Variational Quantum Search

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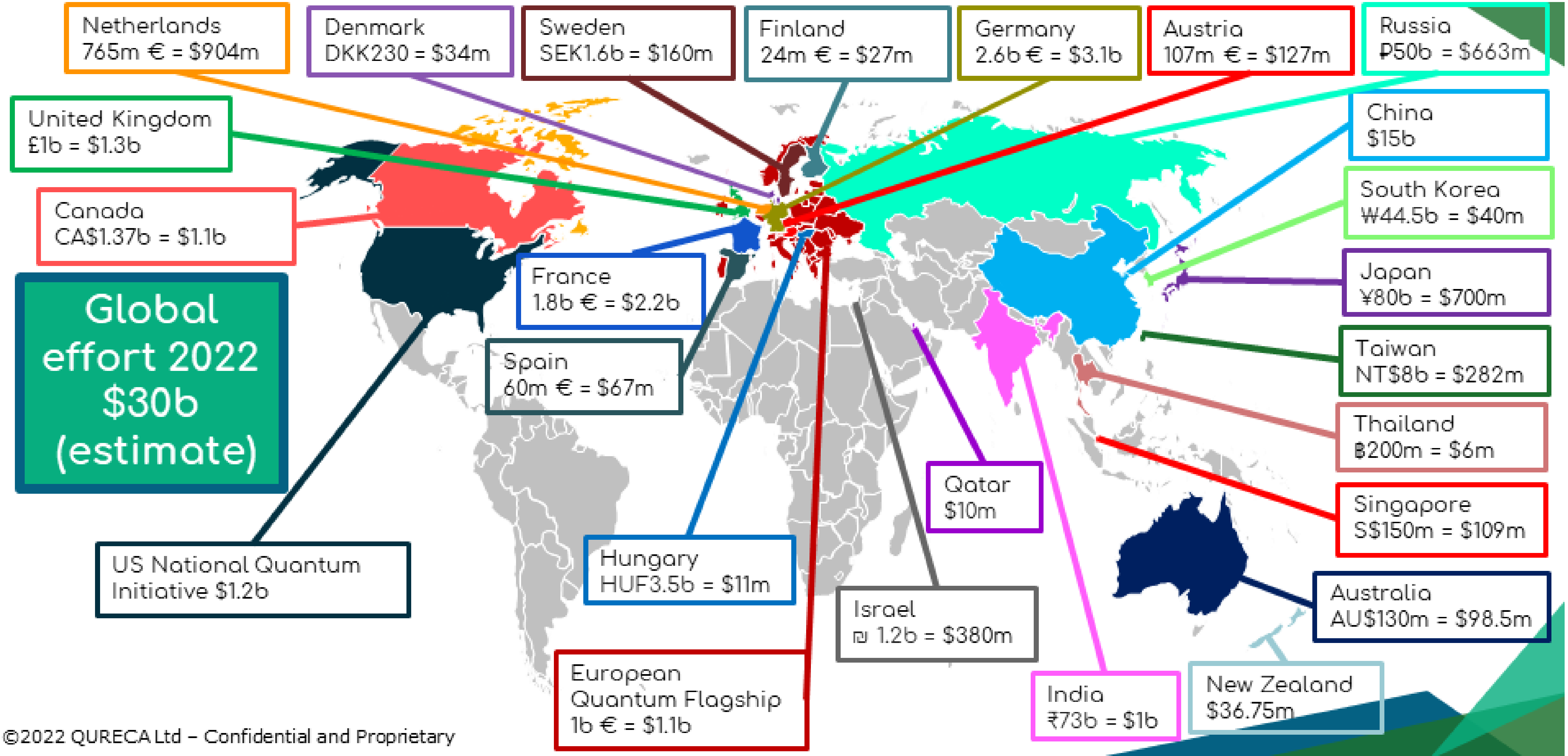
Quantum Effort Worldwide

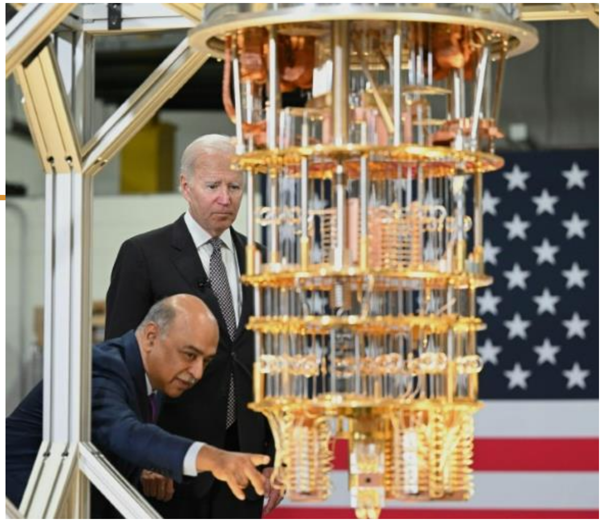


Source: https://qureca.com/overview-on-quantum-initiatives-worldwide-update-2022/

Quantum Computing Race - Corporations

Source: https://hsrc.biz/reports/quantum-computing-market-technologies/

Introduction

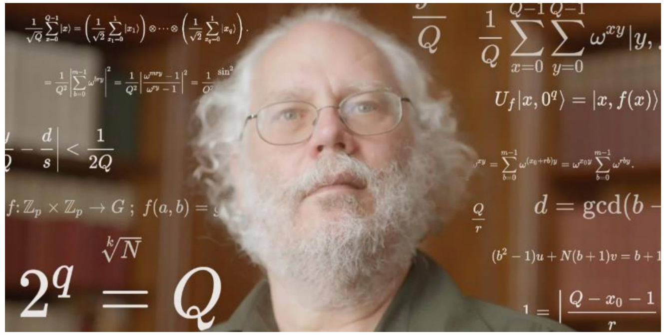


- Quantum computers are developing rapidly.
  - IBM 433 (1121) qubits in Nov. 2022 (Dec. 2023)
  - IBM aims to reach 1386 qubits within two years
- Quantum Supremacy: Google quantum computer 200 seconds that would take the fastest supercomputers about 10,000 years?

- Variational quantum algorithms (VQAs) show great promise in helping to realize systematic *supremacy* of quantum computing
  - they typically require fewer qubits and lower circuit depth
- VQAs have been successfully used in many fields such as optimization, error correction, machine learning, physics and chemistry, etc.

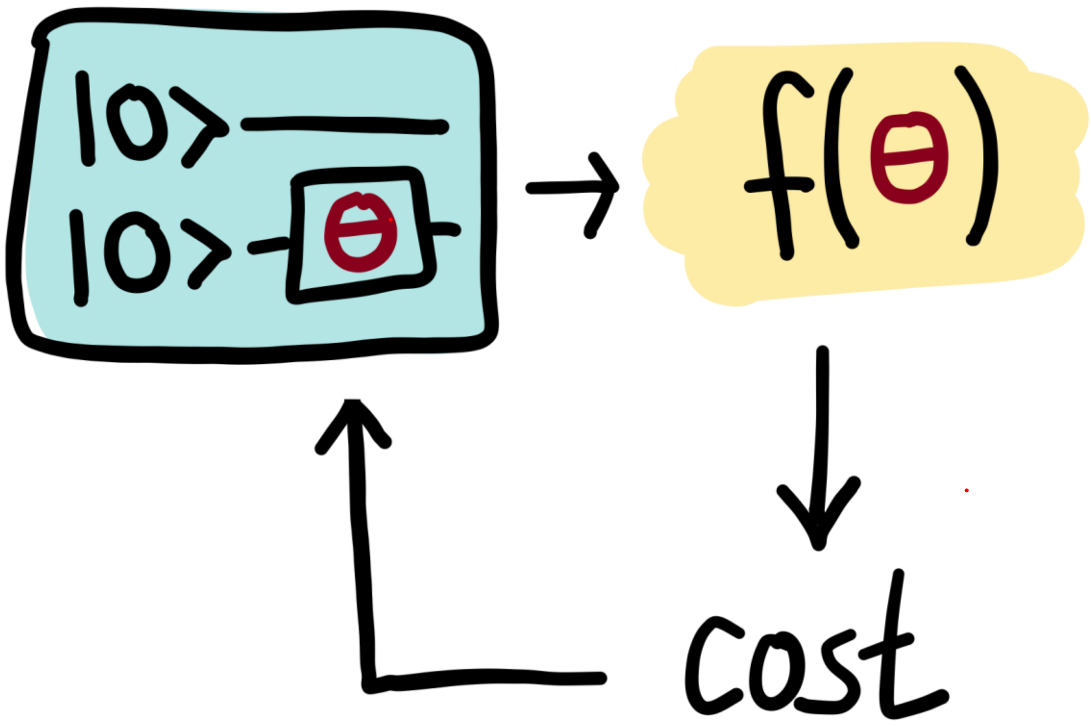
Introduction – Quantum Algorithms

- Pure quantum algorithms can be divided into two categories
  - based upon Shor's quantum Fourier transform
  - based upon Grover's algorithm to perform quantum search
- Grover's algorithm provides quadratic speedup, e.g., search unstructured database
  - The circuit depth increases exponentially with the number of qubits

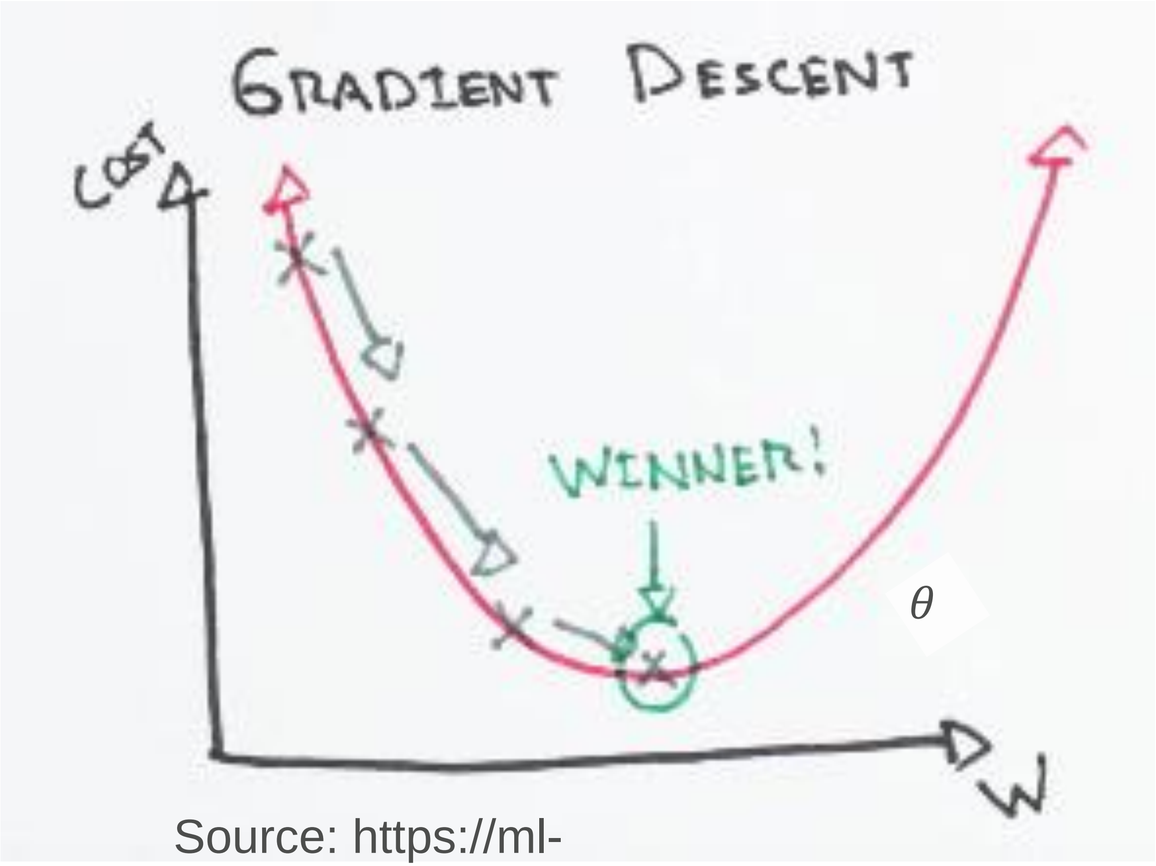


Introduction – Variational Quantum Algorithm

➢Variational Quantum Algorithm



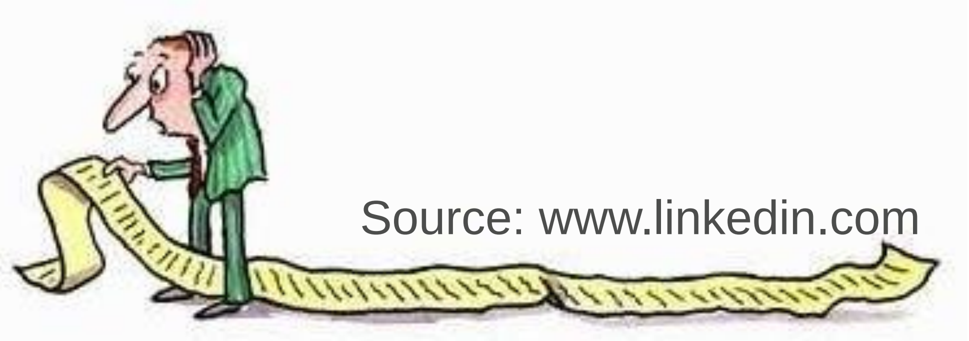
Source: https://pennylane.ai/qml/glossary/variational_circuit.html



Source: https://ml-cheatsheet.readthedocs.io/en/latest/gradient_descent.html

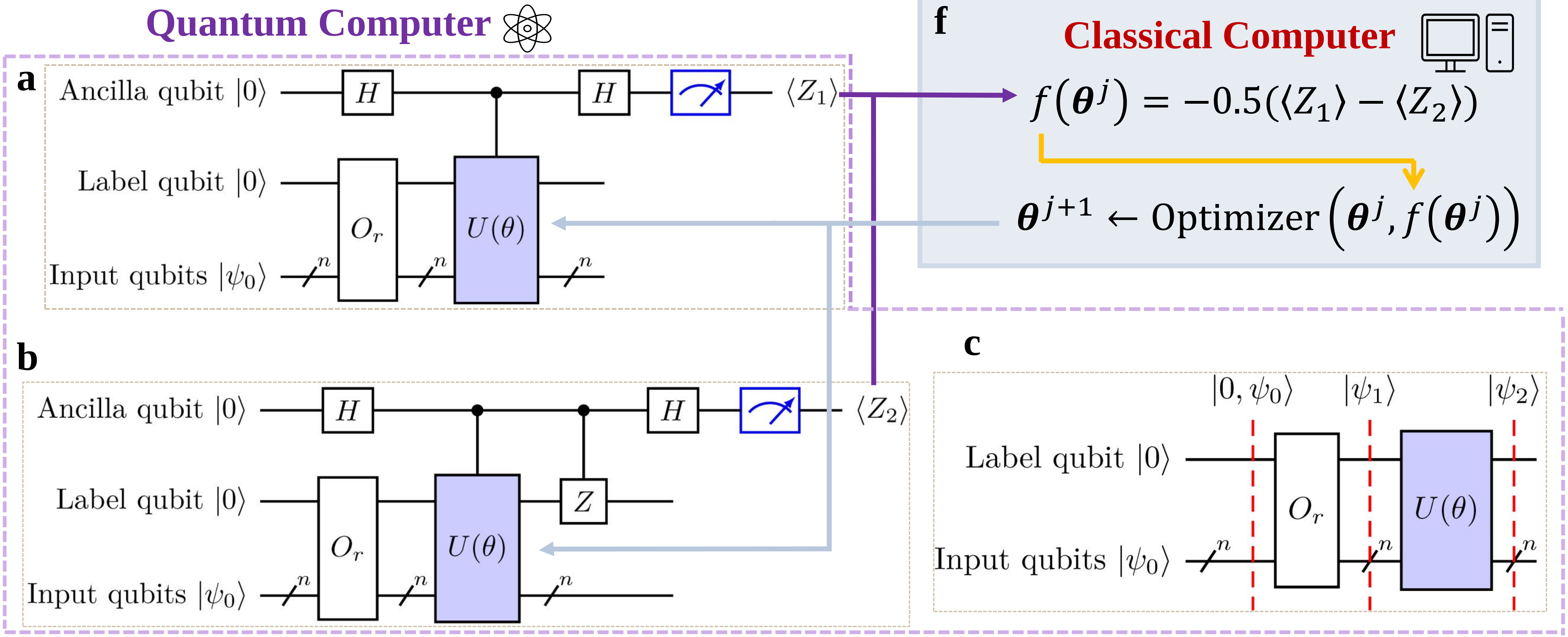
Search an Unstructured Database

- A database has 2^n elements
  - 266 qubits: $2^{266} = 10^{80}$
  - The known universe has 10^{80} atoms



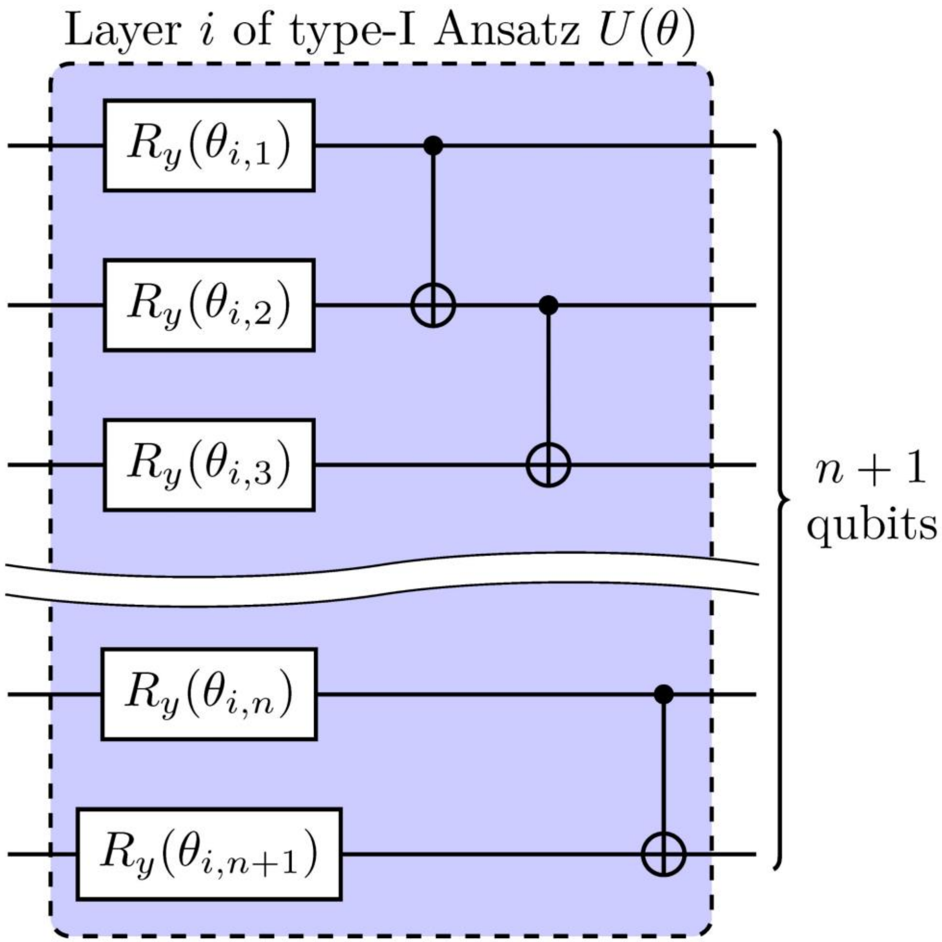
- ***Good element***: the element needs to be found
- Classical method need an average of $2^n/2$ operations
- Grover's search algorithm: $\sqrt{2^n}$ layers of quantum circuits
- Can we do better?
  - VQS algorithm: $O(n)$ layers of quantum circuits

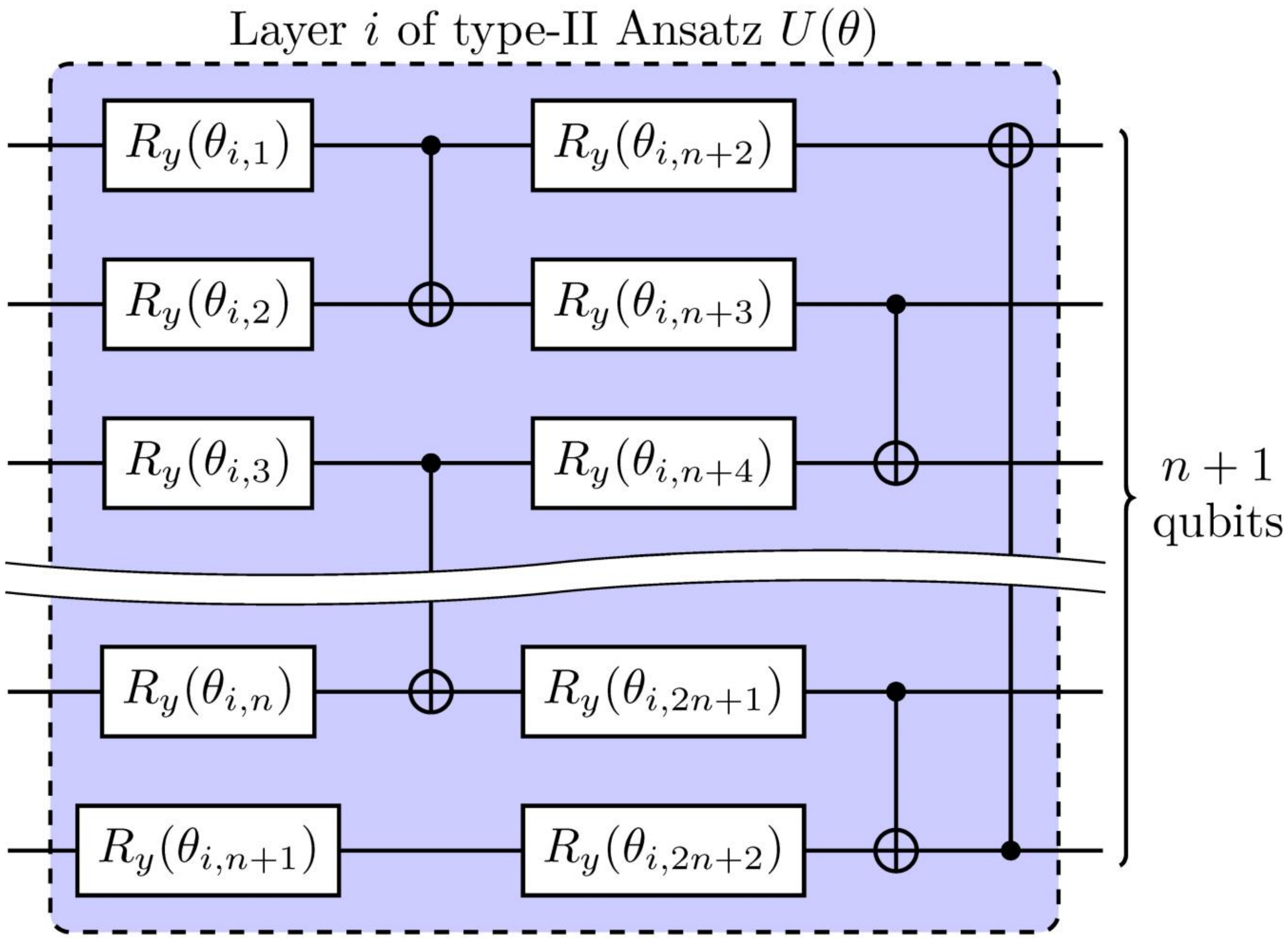
Variational Quantum Search (VQS)



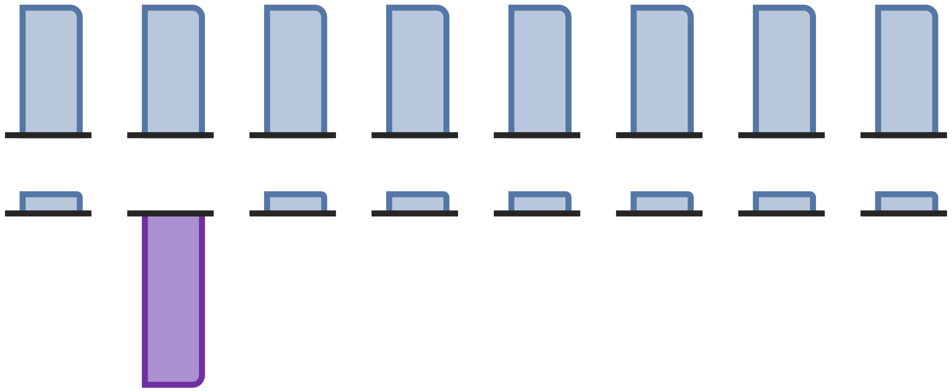
VQS - Ansatz

Hands-on, https://quantum-computing.ibm.com/composer





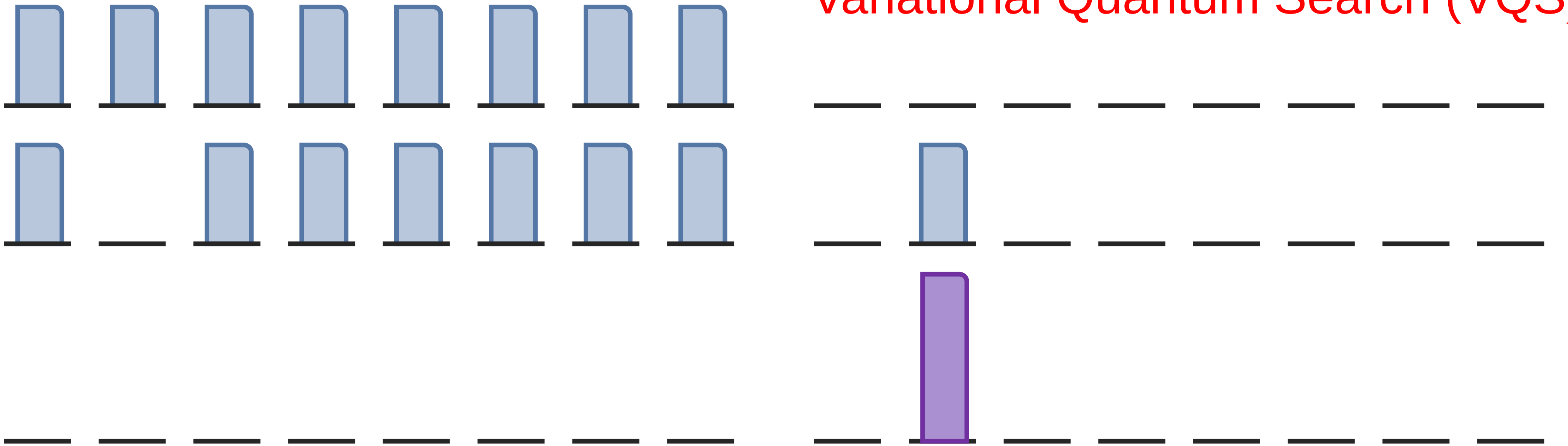
Search an Unstructured Database - 2



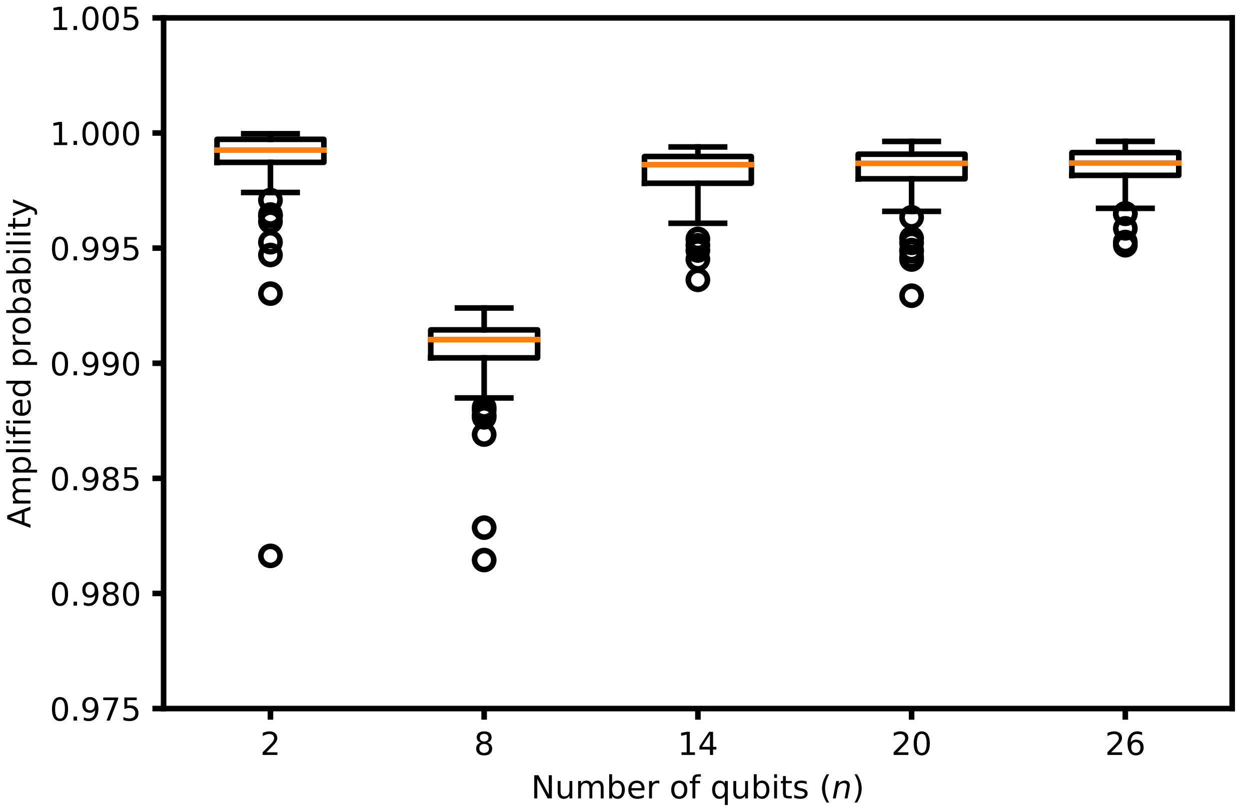
$$\text{probability} = \text{amplitude}^2$$

Grover's search algorithm

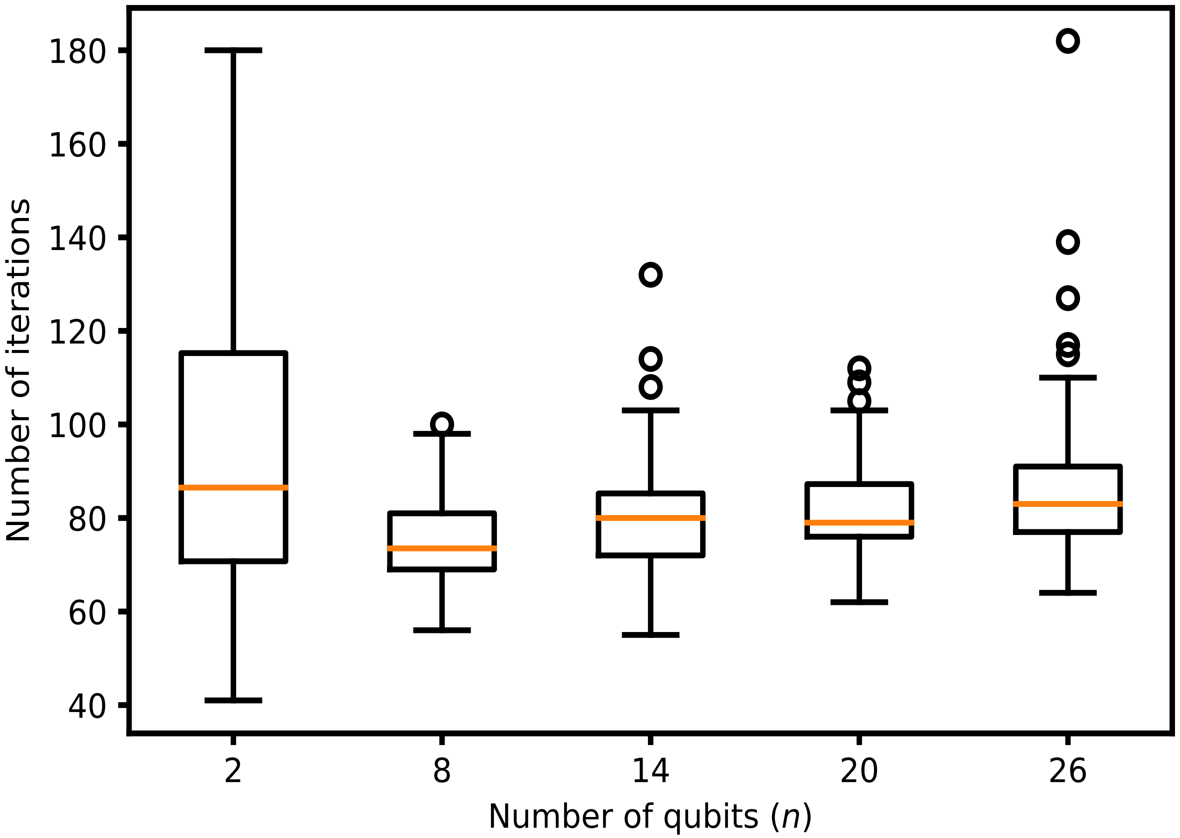
Variational Quantum Search (VQS) algorithm



VQS – Results Using Type-I Ansatz



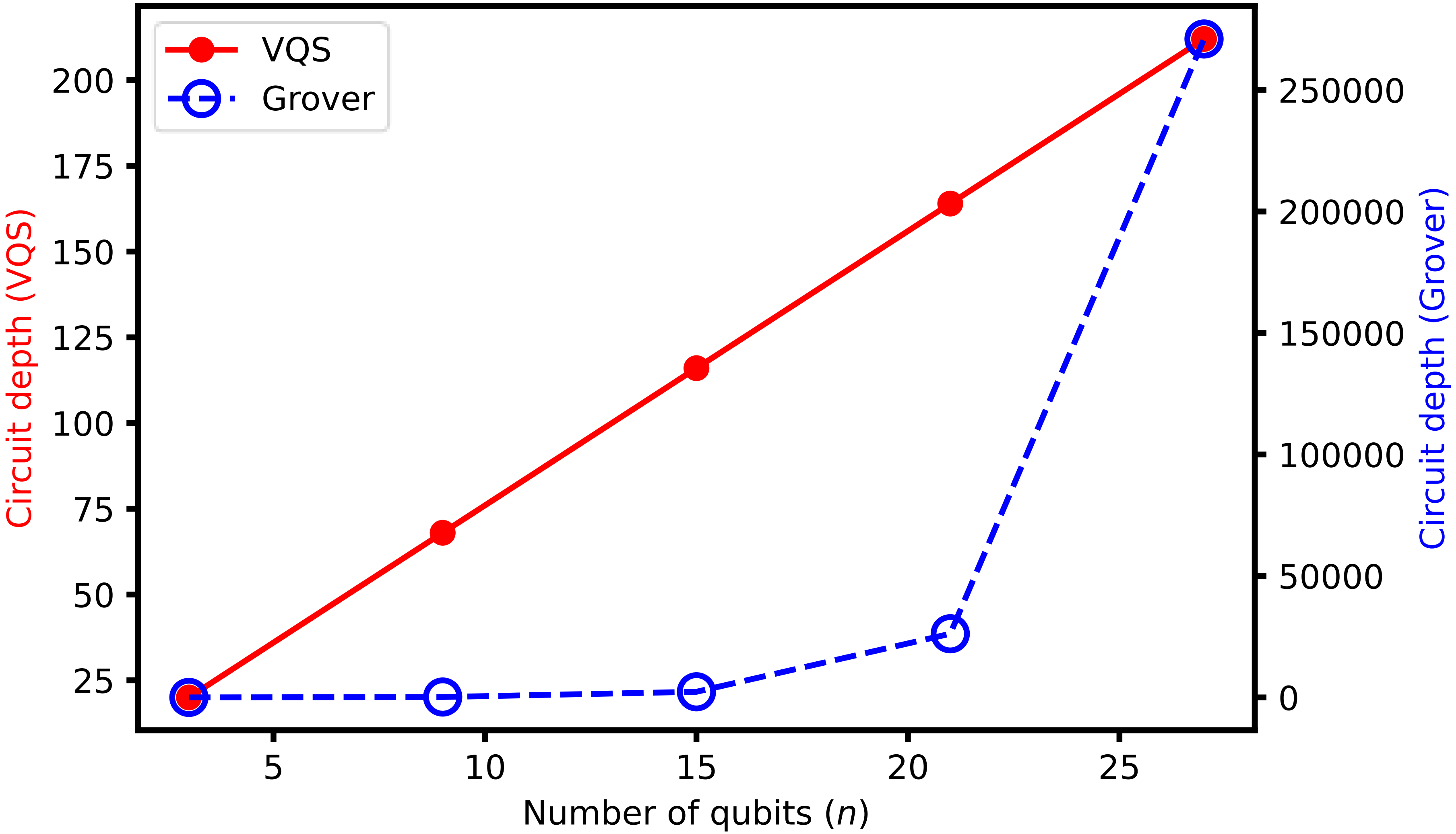
Probability of finding the good element



Number of iterations used to find the optimal parameters in Ansatz

VQS vs. Grover's Algorithm: Circuit Depth

- Exponential advantage!!
- Verified up to 26 qubits due to limitation of GPU memory
- Significant milestone if proved for any number of qubits!



Summary

- ➢Proposed a Variational Quantum Search (VQS) algorithm [15]
  - The maximum circuit depth of VQS increases linearly with the qubit #
    - ✓Exponentially shallower than Grover's algorithm
    - ✓Verified up to 26 qubits, due to limitation of GPU memory
  - We demonstrate that a depth-56 circuit in VQS can replace a depth-270,989 circuit in Grover's algorithm
- ➢Proposed Quantum Feasibility Labeling algorithm [17]
    - ✓Together with VQS, can efficiently solve an NP-complete Vertex Coloring problem

Future Work

- Work on the objective function and optimization method
  - to determine the optimal parameters in the Ansatz (transfer learning)
  - to avoid barren plateau
  - to analyze the local optima and global optimum of the solution space
  - to propose more efficient optimization method
- Validate VQS on larger systems (circuit cutting, MPS, etc.)
- Apply VQS to solve other difficult & interesting problems
  - Optimization problems in power and energy systems, e.g., unit commitment, optimization of DER and energy storage systems, contingency analysis

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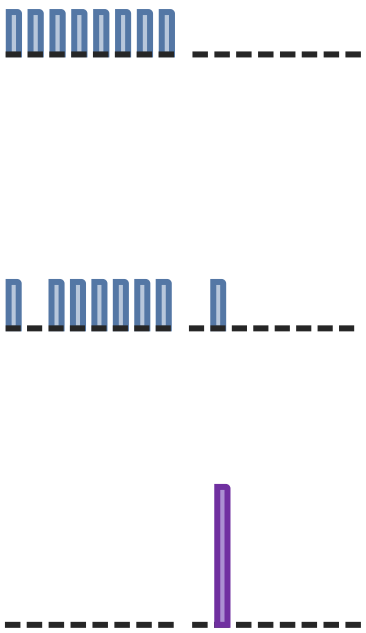
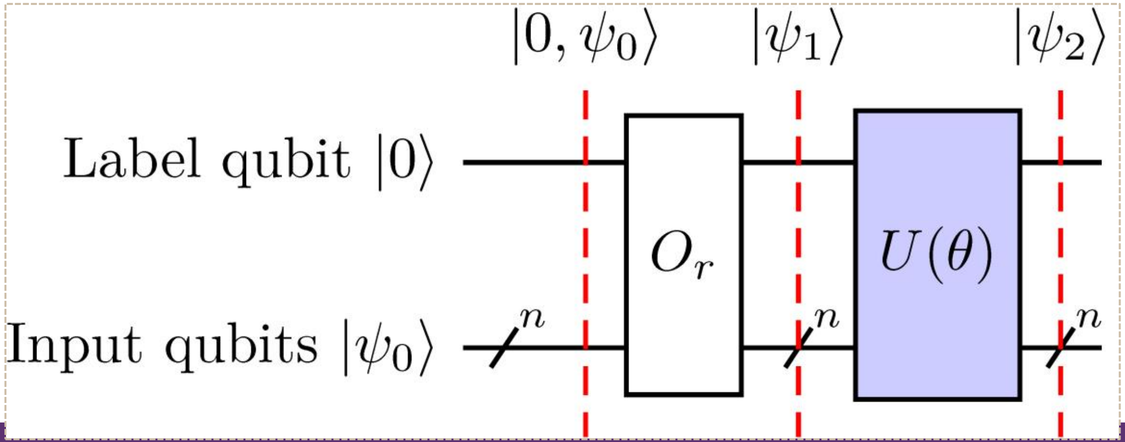
Reference Books/Resources

- Michael A. Nielsen, Isaac L. Chuang. Quantum Computation and Quantum Information: 10th Anniversary Edition
- David Mermin. Quantum computer science: An Introduction
- Xanadu Quantum Codebook [https://codebook.xanadu.ai/]
- IBM Quantum Composer [https://quantum-computing.ibm.com/composer]

VQS - States in Vector Form

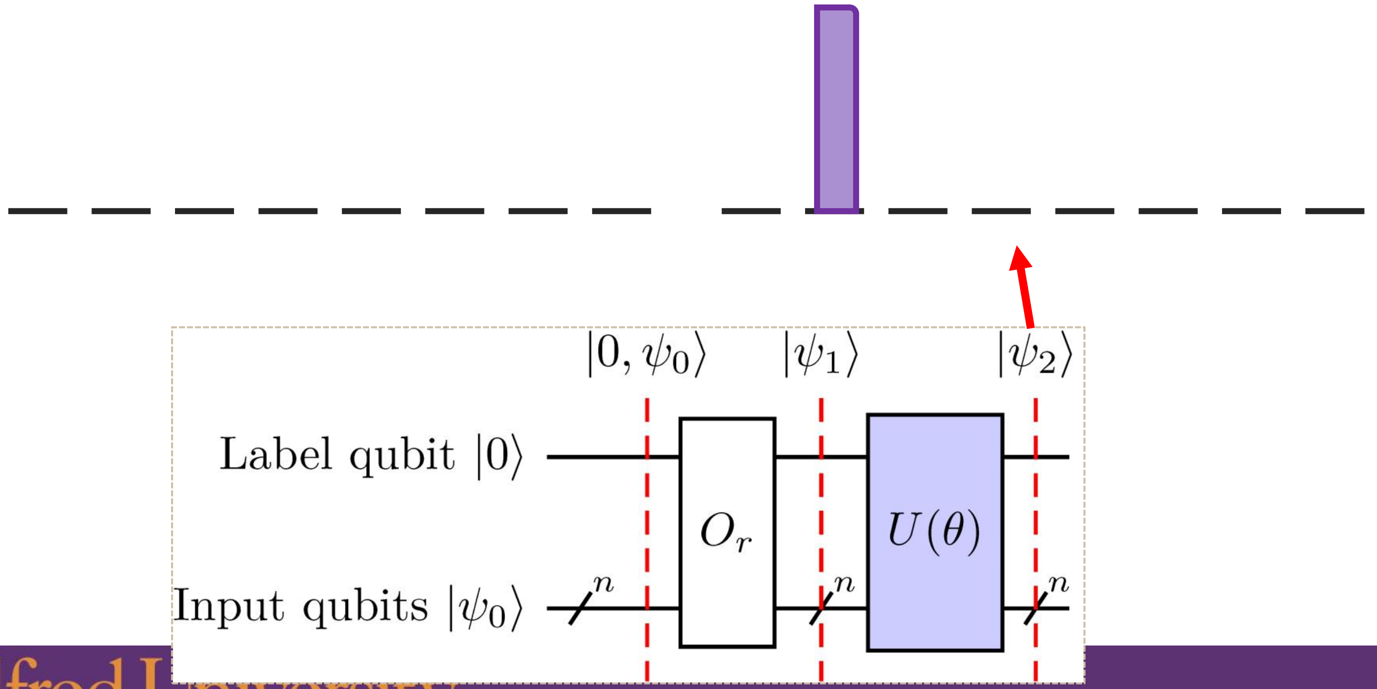
- $|0, \psi_0\rangle = [\underbrace{\alpha_{b_1}, \alpha_{b_2}, \cdots, \alpha_{b_{N_b}}, \alpha_{g_1}, \alpha_{g_2}, \cdots, \alpha_{g_{N_g}}}_{\text{1st half: } N \text{ elements}}, \underbrace{0, 0, \cdots, 0}_{\text{2nd half: } N \text{ elements}}]^T$
- $|\psi_1\rangle = [\underbrace{\alpha_{b_1}, \alpha_{b_2}, \cdots, \alpha_{b_{N_b}}, 0, \cdots, 0}_{\text{1st half: } N \text{ elements}}, \underbrace{0, \cdots, 0, \alpha_{g_1}, \alpha_{g_2}, \cdots, \alpha_{g_{N_g}}}_{\text{2nd half: } N \text{ elements}}]^T$
- $|\psi_2\rangle = [\beta_1, \beta_2, \cdots, \beta_N, \beta_{N+1}, \cdots, \beta_{2N-1}, \beta_{2N}]^T$
- $\sum_{i=1}^{2N} |\beta_i|^2 = 1$





VQS - Objective Function

- The **objective function** is set to $f(\boldsymbol{\theta}) = -0.5(\langle Z_1 \rangle - \langle Z_2 \rangle)$
- This function ensures that when $f(\boldsymbol{\theta})$ is minimized,
  - $|\psi_2\rangle$ is only a superposition of good elements
  - measuring $|\psi_2\rangle$ will obtain good elements with probabilities close to 1



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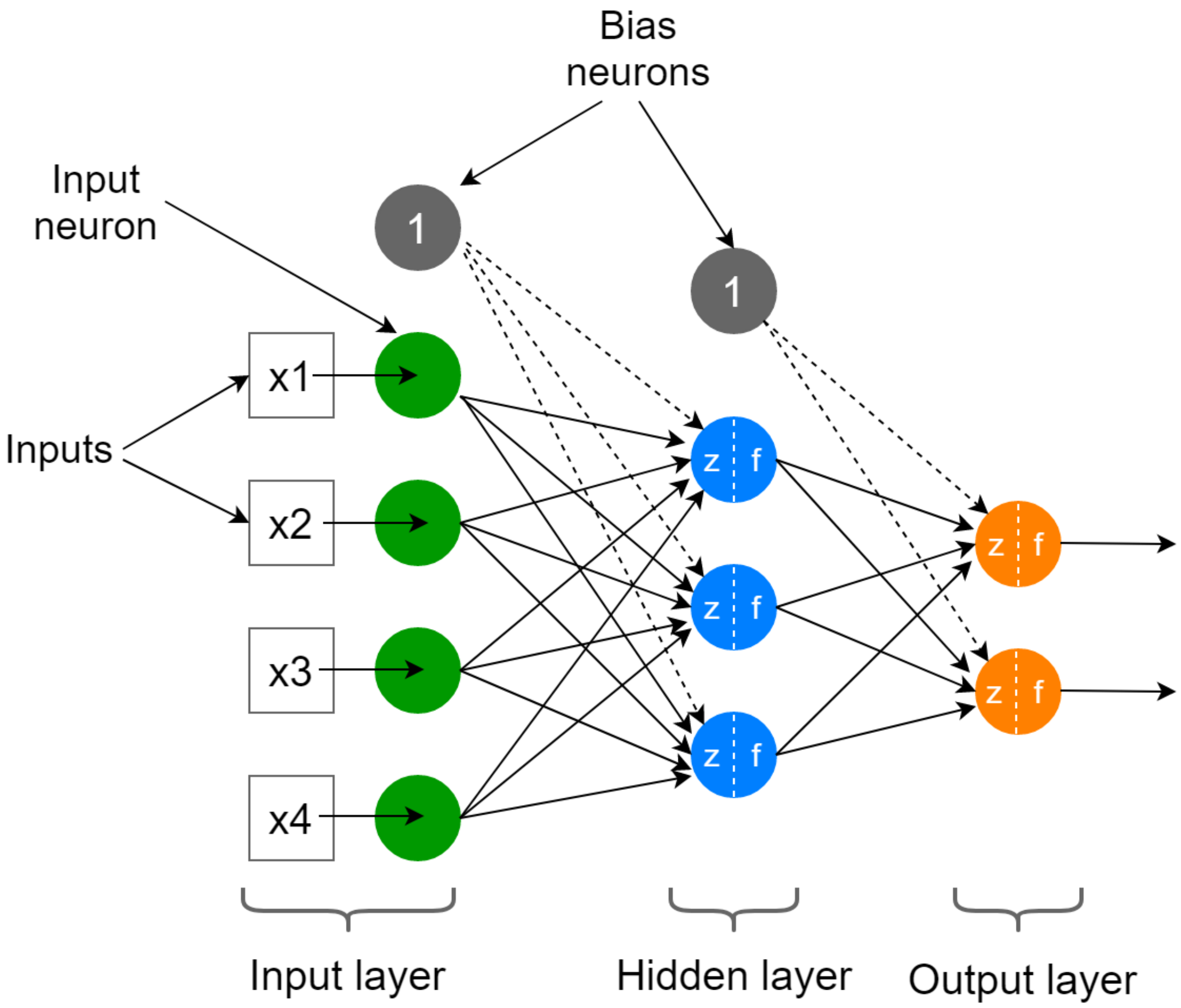
Classical Computer

$$f(\boldsymbol{\theta}^j) = -0.5(\langle Z_1 \rangle - \langle Z_2 \rangle)$$

$$\boldsymbol{\theta}^{j+1} \leftarrow \text{Optimizer}\left(\boldsymbol{\theta}^j, f(\boldsymbol{\theta}^j)\right)$$

Classical Neural Network

- ➢Input layer
- ➢Hidden layers
- ➢Output layer



Source: https://rukshanpramoditha.medium.com/one-hidden-layer-shallow-neural-network-architecture-d45097f649e6