Collaborative Beamforming in Wireless Sensor Networks

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***Abstract*— Collaborative beamforming (CB) is an energy efficient transmission scheme in wireless sensor networks (WSNs). Despite the promising aspects of CB, its practical implementation remains largely unexplored and the challenges, namely frequency, phase, and data synchronization, must be addressed for successful deployment. We propose a CB protocol with a compatible transceiver architecture. The protocol uses a phase-locked loop (PLL) continuously operating in closed-loop for frequency synchronization, a local two-way phase estimation and remote one-way calibration technique for phase synchronization, and a data entrainment technique to achieve data synchronization with minimal data traffic. The efficiency analysis including carrier jitter demonstrates the feasibility of the proposed CB protocol.**

I. Introduction

WSNs have great potentials in a variety of practical applications such as home automation and structural health monitoring [1]. For most anticipated applications, energy efficiency is one of the most important design criteria because it directly affects the reliability and maintenance cost of the wireless sensor nodes. Because of the limited energy, each sensor node has limited computational capability and low transmit power. However, many WSN applications require communication with a distant stationary receiver. Due to the low transmit power, an individual sensor node is not able to communicate directly with the remote receiver.

A special case of cooperative communication intended for energy efficient communication is collaborative beamforming (CB) where single-antenna transceivers collaborate and form a virtual antenna array that focuses the signal transmission to a desired destination [2] - [13]. Given $\mathcal{N}$ sensor nodes participating to the CB, the power gain is proportional to $\mathcal{N}^2$ in an ideal case . For a constant received power, each sensor node can decrase its power by $\mathcal{N}$-fold. Hence, CB enhances the energy efficiency in WSN systems [6].

Despite the promising prospects of CB, its practical implementation remains largely unexplored and several technical challenges must be properly addressed for successful deployment. The challenges include: (a) carrier frequency synchronization with oscillator drift caused by temperature, process variations, and aging, (b) carrier phase estimation and adjustment in multipath environments for a constructive signal addition at a remote receiver, and (c) time consuming and power intensive data sharing step necessary for data synchronization. The imperfections in (a) and (b) result in time-varying, unpredictable and distorted beam patterns [7]. The data misalignment among the participating nodes distorts the transmitted signal. Furthermore, the existing CB approaches require an extensive data traffic because sensor nodes need to decide on the common information for transmission.

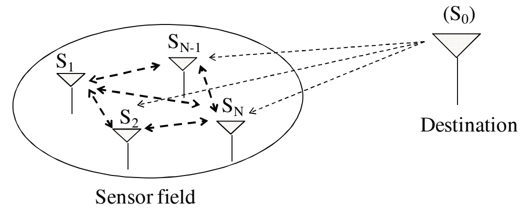

Fig. 1. A randomly deployed WSN system model

In this paper, we propose a new CB protocol and its supporting transceiver architecture that effectively addresses all of the aforementioned issues. We achieve carrier frequency synchronization using one PLL continuously working in a close loop mode and using a frequency division duplexing (FDD). A two-way phase estimation technique is adopted to eliminate the uncertainties in the carrier phase estimation in multi-path environments. We also adopt a data entrainment technique that significantly reduces the data traffic among sensor nodes and simplifies the network operation by eliminating the need to store all data to transmit in each sensor node.

II. CB protocol and compatible transceiver

A. System model

We consider a randomly deployed WSN with N stationary sensor nodes (S_1,...,S_N) and one destination node (S_0) as shown in Fig. 1. An individual sensor node has a single antenna with isotropic beampattern and is not capable of transmitting signals to the destination due to its low fixed transmit power. On the other hand, the destination has sufficient power to transmit signals to the sensor field. Our primary objective is to minimize the power consumption at the sensor nodes.

We assume that sensor nodes do not have the knowledge of their location information. In an actual environment, even if the precise location information of the sensor nodes is known, it cannot be used for the CB operation because of the multipath effect. We model the channel from S_i to S_j for $\forall i, j \in \{0, 1, ..., N\}, i \neq j$ at the frequency f as linear time-invariant (LTI) system with impulse response $h^f_{i,j}(t)$. We

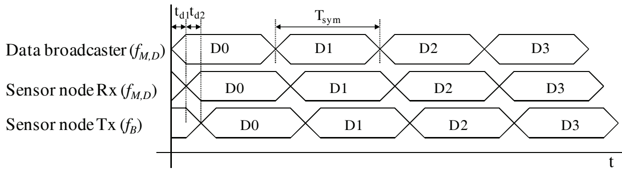

Fig. 2. A simplified timing diagram for data entrainment in CB

assume that the channel is reciprocal i.e. $h^f_{i,j}(t) = h^f_{j,i}(t)$ for the same frequency f and that it does not change significantly during the proposed protocol.

B. Protocol and Operation Sequence

In this part, we elaborate the proposed CB protocol. We divide the CB process into two phases: preparation and beamforming phases. In preparation phase, sensor nodes sense the environment and collect data to be transmitted in the beamforming stage. During the beamforming phase, all participating sensor nodes should maintain carrier frequency synchronization and carrier phase adjustment according to the channel condition to transmit common message to a remote receiver.

1) Preparation Phase: In the preparation phase, sensor nodes collect data and store them to employ the data entrainment technique in the beamforming phase.

Data Entrainment: The existing CB approaches require data sharing and queuing among the participating sensor nodes. All data to be transmitted need to be stored and queued in each node. To mitigate this issue, we use data entrainment technique in the beamforming phase. In this technique, each sensor node collects data from the environment and stores them in its own memory unit. It does not share its data with other sensor nodes. After a network is formed among the sensor nodes, a master node attributes an identification (ID) number to all the sensor nodes. Then, each node successively serves as a data broadcaster in the network. A data broadcaster broadcasts its data to the participating nodes, and they immediately transmit them back after retrieving the received data. The retransmission uses beamforming frequency (f_B) which is different from the frequency ($f_{M,D}$) used by the data broadcaster. A simplified timing diagram is shown in Fig. 2. In this configuration, we note that any node can serve as a data broadcaster. We assume that the data rate of the remote transmission using CB is low enough, so the data retrieval time delay (t_{d1} in Fig. 2) in each sensor node does not significantly affect the performance, and the predictable internal delay (t_{d2}) does not affect the operation. This entrainment technique does not require a data sharing step among the sensor nodes. Consequently, the data entrainment technique significantly reduces the amount of data traffic and time for data sharing and queuing among the participating nodes in preparation phase. We note that with data entrainment high priority information from a certain region of the environment can be extracted immediately.

2) Beamforming Phase: After the preparation phase, each sensor node has its own data to be broadcasted for beamforming stage by applying data entrainment. Two challenges must be addressed in this phase for successful beamforming: carrier frequency synchronization among the participating nodes and carrier phase adjustment in each node for a constructive addition at a remote target receiver. Since we use data entrainment technique, data synchronization among sensor nodes is inherently achieved. The beamforming phase is divided into beacon (wake-up), pre-alignment, and transmission stages.

In the *beacon (wake-up) stage*, the destination node initiates the beamforming phase by broadcasting signal $s_D(t)$:

$$s_D(t) = Ae^{j\{2\pi f_M t + \varphi_0\}} \tag{1}$$

where A is the amplitude, and φ_0 and f_M are the initial phase and frequency of the carrier signal of the destination node, respectively. We note the destination generates $s_D(t)$ by using a crystal-based PLL to maintain a constant phase. Once the sensor nodes detect the beacon signal, they initiate the pre-alignment process.

In the *pre-alignment stage*, the destination and sensor nodes achieve carrier frequency synchronization and carrier phase adjustment matched to a given multi-path channel condition.

The i^{th} sensor node receives the beacon signal as $s_{D,i}(t)$:

$$s_{D,i}(t) = Ah^{f_M}_{0,i}(t)e^{j\{2\pi f_M t + \varphi_0 + \Delta\varphi_{D,i}\}} \tag{2}$$

where $\Delta\varphi_{D,i}$ is the channel phase shift between the destination and the i^{th} sensor node and $h^{f_M}_{0,i}(t)$ is the channel amplitude response for $i \in \{1, 2, ..., N\}$ at the frequency f_M.

Once each sensor node detects $s_{D,i}(t)$ from the destination node, it uses the received signal as a reference for its own PLL. Using f_M as reference and assuming that the communication channel between the destination and sensor nodes has a large SNR, each sensor node generates a carrier with frequency f_B:

$$s_{s,i}(t) = e^{j\{2\pi f_M t + \varphi_0 + \Delta\varphi_{D,i}\}\frac{M}{L}} = e^{j\{2\pi f_B t + \varphi_{B,i}\}} \tag{3}$$

where $f_B = f_M \frac{M}{L}$, $\varphi_{B,i} = (\varphi_0 + \Delta\varphi_{D,i})\frac{M}{L} \mod(2\pi)$, and M and L ($M \neq L$) are integers. We also assume that each sensor node generates $s_{s,i}(t)$ using a second order PLL where the mean value of the steady state phase error is zero. This completes the frequency synchronization required for beamforming transmission since f_B of all sensor nodes are frequency locked to f_M of the destination.

After the frequency synchronization, we need to align the carrier signal phases in every sensor node and adjust them according to the channel delay between the destination and each sensor node. We first find the relative carrier phases of the sensor nodes with respect to a reference node by using a two-way phase estimation technique [7].

For the following part, we assume that S_N is the master node (reference node) and the set of S_i for $i \in \{1, ..., N-1\}$ is the set of the slave nodes. Then, the master node starts broadcasting the generated carrier signal $s_{s,N}(t)$ to the slave nodes and the destination keeps broadcasting $s_D(t)$. The i^{th} slave node receives $s_{sr,i}(t)$:

$$s_{sr,i}(t) = h^{f_B}_{N,i}(t)e^{j\{2\pi f_B t + \varphi_{B,N} + \varphi^{MS}_i\}} \tag{4}$$

where φ^{MS}_i is the channel phase shift between the master node and the i^{th} slave node (superscript signifies the transmission is from master node to slave node). Then, the slave node i compares the phase of $s_{sr,i}(t)$ with the phase of internally generated signal $s_{s,i}(t)$ by using a phase detector (PD), and digitizes and stores the difference $\varphi_{i,d}$:

$$\varphi_{i,d} = \varphi_{B,N} + \varphi_i^{MS} - \varphi_{B,i} \quad mod(2\pi) \tag{5}$$

Then, the master node stops to transmit $s_{s,N}(t)$ and the i-th slave node starts transmitting $s_{s,i}(t)$ to the master node. The master node receives $s_{mr,i}(t)$:

$$s_{mr,i}(t) = h_{i,N}^{f_B}(t)e^{j\{2\pi f_B t + \varphi_{B,i} + \varphi_i^{SM}\}} \tag{6}$$

where φ_i^{SM} is the phase shift between the slave node i and the master node (from i-th slave node to the master node). The master node estimates the phase difference between $s_{s,N}(t)$ and (6) by using a PD and stores the difference $\varphi_{i,p}$:

$$\varphi_{i,p} = \varphi_{B,i} + \varphi_i^{SM} - \varphi_{B,N} \quad mod(2\pi) \tag{7}$$

After the estimation of the phase difference $\varphi_{i,p}$ for all slave node i, the master node sends $\varphi_{i,p}$ to the slave node i. From the reciprocity of the channel: $\varphi_i^{MS} = \varphi_i^{SM}$. Then, the slave node i subtract (5) from (7), and stores:

$$\varphi_{i,c} = \varphi_{i,d} - \varphi_{i,p} = 2\{\varphi_{B,i} - \varphi_{B,N}\} \quad mod(2\pi). \tag{8}$$

which completes the local relative carrier phase estimation.

Next, we need to estimate the channel phase delay between the destination and the sensor nodes. By using a divide-by-L/M circuit the destination node generates $s_B(t)$ as:

$$s_B(t) = Ae^{j\{(2\pi f_M t + \varphi_0)\frac{M}{L}\}} = Ae^{j\{(2\pi f_B t + \varphi_B)\}} \tag{9}$$

In addition to the $s_D(t)$, the destination starts broadcasting $s_B(t)$ to the sensor nodes and the i^{th} sensor node receives

$$s_{B,i}(t) = Ah_{0,i}^{f_B}(t)e^{j\{(2\pi f_B t + \varphi_B + \varphi_i^{DS})\}} \tag{10}$$

where φ_i^{DS} is the channel phase delay between the destination and the i^{th} sensor node. Then, each sensor node estimates the phase difference between $s_{B,i}(t)$ and $s_{s,i}(t)$ by using a PD and stores $\Delta\varphi_i$:

$$\Delta\varphi_i = \varphi_B + \varphi_i^{DS} - \varphi_{B,i} \tag{11}$$

After obtaining the channel phase information between the destination and sensor nodes, each sensor node adjusts its internal signal $s_{s,i}(t)$ with (8) using a programmable delay cell (PDC). The PDC-1 subtracts $\varphi_{i,c}$ from the phase of the $s_{s,i}(t)$. Then, the PDC-2 subtracts $\Delta\varphi_i$ from the new internal carrier and the output phase of the signal transmitted from the i-th slave node becomes:

$$\varphi_{output,i} = 2\varphi_{B,N} - \varphi_B - \varphi_i^{DS} \tag{12}$$

We assume a reciprocal channel, then the received carrier phase at the destination from the slave node i is:

$$\varphi_{received,i} = 2\varphi_{B,N} - \varphi_B \tag{13}$$

which is constant and the same for all slave nodes. This completes the phase adjustment for CB. The destination stops to transmit $s_B(t)$ (keeps broadcasting $s_D(t)$) and enters receiving mode at the frequency f_B, and the sensor network goes into the beamforming transmission stage.

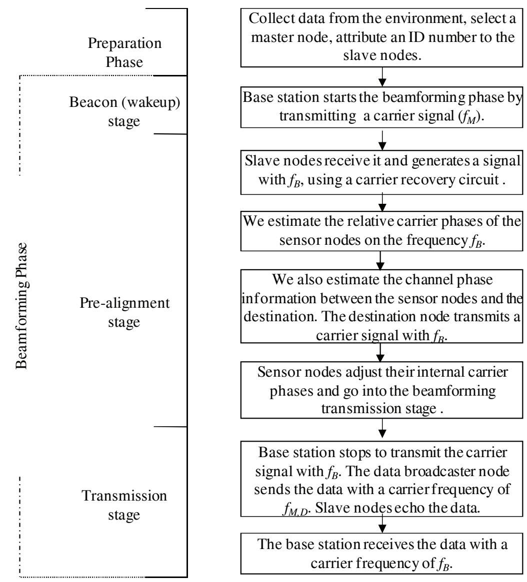

Fig. 3. Operation sequence of the protocol

In the *transmission stage*, the destination node keeps broadcasting the pilot frequency f_M, and the sensor nodes keep using f_M as a reference to generate beamforming carrier frequency (f_B). According to the ID number given by the master node, sensor nodes become data broadcaster sequentially and transmit their stored data to the other sensor nodes on the carrier frequency $f_{M,D}$ ($\neq f_M$ and $\neq f_B$).

The remaining sensor nodes start retrieving the received data carried on $f_{M,D}$. The recovered data is echoed immediately using f_B as a carrier by employing FDD. The received signal by the base station is $r(t)$:

$$r(t) = p(t)\{\sum_{i=1}^{N-1} h_{i,0}(t)e^{2\varphi_{B,N} - \varphi_B} + n(t)\} \tag{14}$$

where $p(t)$ is the baseband data, and $n(t)$ is the additive white Gaussian noise (AWGN). The operation sequence of the protocol is summarized in Fig. 3.

C. Transceiver Architecture

In this section, we present a transceiver architecture compatible with the CB technique described in the previous section. The architecture needs to support both master and slave mode operations as well as data broadcaster mode. Fig. 4 shows a block diagram of the proposed transceiver architecture. The receiver (Rx) has three signal paths for $f_{M,D}$, f_M, and f_B. The path for $f_{M,D}$ retrieves the data from the data broadcaster during beamforming transmission stage. The path for f_M detects the signal with frequency f_M broadcasted by the destination node during the beamforming phase. The path for f_B is used to estimate both the relative carrier phase information between the master node and slave nodes and the channel phase

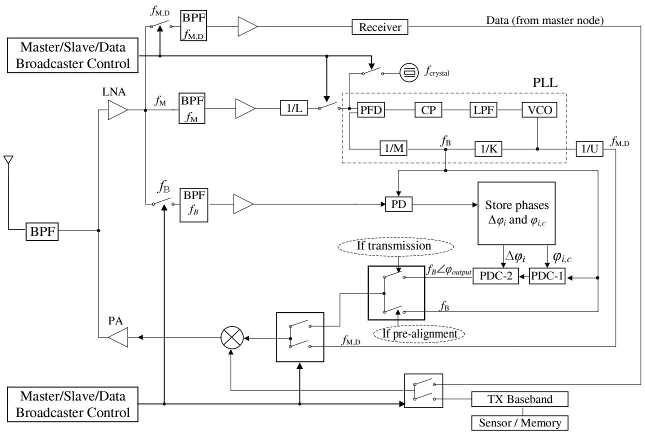

Fig. 4. Proposed transceiver architecture

information between the destination and the sensor nodes. The path includes a PD that estimates phase difference and adjusts the phase of f_B. The transmitter (Tx) retransmits (echoes) the retrieved data using carrier frequency f_B locked to f_M. While in data broadcasting mode, the Tx uses $f_{M,D}$ as a carrier to transmit data stored in local memory and the Rx uses f_M path to be frequency synchronized with the destination node. Since both Rx and Tx work simultaneously with different frequencies, the transceiver front-end uses FDD.

III. Performance Analysis

A. Phase Error Analysis

There are various kinds of error sources that degrade the beamforming gain. We consider the major error source: the phase jitter due to the voltage controlled oscillator (VCO) phase noise. We assume that the communication channel has high SNR and hence, the phase estimation error is not a concern.

Jitter Analysis

In the beamforming phase, the destination node broadcasts a reference signal for frequency synchronization. However, due to the VCO phase noise the broadcasted signal has a timing jitter which results in temporal phase variations. Each sensor node receives this time varying signal and generates its internal carrier. The VCO phase noise of the sensor nodes also causes timing jitter. We define $\varphi_i^{j_1}(t)$ and $\varphi_i^{j_2}(t)$ as the phase error due to the time varying input signal and the VCO phase noise of the sensor node i, respectively. We assume that they are not dependent upon the carrier frequency.

Detailed analyses of the jitter have been focused in many researchers [14]- [19]. Hajimiri [19] derived the timing jitter equation for a first order PLL by modeling the phase noise as a white noise on the VCO control voltage. The variance of the timing jitter σ_j^2 is given as [19]:

$$\sigma_j^2(\tau) = \frac{N_0 K_v^2}{2w_0^2}\tau \qquad (15)$$

where $N_0/2$ is the double sideband input white noise, K_v is the VCO gain, τ is the delay from the reference edge, and w_0 is the center frequency. In a PLL, $max\{\tau\} = 1/w_{3-dB}$ where $w_{3-dB} = 2\pi f_{3-dB}$, and f_{3-dB} is the loop bandwidth. Hence, the maximum phase jitter deviation is bounded by:

$$\left|\varphi_i^j(t)\right| \leq \sqrt{\frac{1}{16\pi^3}\frac{N_0}{f_{3-dB}}}K_v \qquad (16)$$

We note that $\varphi_i^{j_2}(t) = \varphi_i^j(t)$ by definition.

Secondly, the transfer function of the second order PLL has a low-pass characteristic [20]. If the input varies rapidly, the PLL filters the jitter, i.e. the output tracks the input to a lesser extent. Hence, $\varphi_i^{j_1}(t) \leq \varphi_i^j(t) = \varphi_i^{j_2}(t)$.

Total Phase Error Analysis

Next, we derive the total phase error of the proposed protocol due to the timing jitter. After i^{th} sensor node receives the signal transmitted by the master node as given in (3) for $i = N$, it estimates the phase of the received signal $\hat{\varphi}_{i,d}$:

$$\hat{\varphi}_{i,d} = \varphi_{i,d} + \varphi_i^p(t_1) + \varphi_i^p(t_2) \qquad (17)$$

where $\varphi_i^p(t_\ell) \triangleq \varphi_i^{j_1}(t_\ell) + \varphi_i^{j_2}(t_\ell)$ is the estimation error in the slave node i at time $t = t_\ell$ and $\varphi_i^p(t_\ell)$ for $\ell = 1, 2$ in (17) comes from the fact that both of the compared signals have jitter components. Similarly, when the sensor node i transmits its internal carrier to the master node, the master node estimates $\hat{\varphi}_{i,p}$:

$$\hat{\varphi}_{i,p} = \varphi_{i,p} + \varphi_i^p(t_3) + \varphi_i^p(t_4) \qquad (18)$$

Then, the stored estimated phase value $\hat{\varphi}_{i,c}$ becomes:

$$\hat{\varphi}_{i,c} = \varphi_{i,c} + \sum_{\ell=1}^{4} \varphi_i^p(t_\ell) \qquad (19)$$

When the destination node sends a carrier with f_B for the channel phase information estimation, the estimated phase value $\widehat{\Delta\varphi_i}$ becomes,

$$\widehat{\Delta\varphi_i} = \Delta\varphi_i + \varphi_i^{j_1}(t_6) + \varphi_i^p(t_5) \qquad (20)$$

because the received signal from the destination does not have $\varphi_i^{j_2}(t)$ component. Then, the received carrier phase to the destination from a sensor node i is $\hat{\varphi}_{received,i}$:

$$\hat{\varphi}_{received,i} = \varphi_{received,i} + \varphi_i^{j_1}(t_6) + \sum_{\ell=1,\ell\neq 6}^{7} \varphi_i^p(t_\ell) \qquad (21)$$

The received signal by the destination node becomes:

$$r(t) = p(t)\{\sum_{i=1}^{N-1} h_{i,0}(t)e^{\hat{\varphi}_{received,i}} + n(t)\} \qquad (22)$$

For the beamforming efficiency analysis, we assume that the transmitted signals by the sensor nodes arrive at destination with equal amplitudes i.e. $h_{i,0}(t) = c$ and we normalize the received signal with this constant.

Next, we estimate the total phase jitter variance for beamforming gain analysis. We assume that the phase jitters in

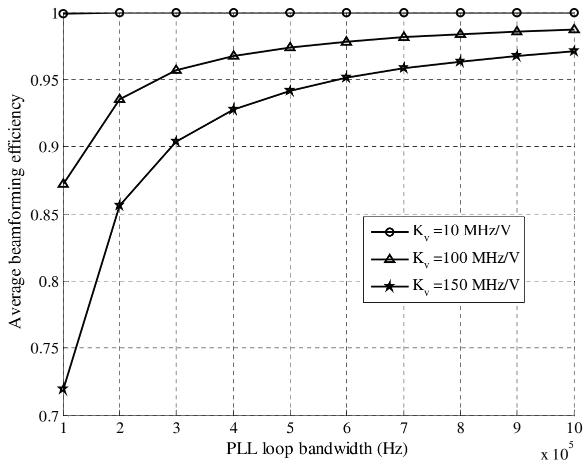

Fig. 5. Beamforming gain v.s. PLL loop bandwidth

different sensor nodes and at different instants of time are uncorrelated and that the phase jitters have normal distributions with mean zero and standard deviation given in (16). Thus, the phase jitter variance of total phase error, (21), is $\sigma_T^2 = 7\sigma_{j1}^2 + 6\sigma_{j2}^2$. Then, the upper bound of the maximum variance using equality in (16) is found to be

$$\sigma_{T,max}^2 = \frac{13}{16\pi^3}\frac{N_0 K_v^2}{f_{3-dB}} \tag{23}$$

Above equations show that increasing the PLL loop bandwidth decreases the phase error $\varphi_i^j(t)$.

B. Beamforming Gain Analysis

We define the average beamforming gain β for $N \to \infty$ as the square root of the normalized received power when the total transmit power $P_T = 1$:

$$\beta = E\{e^{j(\Delta\phi)}\} \tag{24}$$

where $E\{.\}$ denotes the expectation operator. We know that $\Delta\phi$ has a normal distribution with mean zero and standard deviation $\sigma_{T,max}$. As an example, we simulate the beamforming gain with respect to the PLL loop bandwidth for $N_0 = 10^{-10}\ V^2/Hz$, $f_B = 600$ MHz, $K_v = 10{-}200$ MHz/V, and 1 MHz $\geq f_{3-dB} \geq$ 100 kHz. As shown in Fig. 5, for these typical conditions, the beamforming efficiency is always better than 72%. K_v is the most significant parameter because the phase jitter deviation is linearly proportional to K_v. We note that increasing the PLL loop bandwidth increases the average beamforming gain. Hence, the degradation due to the K_v can be partially compensated by increasing loop bandwidth. Fig. 5 provides the design parameters of the PLL for a given target beamforming gain.

IV. Conclusion

We present a CB protocol and compatible transceiver in WSNs that effectively addresses the frequency, phase and data synchronization issues. It uses a beacon based locking technique for frequency synchronization that does not suffer from frequency drift and does not require simultaneous operations of transmitters and receivers at the same frequency. The phase synchronization issue is addressed using a local two-way phase initialization and a remote one way calibration. We address the data synchronization issue using data entrainment that significantly reduces data traffic. The analysis shows that the carrier signal jitter significantly affects the beamforming gain, and a reasonable gain (>70%) can be achieved with typical radio design parameters.

V. Acknowledgement

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