix} \ddot{x}_g,
$$

$$
= Az_1 + Bu + Ez_1.
$$

Assuming that the actuator displacement (i.e., the relative displacement of the first floor) $x_1$, the absolute accelerations of each floor, $\ddot{x}_1a, \ddot{x}_2a, \ddot{x}_3a$, and the applied control force, $f$, are measured, the measurement equation is

$$
y = \begin{bmatrix}
x_1 \\
x_{1a} \\
x_{2a} \\
x_{3a} \\
f
\end{bmatrix} = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix} z_1 + \begin{bmatrix}
v_1 \\
v_2 \\
v_3 \\
v_4 \\
v_5 \\
\end{bmatrix},
$$

$$
= Cz_1 + v
$$

The vector $v$ contains the noises in each measurement. The model given in Eqs. (25) and (26) (Model 1) includes actuator dynamics and CSI.

Model 2 for this example is found by setting $\dot{f} = 0$ in Eq. (24). The resulting algebraic relation for the force is then given by

$$
f = \frac{A\gamma k_q}{k_c} u - \frac{A\gamma k_q}{k_c} x_1 - \frac{A^2}{k_c} \dot{x}_1.
$$

Using the state vector $z_2 = [x' \ x' \ f]'$, the state equation reduces to
Using the same measured outputs as above, the measurement equation can be written

\[
z_2 = \begin{bmatrix} 0 & I \\ -M_s^{-1} K_s & -M_s^{-1} C_s \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ -A_k q \gamma & \frac{-A^2}{m_1 k_c} \\ m_1 k_c & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ -A \gamma \frac{q M_s^{-1} B_s}{k_c} \end{bmatrix} u + \begin{bmatrix} 0 \\ -\Gamma_s \end{bmatrix} \ddot{x}_g. \tag{28}
\]

Using the same measured outputs as above, the measurement equation can be written

\[
y = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -M_s^{-1} K_s & -M_s^{-1} C_s \\ -A_k \gamma \frac{q}{k_c} & 0 & -A^2 \frac{1}{m_1 k_c} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ -A_k \gamma \frac{q M_s^{-1} B_s}{k_c} \end{bmatrix} u + v. \tag{29}
\]

Model 3 considers \( G_{fu} \) to be constant in magnitude and phase. The constant gain, \( K_0 \) is determined from the complete model given in Eqs. (25) and (26). Using the state vector \( z_3 = [x' \ x']' \) and the same measurements as above, the state-space representation reduces to

\[
z_3 = \begin{bmatrix} 0 & I \\ -M_s^{-1} K_s & -M_s^{-1} C_s \end{bmatrix} z_3 + \begin{bmatrix} 0 \\ -A \gamma \frac{q M_s^{-1} B_s}{k_c} \end{bmatrix} K_0 u + \begin{bmatrix} 0 \\ -\Gamma_s \end{bmatrix} \ddot{x}_g, \tag{30}
\]

\[
y = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -M_s^{-1} K_s & -M_s^{-1} C_s \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} z_3 + \begin{bmatrix} 0 \\ -A \gamma \frac{q M_s^{-1} B_s}{k_c} \end{bmatrix} K_0 u + v. \tag{31}
\]

The structural parameters for the three degree-of-freedom model reported in Chung, et. al. (1989) were employed in this example with an active bracing system. The parameters associated with the control actuator were chosen to correspond to the model presented previously. Here, the ground acceleration was modeled as a broadband disturbance with a constant two-sided spectral density of magnitude \( S_0 = 1.53 \text{ cm}^2/\text{s}^3 \) \( (2.37 \times 10^{-1} \text{ in}^2/\text{s}^3) \). The structural responses to this disturbance are shown in Table 2 for the type A controllers (i.e., weighting the displacements of the structure) and in Table 3 for type B controllers (i.e., weighting the accelerations of the structure). Here, the uncontrolled configuration has the active bracing system completely removed from the structure. The zeroed controller corresponds to the case in which the active bracing system is attached, but the input command signal is set equal to zero. This configuration is included because it has been used as a basis for comparison in previously reported control studies.

Examining the zeroed control case, one observes that the stiffness of the structure increases (as compared to the uncontrolled case), thus causing the first floor displacement to decrease. Notice that if the relative displacements are weighted (Table 2), the controller which includes actuator dynamics and CSI produces noticeably better results than either of the other two controllers. For the high authority controller (Case 2), the closed-loop system created with Controller 3A becomes unstable before the chosen force level is reached.

When absolute accelerations are weighted, the responses using Controller 1B (i.e., including actuator dynamics and CSI, and designed based on Model 1) and Controller 2B (i.e., designed based on Model 2 which considers the fluid to be incompressible) are very close for both the low and high authority cases.
These results are considerably better than those corresponding to Controller 3B (i.e., neglecting the actuator dynamics and CSI).

**Table 2: Comparison of RMS responses for active bracing system with weighting on the relative displacements.**

<table>
<thead>
<tr>
<th>Configuration</th>
<th>$\sigma_{x_1}$ (cm)</th>
<th>$\sigma_{x_2}$ (cm)</th>
<th>$\sigma_{x_3}$ (cm)</th>
<th>$\sigma_{\dot{x}_1}$ (cm/s$^2$)</th>
<th>$\sigma_{\dot{x}_2}$ (cm/s$^2$)</th>
<th>$\sigma_{\dot{x}_3}$ (cm/s$^2$)</th>
<th>$\sigma_f$ (N)</th>
</tr>
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<tbody>
<tr>
<td>(a) nominal configurations</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Uncontrolled</td>
<td>3.478e-1</td>
<td>7.087e-1</td>
<td>9.058e-1</td>
<td>249.6</td>
<td>199.1</td>
<td>243.2</td>
<td>—</td>
</tr>
<tr>
<td>Zeroed control</td>
<td>3.104e-2</td>
<td>2.720e-1</td>
<td>4.234e-1</td>
<td>159.6</td>
<td>138.0</td>
<td>152.7</td>
<td>2668</td>
</tr>
<tr>
<td>(b) Case 1: low authority controller</td>
<td></td>
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<td></td>
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<tr>
<td>Controller 1A</td>
<td>5.077e-2</td>
<td>1.135e-1</td>
<td>1.783e-1</td>
<td>269.7</td>
<td>126.8</td>
<td>101.5</td>
<td>2687</td>
</tr>
<tr>
<td>Controller 2A</td>
<td>4.140e-2</td>
<td>1.400e-1</td>
<td>2.183e-1</td>
<td>268.2</td>
<td>143.3</td>
<td>114.2</td>
<td>2687</td>
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<tr>
<td>Controller 3A</td>
<td>2.982e-2</td>
<td>2.677e-1</td>
<td>4.178e-1</td>
<td>164.5</td>
<td>137.7</td>
<td>151.7</td>
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<tr>
<td>(c) Case 2: high authority controller</td>
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<tr>
<td>Controller 1A</td>
<td>6.528e-2</td>
<td>8.669e-2</td>
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<td>109.0</td>
<td>3802</td>
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<td>Controller 2A</td>
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<td>118.2</td>
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<tr>
<td>Controller 3A</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
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</table>

**Table 3: Comparison of RMS responses for active bracing system with weighting on the absolute accelerations.**

<table>
<thead>
<tr>
<th>Configuration</th>
<th>$\sigma_{x_1}$ (cm)</th>
<th>$\sigma_{x_2}$ (cm)</th>
<th>$\sigma_{x_3}$ (cm)</th>
<th>$\sigma_{\ddot{x}_1}$ (cm/s$^2$)</th>
<th>$\sigma_{\ddot{x}_2}$ (cm/s$^2$)</th>
<th>$\sigma_{\ddot{x}_3}$ (cm/s$^2$)</th>
<th>$\sigma_f$ (N)</th>
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<tr>
<td>(a) nominal configurations</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Uncontrolled</td>
<td>3.478e-1</td>
<td>7.087e-1</td>
<td>9.058e-1</td>
<td>249.6</td>
<td>199.1</td>
<td>243.2</td>
<td>—</td>
</tr>
<tr>
<td>Zeroed control</td>
<td>3.104e-2</td>
<td>2.720e-1</td>
<td>4.234e-1</td>
<td>159.6</td>
<td>138.0</td>
<td>152.7</td>
<td>2668</td>
</tr>
<tr>
<td>(b) Case 1: low authority controller</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Controller 1B</td>
<td>2.560e-1</td>
<td>3.462e-1</td>
<td>3.746e-1</td>
<td>212.6</td>
<td>33.86</td>
<td>41.33</td>
<td>2687</td>
</tr>
<tr>
<td>Controller 3B</td>
<td>1.442e-1</td>
<td>2.437e-1</td>
<td>3.256e-1</td>
<td>141.2</td>
<td>105.3</td>
<td>108.6</td>
<td>2687</td>
</tr>
<tr>
<td>(c) Case 2: high authority controller</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Controller 1B</td>
<td>3.416e-1</td>
<td>4.366e-1</td>
<td>4.567e-1</td>
<td>32.46</td>
<td>30.20</td>
<td>35.43</td>
<td>3802</td>
</tr>
<tr>
<td>Controller 2B</td>
<td>3.404e-1</td>
<td>4.364e-1</td>
<td>2.562e-1</td>
<td>34.47</td>
<td>29.97</td>
<td>34.95</td>
<td>3802</td>
</tr>
<tr>
<td>Controller 3B</td>
<td>2.568e-1</td>
<td>3.426e-1</td>
<td>3.988e-1</td>
<td>138.2</td>
<td>103.7</td>
<td>106.5</td>
<td>3802</td>
</tr>
</tbody>
</table>
One should point out that the systems employing controllers designed based on Model 2 or Model 3 can quickly become unstable if one tries to reduce the RMS responses through increasing the weighting on the relative displacements or the absolute accelerations in the performance function. However, by accounting for the actuator dynamics/CSI a significantly more authoritative control design (i.e., higher performance) can be achieved without such an instability occurring.

The results given in Tables 2 and 3 indicate that, for a given level of RMS control action, the ability of the controller to reduce the relative displacements of the structure is greatest when the relative displacements of the structure are weighted in the control performance function. Similarly, these tables indicate that the absolute accelerations of the structure are most efficiently reduced when the absolute accelerations of the structure are weighted.

**Example 2: Active Tendon System**

Consider the single story structure subjected to a one-dimensional earthquake excitation $\ddot{x}_g$ with an active tendon system as shown in Fig. 10. In this system a tendon/pulley system is used to transmit the force generated by the hydraulic actuator to the first floor of the structure. A stiff frame is included to connect the actuator to the four pretensioned tendons. The linearized equations of motion are

\begin{align}
m\dddot{x} + \left( c + c_0 \cos^2 \theta \right) \ddot{x} + \left( k + k_0 \cos^2 \theta \right) x + c_0 \left( \cos \theta \right) \dot{a} + k_0 \left( \cos \theta \right) a &= -m\dddot{x}_g \tag{32} \\
m_0 \dddot{a} + c_0 \ddot{a} + k_0 a + c_0 \cos \theta \ddot{x} + k_0 \cos \theta x &= -m_0 \dddot{x}_g + f \tag{33}
\end{align}

where $m$ is the mass of the building, $m_o$ is the combined mass of the stiff frame and the actuator rod/piston, $c$ and $k$ are the damping and stiffness coefficients of the structure, respectively, $c_0$ and $k_0$ are the total damping and stiffness of the four tendons, $x$ is the displacement of the building relative to the ground, $a$ is the displacement of the actuator, and $f$ is the force applied by the hydraulic actuator to the rigid frame. In Fig. 10, the force designated $f_o$ is the force in the tendons.

Under unity gain feedback of the actuator position, the dynamics of the hydraulic system can be written as

$$
\dot{f} = \frac{2\beta}{V} \left( A_k f - A c_{\dot{a}} - A_c \dot{a} \right).
$$

(34)

Defining the state vector of the system as $z_1 = [a \ \dot{a} \ \dot{x} \ f]'$, the state equation is

![Figure 10. Single story building with active tendon system.](image-url)
The measurements are chosen to include the displacement of the actuator and the absolute acceleration of the building, i.e., \( \mathbf{y} = [a \ \dot{a} \ x \ \ddot{x}]' \). The measurement equation is thus

\[
\mathbf{y} = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
\frac{k_0}{m} \cos \theta & \frac{c_0}{m} \cos \theta & \frac{k_0}{m} \cos^2 \theta + k & \frac{c_0}{m} \cos^2 \theta + c & 0 \\
\frac{2\beta}{V} \frac{A}{k_c} & \frac{2\beta}{V} A^2 & 0 & 0 & -\frac{2\beta}{V} k_c
\end{bmatrix} \mathbf{z}_1 + \mathbf{v}
\]

where the vector \( \mathbf{v} \) contains the noises in each measurement. The model in Eqs. (35) and (36) includes actuator dynamics and CSI and is designated Model 1.

Model 2 is found by setting \( \dot{f} = 0 \) in Eq. (34). Using this approach will result in a controller that is equivalent to that used in Reinhorn, et. al. (1989). The resulting algebraic relation for the force is then given by

\[
f = \frac{Ak_q \gamma}{k_c} (u - a) - \frac{A^2}{k_c} \dot{a} \ .
\]

Using the state vector \( \mathbf{z}_2 = [a \ \dot{a} \ x \ \ddot{x}]' \) and the same measured outputs as above, the state-space representation of Model 2 is

\[
\dot{\mathbf{z}}_2 = \begin{bmatrix}
\frac{k_0}{m} & \frac{k_0}{m} \cos \theta & \frac{k_0}{m} \cos^2 \theta + k & \frac{k_0}{m} \cos^2 \theta + c & 0 \\
0 & 0 & 0 & 0 & 1 \\
\frac{c_0}{m} \cos \theta & \frac{c_0}{m} \cos \theta & \frac{c_0}{m} \cos^2 \theta + k & \frac{c_0}{m} \cos^2 \theta + c & 0 \\
\frac{2\beta}{V} \frac{A}{k_c} & \frac{2\beta}{V} A^2 & 0 & 0 & -\frac{2\beta}{V} k_c
\end{bmatrix} \mathbf{z}_2 + \mathbf{v}
\]

\[
\mathbf{y} = \begin{bmatrix}
1 & 0 & 0 & 0 \\
\frac{k_0}{m} \cos \theta & \frac{k_0}{m} \cos \theta & \frac{k_0}{m} \cos^2 \theta + k & \frac{k_0}{m} \cos^2 \theta + c
\end{bmatrix} \mathbf{z}_2 + \mathbf{v}
\]
Model 3 is found by assuming the transfer function $G_{fu}$ is constant in magnitude and has zero phase. Using the state vector $z_3 = [a \  \dot{a} \ x \ \dot{x}]'$ and the same measurements as above, the state-space representation for Model 3 is determined as

$$ z_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{k_0}{m} & -\frac{c_0}{m} & \frac{k_0}{m} \cos \theta & -\frac{c_0 \cos \theta}{m} \\ 0 & 0 & 0 & 1 \\ \frac{k_0}{m} \cos \theta & -\frac{c_0 \cos \theta}{m} & \frac{k_0}{m} \cos^2 \theta + k & -\frac{c_0 \cos^2 \theta + c}{m} \end{bmatrix} \begin{bmatrix} 0 \\ K_0 \\ 0 \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ -1 \end{bmatrix} \dot{x}_g $$

$$ y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{k_0}{m} \cos \theta & -\frac{c_0 \cos \theta}{m} & \frac{k_0}{m} \cos^2 \theta + k & -\frac{c_0 \cos^2 \theta + c}{m} \end{bmatrix} z_3 + v $$

In this example, the structural parameters were chosen to correspond to the SDOF test structure described in Chung, et. al (1988). Their values were: $m = 2,924 \text{ kg} \ (16.69 \text{ lb-s}^2/\text{in})$, $c = 15.8 \text{ N \cdot s/cm} \ (9.025 \text{ lb-s/in})$, $k = 13,895 \text{ N/m} \ (7934 \text{ lb-in})$, $\theta = 36$ degrees, $k_o = 14,879 \text{ N/m} \ [8496 \text{ lb-in} \ (i.e., \ 2124 \times 4)]$, and $c_o = 0$. The mass of the frame, $m_o$, was chosen to be 0.417 lb-s$^2$/in. The ground acceleration was modeled as a broadband excitation with a constant two-sided spectral density of magnitude $S_0 = 0.2568 \text{ cm}^2/\text{s}^3 \ (3.98 \times 10^{-2} \text{ in}^2/\text{s}^3)$. In all cases, the response calculations were based on the model in Eqs. (35) and (36). Here, the uncontrolled configuration refers to the case in which the tendons are present, but are fixed to the ground. The zeroed configuration refers to the case in which the actuator is attached to the tendons, but the command signal of the control actuator is set equal to zero.

The response statistics are provided in Table 4 for the various type A controllers (displacement weighting) and in Table 5 for the various type B controllers (acceleration weighting). Notice that in the case of displacement weighting (Table 4), a stable controller for Model 2 cannot be designed at the force levels chosen. Also, application of both the low and high authority controllers designed using Model 3 (Controller 3A) has a detrimental effect on the displacement response of the system compared to the uncontrolled system.

All of the control designs which minimize the absolute acceleration produce a significant reduction in the acceleration as well as the displacement (see Table 5), except the controller which is based on Model 3 (Controller 3B). The low authority controller designed based on Model 3 (Controller 3B) has little effect of the acceleration response of the structure compared to the uncontrolled case, and the high authority controller actually increases the acceleration response. At this force level, the control design based on Model 2 (Controller 2B) performs comparably to Controller 1B, but this controller becomes unstable if the weighting on the acceleration is increased above this value. As in the previous example, the displacements are most efficiently reduced when the displacements are weighted, and the accelerations are most efficiently reduced when the accelerations are weighted.

To better understand the effects of CSI and actuator dynamics in this example, the transfer function from the actuator command to the actuator displacement is provided in Fig. 11 for the active tendon system. Notice that a complex pole pair is present at approximately 90 Hz due to the stabilizing position feedback. Also, the poles of the uncontrolled structure (i.e., with the actuator removed and the tendons attached to the ground) have become zeros of the actuator transfer function, $G_{fu}$. The transfer functions from the actuator command to the actuator force, $G_{fu}$, and to the tendon force, $G_{f_o u}$, are shown in Fig. 12. Compa-
Table 4: Comparison of RMS values of controlled responses for the active tendon system with weighting on the relative displacement.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>$\sigma_x$ (cm)</th>
<th>$\sigma_a$ (cm)</th>
<th>$\sigma_{x_a}$ (cm/s²)</th>
<th>$\sigma_f$ (N)</th>
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</thead>
<tbody>
<tr>
<td>Uncontrolled</td>
<td>4.295e-2</td>
<td>–</td>
<td>34.73</td>
<td>–</td>
</tr>
<tr>
<td>Zeroed Control</td>
<td>2.748e-2</td>
<td>3.065e-3</td>
<td>21.18</td>
<td>299.4</td>
</tr>
<tr>
<td><strong>Case 1: Low Authority Controller</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Controller 1A</td>
<td>3.752e-3</td>
<td>2.457e-2</td>
<td>12.5</td>
<td>575.4</td>
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<td>3.692e-2</td>
<td>4.900e-2</td>
<td>21.74</td>
<td>575.4</td>
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<td>Controller 3A</td>
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<td>9.324e-3</td>
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<td>2.687e-2</td>
<td>12.7</td>
<td>1829</td>
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<td>Controller 2A</td>
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<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Controller 3A</td>
<td>1.111e-1</td>
<td>3.484e-2</td>
<td>104.0</td>
<td>1829</td>
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</table>

Table 5: Comparison of RMS values of controlled responses for the active tendon system with weighting on the absolute acceleration.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>$\sigma_x$ (cm)</th>
<th>$\sigma_a$ (cm)</th>
<th>$\sigma_{x_a}$ (cm/s²)</th>
<th>$\sigma_f$ (N)</th>
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<tr>
<td>Uncontrolled</td>
<td>4.295e-2</td>
<td>–</td>
<td>34.73</td>
<td>–</td>
</tr>
<tr>
<td>Zeroed Control</td>
<td>2.748e-2</td>
<td>3.065e-3</td>
<td>21.18</td>
<td>299.4</td>
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<tr>
<td><strong>Case 1: Low Authority Controller</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Controller 1B</td>
<td>3.748e-2</td>
<td>6.871e-2</td>
<td>3.686</td>
<td>575.4</td>
</tr>
<tr>
<td>Controller 2B</td>
<td>3.722e-2</td>
<td>6.845e-2</td>
<td>3.736</td>
<td>575.4</td>
</tr>
<tr>
<td>Controller 3B</td>
<td>3.043e-2</td>
<td>5.653e-2</td>
<td>14.84</td>
<td>575.4</td>
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<td><strong>Case 2: High Authority Controller</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Controller 1B</td>
<td>1.091e-1</td>
<td>2.111e-1</td>
<td>2.606</td>
<td>1829</td>
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<tr>
<td>Controller 2B</td>
<td>1.077e-1</td>
<td>2.086e-1</td>
<td>2.770</td>
<td>1829</td>
</tr>
<tr>
<td>Controller 3B</td>
<td>1.058e-1</td>
<td>2.061e-1</td>
<td>16.47</td>
<td>1829</td>
</tr>
</tbody>
</table>
ing these two transfer functions, it is evident that the two forces are not the same. Due to the presence of a complex zero pair in \( G_{fu} \), the two transfer functions are significantly different in magnitude at high frequencies, and above approximately 23 Hz they are 180 degrees out of phase.
Example 3: Active Mass Driver

Consider the single story structure subjected to a one-dimensional earthquake excitation $\ddot{x}_g$ with an active mass driver as shown in Fig. 13. The equations of motion are

$$M\ddot{x}_1 + (c_1 + c_2)\dot{x}_1 - c_2\dot{x}_2 + (k_1 + k_2)x_1 - k_2x_2 = -f - M\ddot{x}_g,$$

$$m\ddot{x}_2 + c_2(\dot{x}_2 - \dot{x}_1) + k_2(x_2 - x_1) = f - m\ddot{x}_g,$$

where $M$ is the mass of the building, $m$ is the mass of the AMD (including the mass of the actuator rod/piston), $x_1$ and $x_2$ are the displacement relative to the ground of the building and the moving mass, respectively, $c_1$ and $k_1$ are the damping and stiffness coefficients of the building, respectively, $c_2$ and $k_2$ are the damping and stiffness coefficients of the active mass driver system, respectively, and $f$ is the control force applied by the hydraulic actuator.

From above, the equation governing the dynamics of the hydraulic system under unity-gain feedback of the actuator displacement (i.e., $x_2 - x_1$) can be written as

$$\dot{f} = \frac{2\beta}{V} \left( Ak_q \gamma (u - (x_2 - x_1)) - k_c - A^2 (\dot{x}_2 - \dot{x}_1) \right).$$

Defining the state vector of the system as $z_1 = [x_1 \ x_2 \ \dot{x}_1 \ \dot{x}_2 \ f]'$, the equations of motion can be written in matrix form as

$$\dot{z}_1 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ \frac{(k_1 + k_2)}{M} & \frac{k_2}{M} & \frac{(c_1 + c_2)}{M} & \frac{c_2}{M} & \frac{1}{M} \\ \frac{k_2}{m} & \frac{k_2}{m} & \frac{c_2}{m} & \frac{c_2}{m} & \frac{1}{m} \\ \frac{2\beta Ak_q \gamma}{V} & \frac{2\beta Ak_q \gamma}{V} & \frac{2\beta A^2}{V} & \frac{2\beta A^2}{V} & \frac{2\beta k_c}{V} \end{bmatrix} z_1 + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \frac{2\beta A k_q \gamma}{V} \end{bmatrix} u + \begin{bmatrix} 0 \\ 0 \\ -1 \ddot{x}_g \end{bmatrix}.\quad(45)$$

Figure 13. Single story building with active mass driver.
The measurements are assumed to be: displacement of the first floor mass relative to the ground, displacement of the AMD relative to the first floor mass, the absolute accelerations of both masses, and the force applied by the actuator, i.e., \( y = [x_1 (x_2 - x_1) \dot{x}_a_1 \dot{x}_a_2 f]' \). Thus, the measurement equation is

\[
y = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ \frac{(k_1 + k_2)}{M} & \frac{k_2}{M} & \frac{(c_1 + c_2)}{M} & \frac{c_2}{M} & 1 \\ \frac{k_2}{m} & \frac{k_2}{m} & \frac{c_2}{m} & \frac{c_2}{m} & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} z_1 + \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix},
\]

where \( v_i \) is the noise in the \( i \)th measurement. This model of the AMD system accounts for both the dynamics of the actuator and CSI and is designated Model 1.

Model 2 is formed by setting \( f = 0 \) in Eq. (44), resulting in the algebraic relation for the force given by

\[
f = \frac{A\gamma k q}{k_c} u - \frac{A\gamma k q}{k_c} (x_2 - x_1) - \frac{A^2}{k_c} (\dot{x}_2 - \dot{x}_1) .
\]

Using the state vector \( z_2 = [x_1 x_2 \dot{x}_1 \dot{x}_2]' \) and the same measured outputs as above, the state-space representation of this model is

\[
z_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{(k_1 + k_2) + d_1}{M} & \frac{k_2 + d_1}{M} & \frac{(c_1 + c_2) + d_2}{M} & \frac{c_2 + d_2}{M} \\ \frac{k_2 + d_1}{m} & \frac{k_2 + d_1}{m} & \frac{c_2 + d_2}{m} & \frac{c_2 + d_2}{m} \end{bmatrix} z_2 + \begin{bmatrix} 0 \\ 0 \\ \frac{-d_1}{M} \\ \frac{d_1}{m} \end{bmatrix} u + \begin{bmatrix} 0 \\ 0 \\ \dot{x}_g \end{bmatrix},
\]

where \( d_1 = A\gamma k q / k_c \), and \( d_2 = A^2 / k_c \).

Model 3 is formed by assuming \( G_{fu} \) has a constant magnitude and zero phase. Using the state vector \( z_3 = [x_1 x_2 \dot{x}_1 \dot{x}_2]' \) and the same measurements as above, the state-space representation for Model 3 is

\[
y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ \frac{(k_1 + k_2) + d_1}{M} & \frac{k_2 + d_1}{M} & \frac{(c_1 + c_2) + d_2}{M} & \frac{c_2 + d_2}{M} \\ \frac{k_2 + d_1}{m} & \frac{k_2 + d_1}{m} & \frac{c_2 + d_2}{m} & \frac{c_2 + d_2}{m} \end{bmatrix} z_2 + \begin{bmatrix} 0 \\ 0 \\ \frac{-d_1}{M} \\ \frac{d_1}{m} \end{bmatrix} u + v
\]

where \( d_1 = A\gamma k q / k_c \), and \( d_2 = A^2 / k_c \).
In this example, the structural parameters were chosen to correspond to the experimental model in the SDOF configuration described in the experimental verification section. The values were: $M = 245$ kg ($1.4$ lb-s$^2$/in), $c_1 = 3.06$ N-s/m (1.75 lb-s/in), and $k_1 = 4,991$ N/m (2850 lb/in). The AMD was chosen such that the mass was 2% that of the structure, the stiffness $k_2 = 99.8$ N/m (57 lb/in) was chosen to tune the AMD to the natural frequency of the structure, and the damping $c_2$ was considered to be negligible and set equal to zero. Using the constants for the hydraulic actuator model presented in experimental verification section of this paper and the above parameters, the matrices in the above models were formed. For all of the controllers, the model on which the responses calculations were based is given in Eqs. (45) and (46). In each case, the ground acceleration was modeled as a broadband excitation with a constant two-sided spectral density of magnitude $S_0 = 3.83 \times 10^{-1}$ in$^2$/s$^3$.

Herein, the controlled response using the AMD is compared to the uncontrolled structure (i.e., with the AMD removed). The zeroed control configuration is not presented for the AMD system because the results are similar to the responses of the uncontrolled configuration. The response statistics for the various type A controllers are provided in Table 6. For the high authority controller (Case 2), notice that while requiring the same RMS control force, Controller 1A reduced the relative displacement of the first floor by 51.3%, whereas Controller 3A only produced a 38.5% reduction in relative displacement. The system formed using Controller 2A became unstable before this force level was achieved. Response statistics for the three type B controllers are shown in Table 7. Notice that the RMS absolute accelerations for Controllers 1B and 2B are the very close in both the high and low authority cases, and these controllers produce significantly better results than Controller 3B which considers the actuator transfer function to be constant over all frequencies. Also notice that for this example, the relative displacement response with Controller 1B (acceleration weighting) is nearly the same as for Controller 1A (displacement weighting) when the same amount of actuator force is employed. For this example, it would be slightly more beneficial to use a control strategy which weights the acceleration of the structure than one which weights the relative displacement. In addition, the controlled systems formed by using Controllers 2A, 2B, 3A, and 3B become unstable at high displacement/acceleration weighting in the performance function (i.e., high authority control). If the compressibility of the hydraulic fluid is accounted for, a more authoritative (i.e., higher performance) controller can be designed without such an instability occurring.
Table 6: Comparison of RMS values of controlled responses for the AMD model with weighting on the relative displacement.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>$\sigma_{x_1}$ (cm)</th>
<th>$\sigma_{x_1-x_2}$ (cm)</th>
<th>$\sigma_{x_1}$ (cm/s²)</th>
<th>$\sigma_f$ (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) nominal configuration</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Uncontrolled</td>
<td>6.787e-2</td>
<td>—</td>
<td>140.1</td>
<td>—</td>
</tr>
<tr>
<td>(b) Case 1: low authority controller</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Controller 1A</td>
<td>4.321e-2</td>
<td>1.094e-1</td>
<td>86.49</td>
<td>27.15</td>
</tr>
<tr>
<td>Controller 2A</td>
<td>5.509e-2</td>
<td>5.100e-2</td>
<td>110.1</td>
<td>27.15</td>
</tr>
<tr>
<td>Controller 3A</td>
<td>5.255e-2</td>
<td>7.173e-2</td>
<td>104.1</td>
<td>27.15</td>
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<tr>
<td>(c) Case 2: high authority controller</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Controller 1A</td>
<td>3.353e-2</td>
<td>1.866e-1</td>
<td>67.51</td>
<td>46.84</td>
</tr>
<tr>
<td>Controller 2A</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Controller 3A</td>
<td>4.229e-2</td>
<td>1.463e-1</td>
<td>83.01</td>
<td>46.84</td>
</tr>
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</table>

Table 7: Comparison of RMS values of controlled responses for the AMD model with weighting on the absolute acceleration.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>$\sigma_{x_1}$ (cm)</th>
<th>$\sigma_{x_1-x_2}$ (cm)</th>
<th>$\sigma_{x_1}$ (cm/s²)</th>
<th>$\sigma_f$ (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) nominal configuration</td>
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<td></td>
<td></td>
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</tr>
<tr>
<td>Uncontrolled</td>
<td>6.787e-2</td>
<td>—</td>
<td>140.1</td>
<td>—</td>
</tr>
<tr>
<td>(b) Case 1: low authority controller</td>
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<td></td>
<td></td>
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<tr>
<td>Controller 1B</td>
<td>4.181e-2</td>
<td>1.197e-1</td>
<td>83.26</td>
<td>27.15</td>
</tr>
<tr>
<td>Controller 2B</td>
<td>4.181e-2</td>
<td>1.196e-1</td>
<td>83.06</td>
<td>27.15</td>
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<tr>
<td>Controller 3B</td>
<td>5.936e-2</td>
<td>4.338e-2</td>
<td>117.7</td>
<td>27.15</td>
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<tr>
<td>Controller 1B</td>
<td>3.272e-2</td>
<td>2.004e-1</td>
<td>64.85</td>
<td>46.84</td>
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<tr>
<td>Controller 2B</td>
<td>3.277e-2</td>
<td>1.999e-1</td>
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<tr>
<td>Controller 3B</td>
<td>4.663e-2</td>
<td>1.113e-1</td>
<td>91.92</td>
<td>46.84</td>
</tr>
</tbody>
</table>
Conclusions

The role of control-structure interaction in the design of protective systems has been investigated, and the importance of accounting for actuator dynamics and control-structure interaction has been demonstrated. For the case of hydraulic control actuation, a natural velocity feedback interaction path has been shown to exist. This feedback, together with the stabilizing displacement (and/or force, velocity) feedback, causes control-structure interaction to be intrinsic to the device. The dynamic model presented for the hydraulic actuator was verified experimentally, as well as the predicted control-structure interaction behavior. Examples employing seismically excited structures have been provided which show that considering actuator dynamics and control-structure interaction in the design of a controller significantly improves the performance of the controlled system.

The following conclusions from this work should be emphasized:

• When the feedback interaction path is present, the poles of the structure will appear as zeros of the transfer function from the command input to the force applied to the structure. This result occurs regardless of how fast the dynamics of the actuator are (including the case when $G_a$ is a constant gain).
• For actuators attached to lightly damped structures in which the feedback interaction path is present, the ability of the actuator to apply forces at the structure’s natural frequencies is greatly limited. The actuator cannot apply a force at the natural frequencies of an undamped structure.
• Hydraulic actuators, both active and semi-active, have an implicit feedback interaction path that occurs due to the natural velocity feedback of the actuator response. This interaction occurs for actuators configured in both displacement, velocity and/or force control.
• Simple models can be employed to represent the dynamics of the hydraulic actuator and the associated control-structure interaction.
• Most researchers in the control of civil engineering structures have neglected the dynamics of the actuator, as well as the control-structure interaction effect. This approach is equivalent to assuming that the transfer function $G_{fu}$ is constant in magnitude with zero phase. In general, neither the phase nor the magnitude of $G_{fu}$ will be constant. Neglecting phase differences between the command input $u$ and the resulting force $f$, will result in a time lag associated with generation of the control forces. Appropriate modeling of the actuator dynamics and control-structure interaction accommodates this time lag.
• In a structural control system for a given level of control action, neglecting the actuator dynamics generally results in larger responses than in the case where the control-structure interaction is considered. Also, neglecting actuator dynamics and control-structure interaction results in less achievable performance of the controller because the closed loop system more easily becomes unstable.
• Better results are obtained if the compressibility of the hydraulic fluid is taken into account than if the fluid is treated as incompressible. Also, in the latter case, the achievable performance level of the controller is significantly reduced due to instabilities created in the closed-loop system.
• For the examples of active bracing and active tendon systems considered herein, the structural response quantities are more efficiently reduced if they are directly weighted in the control performance function. Thus, to reduce the absolute structural accelerations, one should directly weight these acceleration responses in the control performance function.
• For the active mass driver example, the relative displacements and absolute accelerations are most efficiently reduced by weighting the absolute accelerations of the structure.
• In general, modeling errors resulting from neglecting actuator dynamics and control-structure interaction can be expected to decrease both the stability and performance robustness of the controlled structure.

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References


