Semi-Active Control Strategies for MR Dampers:
A Comparative Study

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Abstract

This paper presents the results of a study to evaluate the performance of a number of recently proposed semi-active control algorithms for use with multiple MR dampers. A variety of control algorithms used in recent semi-active control studies are considered including the Lyapunov controller, decentralized bang-bang controller, modulated homogeneous friction algorithm, and a clipped optimal controller. Each algorithm is formulated for use with the MR damper. Additionally, each algorithm uses measurements of the absolute acceleration and device displacements for determining the control action to ensure that the algorithms would be implementable on a physical structure. The performance of the algorithms is compared through a numerical example, and the advantages of each algorithm are discussed. The numerical example considers a six story structure controlled with MR dampers on the lower two floors. In simulation, an El Centro earthquake is used to excite the system, and the reduction in the drifts, accelerations, and relative displacements throughout the structure is examined.

1. Introduction

Magnetorheological (MR) dampers have, over the last several years, been recognized as having a number of attractive characteristics for use in vibration control applications (Kamath and Wereley, 1995, 1997a-b; Gordanejad, 1999; Weiss, et. al., 1994; Ginder, et. al., 1996; Spencer et al., 1997b; Spencer and Sain, 1997; Dyke and Spencer, 1996; Dyke et al., 1998). MR fluids were developed in the 1940’s (Winslow, 1947; Rainbow, 1948), and consist of a suspension of iron particles in a carrier medium such as oil. Application of a magnetic field to the fluid causes the particles to align and interparticle bonds increase the resistance of the fluid, turning the fluid into a semi-solid (Weiss, et. al., 1994; Ginder, et. al., 1996). MR dampers are relatively inexpensive to manufacture because the fluid properties are not sensitive to contaminants. Other attractive features include their small power requirements, reliability, and stability. Requiring only 20–50 watts of power, these devices can operate with a battery, eliminating the need for a large power source or generator. Because the device forces are adjusted by varying the

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strength of the magnetic field, no mechanical valves are required, making a highly reliable device. Additionally, the fluid itself responds in milliseconds, which allows for the development of devices with a high bandwidth.

MR devices are classified as semi-active devices. Semi-active devices are characterized by their ability to dynamically vary their properties with a minimal amount of power (Housner et al., 1997; Spencer and Sain, 1997). Because semi-active devices can only absorb or store vibratory energy in a structure by reacting to its motion, they are considered to be stable (in a bounded-input, bounded-output sense). Thus, semi-active devices are expected to offer effective performance over a variety of amplitude and frequency ranges.

MR dampers have demonstrated promise for civil engineering applications in both analytical and experimental studies. Spencer et al., (1997d) developed a phenomenological model for an MR damper based on the Bouc-Wen hysteresis model (Wen, 1976). This model was subsequently used to demonstrate the capabilities of MR dampers (Dyke et al., 1996a–d). Further, Dyke, et al. (1998) conducted an experiment using a single MR damper to control a three-story structure. An evaluation of these control strategies was conducted for use with a single MR damper (Dyke and Spencer, 1997). In a numerical example, a linear multi-story building was controlled with a single MR damper. The results demonstrated that the performance of the controlled system is highly dependent on the choice of algorithm. Further studies have examined the effectiveness of the clipped-optimal controller for multi-input MR control systems (Yi et al., 1998, 1999a,b; Dyke et al., 1999; Dyke and Spencer, 1996). Additionally, a 20-ton MR damper is being tested at the University of Notre Dame (Spencer et al., 1997b; Carlson and Spencer, 1996b).

One challenge in the use of semi-active technology is in developing nonlinear control algorithms that are appropriate for implementation in full-scale structures. A number of control algorithms have been adopted for semi-active systems. In one of the first examinations of semi-active control, Karnopp et al. (1974) proposed a “skyhook” damper control algorithm for a vehicle suspension system and demonstrated that this system offers improved performance over a passive system when applied to a single-degree-of-freedom system. Feng and Shinozukah (1990) developed a bang-bang controller for a hybrid controller on a bridge. More recently a control strategy based on Lyapunov stability theory has been proposed for ER dampers (Brogan, 1991; Leitmann, 1994). The goal of this algorithm is to reduce the responses by minimizing the rate of change of a Lyapunov function. McClamroch and Gavin (1995) used a similar approach to develop a decentralized bang-bang controller. This control algorithm acts to minimize the total energy in the structure. A modulated homogeneous friction algorithm (Inaudi, 1997) was developed for a variable friction device. Clipped-optimal controllers have also been proposed and implemented for semi-active systems (Sack and Patten, 1994; Sack et al., 1994; Dyke, 1996a–d).

The effective utilization of multiple control devices is an important step in the examination of semi-active control algorithms. A typical control system for a full-scale structure is expected to have control devices distributed throughout a number of floors. Because of the inherent nonlinear nature of these devices, one of the challenging aspects of utilizing this technology to achieve high levels of perfor-
mance is in the development of appropriate control algorithms. The proper selection of a control algorithm may be dependent on the type of nonlinearity present in the semi-active device, the available feedback measurements, or the number of devices to be implemented in the structure.

The purpose of this study is to evaluate a selection of control algorithms for use in multi-input semi-active structural control systems. Four recently proposed semi-active control algorithms are discussed including the decentralized bang-bang controller, the Lyapunov controller, the clipped-optimal controller, and the modulated homogeneous friction controller. In addition to these, a related fifth algorithm, referred to herein as the maximum energy dissipation algorithm, is also considered. These algorithms are formulated for use with MR dampers. In a numerical example, a six-story building model with MR dampers on the bottom two floors is used to compare the performance of the proposed control algorithms. This example was selected to represent the experimental system in the Washington University Structural Control and Earthquake Engineering Lab (http://www.seas.wustl.edu/research/quake/). An experimentally-verified phenomenological model based on the Bouc-Wen model is used to simulate the behavior of the MR damper. The responses of the excited system are examined for each algorithm and the performance of the various control algorithms on the multi-input system are compared.

2. Shear Mode Magnetorheological Damper Modeling

Magnetorheological dampers are semi-active devices that use magnetorheological fluids to construct a versatile damping device. Because the strength of the magnetic field controls the yield stress of the fluid, devices utilizing MR fluids are expected to be applicable for a wide range of situations. For civil engineering applications MR devices are attractive because they require only a battery for power and are quite reliable (Kamath and Wereley, 1995, 1997a-b; Kamath, et. al., 1996, 1997; Gordaninejad, 1999; Carlson and Spencer, 1996a-b; Spencer et al., 1997b; Yi et al., 1998, 1999; Dyke, et al. 1999a-b). Furthermore, they are relatively inexpensive to manufacture and maintain, and their insensitivity to temperature fluctuations makes them suitable for both indoor and outdoor applications (Carlson, 1994; Carlson and Weiss, 1994; Carlson, et. al. 1996).

A prototype shear mode MR damper was obtained from the Lord Corporation for experimental testing (Yi et al., 1998, 1999a,b; Dyke, et al. 1999). A schematic diagram of the prototype device is shown in Fig. 1. The device consists of two steel parallel plates. The dimensions of the device are 4.45×1.9×2.5 cm³ (1.75×0.75×1.0 in³). The magnetic field produced in the device is generated by an electromagnet consisting of a coil at one end of the device. Forces are generated when the moving plate, coated with a thin foam saturated with MR fluid, slides between the two parallel plates. The outer plates of the device are 0.635 cm (0.25 in) apart, and the force capacity of the device is dependent on the strength of the fluid and on the size of the gap between the side plates and the center plate. A center plate with a thickness of 0.495 cm (0.195 in) is employed, resulting in a gap of
Tests were conducted on the experimental prototype MR damper. A hydraulic actuator was used to drive the damper, and the displacement and force were measured. The velocities were calculated using a central differences approximation. Sinusoidal, triangular, and square displacement command signals were used. Various constant and time-varying voltages were applied to the prototype MR damper to observe the characteristics of the MR damper. Typical hysteresis loops for the shear mode MR damper obtained through experimental testing are provided in Figure 2. The response of the MR damper due to a 1.5 Hz sinusoid with an amplitude of 1.5 cm is shown for constant voltage levels, 0 V, 1.0 V, 2.0 V, and 3.0 V, being applied to the pulse width modulation circuit used with the MR damper. The force-displacement hysteresis loop is shown in Fig. 2a and the force-velocity hysteresis loop is shown in Fig. 2b. Further details on the experimental behavior of the MR damper are provided in Yi et al. (1998, 1999a,b) and Dyke, et al. (1999).

Adequate modeling of the control devices is essential for the accurate prediction of the behavior of the controlled system. The simple mechanical model shown
in Fig. 3 was developed and shown to accurately predict the behavior of a shear-mode MR damper over a wide range of inputs (Yi, et al., 1998, 1999a,b; Dyke et al, 1999). This phenomenological model was developed based on a previous model used for a MR damper (Spencer et al., 1997d).

The equations governing the force $f$ predicted by this model are

$$f = c_0 \dot{x} + \alpha z \quad \text{(1)}$$

$$\dot{z} = -\gamma |\dot{x}|z|^{n-1} - \beta |x|^n + A \dot{x} \quad \text{(2)}$$

where $z$ is an evolutionary variable that accounts for the history dependence of the response. The model parameters depend on the voltage $v$ to the current driver as follows

$$\alpha = \alpha_a + \alpha_b u \quad \text{and} \quad c_0 = c_{0a} + c_{0b} u \quad \text{(3)}$$

where $u$ is given as the output of the first-order filter

$$\dot{u} = -\eta(u - v). \quad \text{(4)}$$

Eq. (4) is used to model the dynamics involved in reaching rheological equilibrium and in driving the electromagnet in the MR damper (Yi et al, 1998, 1999a,b; Dyke et al., 1999). This MR damper model is used herein to model the behavior of the MR damper.

3. Control Algorithms

Consider a seismically excited structure controlled with $n$ MR dampers. Assuming that the forces provided by the control devices are adequate to keep the response of the primary structure from exiting the linear region, the equations of motion can be written as

$$M_s \ddot{x} + C_s \dot{x} + K_s x = \Lambda f - M_s \Gamma \dot{x}_g \quad \text{(5)}$$

where $x$ is a vector of the relative displacements of the floors of the structure, $\dot{x}_g$ is a one-dimensional ground acceleration, $f = [f_1, f_2, \ldots f_n]^T$ is the vector of measured control forces, defined by Eqs. (1-4), generated by the $n$ MR dampers, $\Gamma$ is a column vector of ones, and $\Lambda$ is a vector determined by the placement of the MR dampers in the structure. This equation can be written in state-space form as
\[ z = Az + Bf + E_x g \]  
\[ y = Cz + Df + v \]  

where \( z \) is the state vector, \( y \) is the vector of measured outputs, and \( v \) is the measurement noise vector. For these applications, the measurements typically available for control force determination include the absolute acceleration of selected points on the structure, the displacement of each control device, and a measurement of each control force.

A variety of approaches have been proposed in the literature for the control of semi-active devices. Subsequently, a selection of these approaches will be presented and evaluated in a numerical example. In developing the control laws, note that it is not possible to directly command the \( i \)th MR damper to generate a specified force, \( f_i \), because the response of the MR damper is dependent on the local motion of the structure where the MR damper is attached. However, the forces produced by the MR damper may be increased or decreased by adjusting the value of the voltage applied to the current driver \( v_i \). Based on this observation in the model, the following guidelines are used in developing the control laws: i) the control voltage to the \( i \)th device is restricted to the range \( v_i = [0, V_{max}] \), and ii) for a fixed set of states, the magnitude of the applied force \( |f_i| \) increases when \( v_i \) increases, and decreases when \( v_i \) decreases. Furthermore, the first order lag in the device model (representing the dynamics involved in the current driver and electromagnet) limits the rate at which the MR effect is realized. Thus, in developing the control laws, one must consider the fact that the force varies continuously even when a step command signal is applied.

### 3.1 Control Based on Lyapunov Stability Theory

In some cases it is possible to employ Lyapunov’s direct approach to stability analysis in the design of a feedback controller (Brogan, 1991). The approach requires the use of a Lyapunov function, denoted \( V(z) \), which must be a positive definite function of the states of the system, \( z \). Let us assume that the origin is a stable equilibrium point. According to Lyapunov stability theory, if the rate of change of the Lyapunov function, \( \dot{V}(z) \), is negative semi-definite, the origin is stable i.s.L. (in the sense of Lyapunov). Thus, in developing the control law, the goal is to choose control inputs for each device that will result in making \( \dot{V} \) as negative as possible. An infinite number of Lyapunov functions may be selected, that may result in a variety of control laws.

Leitmann (1994) applied Lyapunov’s direct approach for the design of a semi-active controller. In this approach, a Lyapunov function is chosen of the form
where $||z||_p$ is the $P$-norm of the states defined by

$$
||z||_p = [z^T P z]^{1/2}
$$

and $P$ is a real, symmetric, positive definite matrix. In the case of a linear system, to ensure $\dot{V}$ is negative definite, the matrix $P$ is found using the Lyapunov equation

$$
A^T P + PA = -Q_p
$$

for a positive definite matrix $Q_p$. The derivative of the Lyapunov function for a solution of Eq. (6) is

$$
\dot{V} = -\frac{1}{2} z^T Q_p z + z^T P f + z^T P E x_g
$$

The only term which can be directly effected by a change in the control voltage is the middle term which contains the force vector $f$. Thus, the control law which will minimize $\dot{V}$ is

$$
v_i = V_{max} H((-z)^T P B_i f_i)
$$

where $H(\cdot)$ is the Heaviside step function, $f_i$ is the measured force produced by the $i$th MR damper, and $B_i$ is the $i$th column of the $B$ matrix in Eq. (6). Notice that this algorithm is classified as a bang-bang controller, and is dependent on the sign of the measured control force and the states of the system. To implement this algorithm, a Kalman filter is used to estimate the states based on the available measurements ($i.e.$, device displacements, device forces, structural accelerations). Thus, in this algorithm, better performance is expected when measurements of the responses of the full structure are used. However, one challenge in the use of the Lyapunov algorithm is in the selection of an appropriate $Q_p$ matrix.

3.2 Decentralized Bang-Bang Control

McClamroch and Gavin (1995) used a similar approach to develop the decentralized bang-bang control law for use with an electrorheological damper. In this approach, the Lyapunov function was chosen to represent the total vibratory energy in the structure (kinetic plus potential energy), as in
Using Eq. (5), the rate of change of the Lyapunov function is then
\[ V' = \frac{1}{2} x^T K_s x + \frac{1}{2} (x + \Gamma \dot{x}_g)^T M_s (x + \Gamma \dot{x}_g) \]
(13)

In this expression, the only way to directly effect \( V' \) is through the last term containing the force vector \( f \). To control this term and make \( V' \) as large and negative as possible (maximizing the rate at which energy is dissipated), the following control law is chosen
\[ v_i = V_{max} H(- (x + \Gamma \dot{x}_g)^T \Lambda f_i) \]
(15)

where \( \Lambda_i \) is the \( i \)th column of the \( \Lambda \) matrix. Note that, because the only non-zero terms in the \( \Lambda \) matrix are those corresponding to the location of the MR dampers, this control law requires only measurements of the floor velocities and applied forces. Interestingly, when any of the semi-active devices are located between the ground and first floor, the absolute velocity of the first floor is required. When the control device is located in the upper floors, the interstory velocity is needed. Therefore, to implement this control algorithm, one would approximate the absolute velocity (obtain the pseudo velocity) by integrating the absolute acceleration (Spencer et al., 1997a) using
\[ H(s) = \frac{39.5s}{39.5s^2 + 8.89s + 1} \]
(16)

### 3.3 Maximum Energy Dissipation

This control algorithm is presented as a variation of the decentralized bang-bang approach proposed by McClamroch and Gavin. In the decentralized bang-bang approach, the Lyapunov function was chosen to represent the total vibratory energy in the system. Let us instead consider a Lyapunov function which represents the relative vibratory energy in the structure (\( i.e., \) without including the velocity of the ground in the kinetic energy term), as in
\[ V = \frac{1}{2} x^T K_s x + \frac{1}{2} x^T M_s x \]
(17)

Using the same procedure applied to develop the decentralized bang-bang approach, the term which can be directly effected by changes in the control voltage is identified and the following control law is obtained
\[ v_i = V_{max} H(-\dot{x}^T \Lambda_i f_i) \]  

(18)

where \( \Lambda_i \) is the \( i \)th column of the \( \Lambda \) matrix. Note that this equation is also a bang-bang control law. As in the decentralized bang-bang approach, only local measurements (i.e., the velocity and control force) are required to implement this control law. Note that if the semi-active device is not located on the first floor of the structure, the resulting control law will be the same as in the decentralized bang-bang approach. However, if the control device is on the first floor, notice that the control action depends on the relative velocity measurement rather than the absolute velocity which was used in the decentralized bang-bang approach. Both a numerical differentiation of the measured device displacements, and a subtraction of the absolute velocities using Eq. 16 were considered to determine the relative velocities. Numerical differentiation of the measurements of the relative displacement of the first floor was found to yield better results for this control algorithm and was used in this study.

Notice that the resulting control law will command the maximum voltage when the measured force and relative velocity are dissipating energy (producing large dissipative forces), and command the minimum voltage when energy is not being dissipated (producing small forces when the force is not dissipative). Thus, here it has been called the maximum energy dissipation algorithm.

### 3.4 Clipped-Optimal Control

One algorithm that has been shown to be effective for use with the MR damper is a clipped-optimal control approach, proposed by Dyke, et al. (1996c-e). The clipped-optimal control approach is to design a linear optimal controller \( K_c(s) \) that calculates a vector of desired control forces \( f_c = [f_{c1}, f_{c2}, \ldots, f_{cn}]^T \) based on the measured structural responses \( y \) and the measured control force vector \( f \) applied to the structure, i.e.,

\[
\begin{align*}
   f_c &= L^{-1}\left\{ -K_c(s)L \begin{bmatrix} y \\ f \end{bmatrix} \right\} \\
   &= L^{-1}\left\{ -K_c(s)Ly - K_c(s)Lf \right\}
\end{align*}
\]

(19)

where \( L(\cdot) \) is the Laplace transform.

Because the force generated in the MR damper is dependent on the local responses of the structural system, the desired optimal control force \( f_{ci} \) cannot always be produced by the MR damper. Only the control voltage \( v_i \) can be directly controlled to increase or decrease the force produced by the device. Thus, a force feedback loop is incorporated to induce the MR damper to generate approximately the desired optimal control force \( f_{ci} \).
To induce the MR damper to generate approximately the corresponding desired optimal control force \( f_{ci} \), the command signal \( v_i \) is selected as follows. When the \( i \)th MR damper is providing the desired optimal force (i.e., \( f_i = f_{ci} \)), the voltage applied to the damper should remain at the present level. If the magnitude of the force produced by the damper is smaller than the magnitude of the desired optimal force and the two forces have the same sign, the voltage applied to the current driver is increased to the maximum level so as to increase the force produced by the damper to match the desired control force. Otherwise, the commanded voltage is set to zero. The algorithm for selecting the command signal for the \( i \)th MR damper is graphically represented in Fig. 4 and can be stated as

\[
v_i = V_{\text{max}} H(f_{ci} - f_i) \]  

(20)

Although a variety of approaches may be used to design the optimal controller, \( H_2 \) /LQG methods are advocated because of their successful application in previous studies. The approach to optimal control design is discussed in detail in (Dyke et al., 1996a-e).

### 3.5 Modulated Homogeneous Friction

Another semi-active control algorithm considered herein was originally proposed for use with a variable friction damper (Inaudi, 1997). This algorithm is considered herein because there are strong similarities between the behavior of a variable friction device and of the MR damper. In this approach, at every occurrence of a local extrema in the deformation of the device (i.e., when the relative velocity between the ends of the semi-active device is zero), the normal force applied at the frictional interface is updated to a new value. The normal force, \( N_i(t) \), is chosen to be proportional to the absolute value of the deformation of the semi-active device. The control law is written

\[
N_i(t) = g_i |P[\Delta_i(t)]| \]  

(21)

where \( g_i \) is a positive gain, and the operator \( P[\cdot] \) (referred to as the prior-local-peak operator) is defined as
\[ P[\Delta_i(t)] = \Delta_i(t-s), \quad \text{where} \quad s = \{ \min x \geq 0 : \dot{\Delta}_i(t-x) = 0 \}, \quad (22) \]

defining \( \Delta_i(t-s) \) as the most recent local extrema in the deformation of the \( i \)th device.

Because this algorithm was developed for use with a variable friction device, the following modifications were necessary to apply it to the MR damper: i) there is no need to check if the force is greater than \( \mu N_i(t) \), where \( \mu \) is the coefficient of friction, because the MR damper is not subject to static friction, and ii) a force feedback loop was implemented to induce the MR damper to produce approximately the frictional force corresponding to the desired normal force. Thus, the goal is to generate a desired control force with a magnitude

\[ f_{ni} = \mu g_i |P[\Delta_i(t)]| = g_{ni} |P[\Delta_i(t)]| \quad (23) \]

where the proportionality constant \( g_{ni} \) has units of stiffness (N/cm). For further clarification, Figure 5 shows a plot of the typical desired control force produced by this algorithm as a function of the device displacement. \( f_{ni} = g_{ni} \Delta_i \) is shown here as a dashed line because at each peak in the displacement, the magnitude of the desired control force is selected according to this relationship.

As in the clipped-optimal control law, because the force produced by the MR damper cannot be directly commanded, a force feedback loop is used. The measured force is compared to the desired force determined by Eq. 23, and the resulting control law is

\[ v_i = V_{\max} H(f_{ni} - |f_i|). \quad (24) \]

An appropriate choice of \( g_{ni} \) will keep the force \( f_{ni} \) within the operating envelope of each MR damper a majority of the time, allowing the MR damper forces to closely approximate the desired force of each device. However, the optimal value of \( g_{ni} \) is dependent on the amplitude of the ground excitation. Additionally, notice that this control law is quite straightforward to implement because it requires only
measurements of applied force and the relative displacements of the control device.

4. Numerical Example

To evaluate these algorithms for use with the MR damper a numerical example is considered in which a model of a six-story building is controlled with four MR dampers. Two devices are rigidly connected between the ground and the first floor, and two devices are rigidly connected between the first and second floors, as shown in Fig. 6. Each MR damper is capable of producing a force equal to 1.8% the weight of the entire structure, and the maximum voltage input to the MR devices is $V_{\text{max}} = 5V$. The governing equations can be written in the form of Eq. (5) by defining the mass of each floor, $m_i$, as 0.227 N/(cm/sec$^2$) (0.129 lb/(in/sec$^2$)), the stiffness of each floor, $k_i$, as 297 N/cm (169 lb/in), and a damping ratio for each mode of 0.5%. This system is a simple representation of the scaled, six-story, test structure that is being used for experimental control studies at the Washington University Structural Control and Earthquake Engineering Laboratory.

In this example, the structural measurements available for calculating the control action include the absolute accelerations of the structure and the forces produced by the MR devices (i.e., $y = [\dot{x}_a1, \dot{x}_a2, \dot{x}_a3, \dot{x}_a4, \dot{x}_a5, f_1, f_2]^T$). Thus, the governing equations can be written in the form of Eqs. (6-7) by defining

$$A = \begin{bmatrix} 0 & I \\ -M_s^{-1}K_s & -M_s^{-1}C_s \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ M_s^{-1}\Lambda \end{bmatrix}, \quad E = -\begin{bmatrix} 0 \\ \Gamma \end{bmatrix},$$

$$C = \begin{bmatrix} -M_s^{-1}K_s & -M_s^{-1}C_s \end{bmatrix}, \quad D = \begin{bmatrix} -M_s^{-1}\Lambda \end{bmatrix}.$$
The MR damper parameters used in this study are $c_{0a} = 0.0064$ Nsec/cm, $c_{0b} = 0.0052$ Nsec/cmV, $\alpha_a = 8.66$ N/cm, $\alpha_b = 8.86$ N/cmV, $\gamma = 300$ cm$^{-2}$, $\beta = 300$ cm$^{-2}$, $A = 120$, and $n = 2$. These parameters were selected based on the identified model of the shear-mode prototype MR damper tested at Washington University (Yi, et al., 1999a,b).

In simulation, the model of the structure is subjected to the NS component of the 1940 El Centro earthquake. The simulations were performed in MATLAB (1994). Because the building system considered is a scaled model, the amplitude of the earthquake was scaled to ten percent of the full-scale earthquake to represent the magnitude of displacements that would be observed in laboratory experiments with this structure.

The various control algorithms were evaluated using a set of evaluation criteria based on those used in the second generation linear control problem for buildings (Spencer et al., 1997a). The first evaluation criterion is a measure of the normalized maximum floor displacement relative to the ground, given as

$$J_1 = \max_{t, i} \left( \frac{|x_i(t)|}{x_{\text{max}}} \right)$$

(25)

where $x_i(t)$ is the relative displacement of the $i$th floor over the entire response, and $x_{\text{max}}$ denotes the uncontrolled maximum displacement. The second evaluation criterion is a measure of the reduction in the interstory drift. The maximum of the normalized interstory drift is

$$J_2 = \max_{t, i} \left( \frac{|d_i(t)/h_i|}{d_{n\text{max}}} \right)$$

(26)

where $h_i$ is the height of each floor (30 cm), $d_i(t)$ is the interstory drift of the above ground floors over the response history, and $d_{n\text{max}}$ denotes the normalized peak interstory drift in the uncontrolled response. The third evaluation criterion is a measure of the normalized peak floor accelerations, given by

$$J_3 = \max_{t, i} \left( \frac{\ddot{x}_{ai}(t)}{\ddot{x}_{a\text{max}}} \right)$$

(27)

where the absolute accelerations of the $i$th floor, $\ddot{x}_{ai}(t)$, are normalized by the peak uncontrolled floor acceleration, denoted $\ddot{x}_{a\text{max}}$.

The final evaluation criteria considered in this study is a measure of the maximum control force per device, normalized by the weight of the structure, given by
where \( W \) is the total weight of the structure (1335 N).

The corresponding uncontrolled responses are as follows: \( x_{\text{max}} = 1.313 \text{ cm}, d_{n_{\text{max}}} = 0.00981 \text{ cm}, x_{a_{\text{max}}} = 146.95 \text{ cm/sec}^2 \). The resulting evaluation criteria are presented in Table 1 for the control algorithms considered. As indicated in the table, the numbers in parentheses indicate the percent reduction as compared to the best passive case. Additionally, to compare the performance of the various control algorithms, the peak of the interstory drift and absolute acceleration responses for all floors were examined. Figure 7 shows the peak response profile of the entire structure for a variety of cases.

### Table 1. Normalized Controlled Maximum Responses due to Scaled El Centro Earthquake.

<table>
<thead>
<tr>
<th>Control Strategy</th>
<th>( J_1 )</th>
<th>( J_2 )</th>
<th>( J_3 )</th>
<th>( J_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Passive-Off</td>
<td>0.862</td>
<td>0.801</td>
<td>0.904</td>
<td>0.00292</td>
</tr>
<tr>
<td>Passive-On</td>
<td>0.506</td>
<td>0.696</td>
<td>1.41</td>
<td>0.0178</td>
</tr>
<tr>
<td>Lyapunov Controller A</td>
<td>0.686 (+35%)(^{a})</td>
<td>0.788 (+13%)</td>
<td>0.756 (-16%)</td>
<td>0.0178</td>
</tr>
<tr>
<td>Lyapunov Controller B</td>
<td>0.326 (-35%)</td>
<td>0.548 (-21%)</td>
<td>1.39 (+53%)</td>
<td>0.0178</td>
</tr>
<tr>
<td>Decentralized Bang-Bang</td>
<td>0.449 (-11%)</td>
<td>0.791 (+13%)</td>
<td>1.00 (+11%)</td>
<td>0.0178</td>
</tr>
<tr>
<td>Max. Energy Dissipation</td>
<td>0.548 (+8%)</td>
<td>0.620 (-11%)</td>
<td>1.06 (+17%)</td>
<td>0.0121</td>
</tr>
<tr>
<td>Clipped-Optimal A</td>
<td>0.631 (+24%)</td>
<td>0.640 (-8%)</td>
<td>0.636 (-29%)</td>
<td>0.01095</td>
</tr>
<tr>
<td>Clipped-Optimal B</td>
<td>0.405 (-20%)</td>
<td>0.547 (-21%)</td>
<td>1.25 (+38%)</td>
<td>0.0178</td>
</tr>
<tr>
<td>Modulated Homog. Friction</td>
<td>0.421 (-17%)</td>
<td>0.559 (-20%)</td>
<td>1.06 (+17%)</td>
<td>0.0178</td>
</tr>
</tbody>
</table>

\(^{a}\)Numbers in parentheses indicate percent reduction as compared to the best passive case. Negative numbers correspond to a response reduction.

To compare the performance of the semi-active system to that of comparable passive systems, two cases are considered in which the MR dampers are used in a passive mode by maintaining a constant voltage to the devices. The results of both a passive-off (0V) and passive-on (5V) configuration are included. The passive-off system reduces the maximum floor displacement, maximum interstory displacement, and maximum absolute acceleration by 14%, 20%, and 10%, respectively, over the uncontrolled case. The passive-on system is able to further reduce the maximum floor displacement and maximum interstory displacement. However,
notice that the passive-on system results in a larger acceleration than the passive-off system. Figure 7 shows that this occurs because the passive-on system attempts to lock up the first two floors, increasing the drift of the upper floors, and increasing the absolute acceleration of the lower floors of the structure.

For the Lyapunov controller there is no standard method of selecting the $Q_p$ matrix, therefore, several $12 \times 12$ $Q_p$ matrices were arbitrarily chosen and tested. As mentioned previously, the challenge in Lyapunov controller design is in the selection of $Q_p$. Thus, a variety of combinations were tried, and two control designs that achieved good performance in are discussed herein. Lyapunov controller A uses a $Q_p$ matrix with nonzero values in the first row of the matrix and reduces the absolute acceleration by 16.4% over the best passive case. Figure 7 demonstrates that this algorithm reduces the peak absolute accelerations of all floors to about the same level. Lyapunov controller B uses a $Q_p$ matrix with ones in the $(7,1), (8,2), (9,3), (10,4), (11,5)$, and $(12,6)$ positions. This design resulted in a reduction of the maximum floor displacement and maximum interstory displacement by 35.6% and 21.3% respectively over the best passive case. Figure 7 shows that the peak drift at the lower floors is reduced significantly, without locking up these floors. Thus this control algorithm is able to achieve significant reductions in the drift throughout the structure.

The results obtained with the decentralized bang-bang controller show that this algorithm is capable of reducing the maximum floor displacement by 11.3% over the passive results, but is not very effective in reducing the maximum interstory displacement and absolute accelerations of this structure. Notice from Figure 7 that this control algorithm allows the first floor to displace significantly as in a base isolation system, but it locks up the second floor of the structure, resulting in increased absolute accelerations in the lower floors. Thus, the performance achieved with this device could be realized by removing the controllable MR damper between the first and second floors and replacing it with a passive device.

Similarly, the maximum energy dissipation algorithm achieves results that are quite similar to that of the passive-on system. The maximum relative displacement achieved is slightly larger than that of the passive-on system, although the maximum interstory drift is marginally less than that of the passive-on system. The maximum acceleration is not lower than that of the passive-off system. Therefore this control algorithm does not achieve significantly better results than the passive systems. To achieve this performance level, a passive energy dissipation device could be used. Thus, this control algorithm is not recommended.

Two clipped-optimal control designs with different capabilities were considered. Clipped-optimal controller A was designed by placing a moderate weighting $(840 \text{ cm}^{-2})$ on the relative displacements of all floors. Clipped-optimal controller B was designed by placing a higher weighting $(9000 \text{ cm}^{-2})$ on the relative displacements of all floors. The results show that clipped-optimal controller A appears to be quite effective in achieving significant reductions in both the maximum absolute acceleration and interstory displacement over the passive case. In fact, this
controller achieves a 29.6% reduction in acceleration as compared to the better passive case, resulting in the lowest acceleration of all cases considered here. Furthermore, Figure 7 indicates that the accelerations are reduced throughout the structure. If further reductions in displacement are desired in the controller, clipped-optimal control B achieves a reduction in the maximum floor displacement and maximum interstory drift of 20% and 21.4% over the best passive cases, although the absolute accelerations increased. Figure 7 shows that the drifts are quite small at the lower floors and the maximum drift occurs at the third floor of the structure, although the drifts are consistently lower than almost all of the other algorithms. Notice that the clipped-optimal control algorithm allows the designer some versatility depending on the control objectives for the particular structure under consideration.

The modulated homogeneous friction algorithm was designed by choosing a value of \( g_{n} \) of 470 N/cm for this example. This value was selected because it utilizes the full range of forces for the MR device without saturating the range of the MR device. Thus, the desired force is always proportional to the previous local extrema in the device displacement. The results show that in this example the control algorithm achieves high levels of performance. The relative displacement and interstory drifts are reduced by 16.8% and 19.7% over the better passive case, although a small increase in the acceleration is observed.

From Figure 7 observe that, in terms of absolute acceleration, most of the semi-active controllers have qualitatively similar behavior in the upper floors. The smallest acceleration response is achieved with clipped-optimal controller A, and the lowest interstory displacement response is achieved with Lyapunov controller B and clipped-optimal controller B, while the modulated homogeneous friction algorithm achieves quite similar performance.

5. Conclusion

A selection of recently proposed semi-active control algorithms have been evaluated for application in a structural control system using multiple MR dampers. In a numerical example a six-story structure was controlled using MR dampers on the lower two floors. The responses of the system to a scaled El Centro earthquake excitation were found for each controller through a simulation of the system. Each algorithm was implemented using available measurements of the structural system, including device forces and absolute structural accelerations. Each semi-active algorithm resulted in an improvement in performance over the best passive controller in some way, although the resulting responses varied greatly depending on the choice of control algorithm. Based on these results, three of these control algorithms were found to be most suited for use with MR dampers in a multi-input control system. The Lyapunov controller algorithm, the clipped-optimal algorithm, and the modulated homogeneous friction algorithm all achieved significant reductions in the responses. Lyapunov controller B and clipped-optimal controller B achieved virtually identical reductions in maximum interstory displacement (21.4%). The reduction in absolute acceleration was supe-
rior with clipped-optimal controller A (29.6%), and the reduction in relative displacement was superior with the Lyapunov controller B (35.6%). Furthermore, both of these algorithms possess the flexibility to allow the control designer to design for a range of control objectives. The modulated homogeneous friction algorithm achieved significant reduction in the displacements and drifts, although an increase in the accelerations was observed.

Further studies are underway to examine issues related to scaling the performance of MR devices, robustness of semi-active control algorithms to sensor failure, and the development of guidelines for the use of these devices.

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References


