Phase Difference and Frequency Offset Estimation for Collaborative Beamforming in Sensor Networks

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***Abstract*— A fast and power efficient phase difference and frequency offset estimation technique for collaborative beamforming in a wireless sensor network is presented. Common radio blocks are used to implement the phase difference estimation technique and we present an analytical expression for the phase difference estimation accuracy in an AWGN channel. The analysis including the effect of noise and multipath on the estimation accuracy shows the effectiveness of the proposed estimation technique for collaborative beamforming applications.**

I. Introduction

Collaborative beamforming enhances the energy efficiency and capacity in wireless sensor systems. Multiple separated transmitters cooperate to form an antenna array for a common message transmission to a desired distant receiver. For high efficient and ISI (inter-symbol interference) free transmissions, transmitted signals of each node at the receiver are required to be in-phase and time synchronized so that signals add constructively. In other words, signals are required to be highly correlated at the receiver for high beamforming gain. The advantages of collaborative beamforming are well documented in the literature [1] [2]. If the transmitted power and free-space attenuations are the same for each node and transmitted signals by N sensor nodes are highly correlated, we obtain N^2-fold power gain which leads to increase in channel capacity or an N-fold increase in propagation range. For a fixed received power, we obtain an N-fold decrease in the transmitted power.

In this paper, we focus on wireless sensor networks (WSNs) because low-cost, low-power, and mass produced WSNs hold the promise of many practical applications. Ecology monitoring in a rain forest and information transmission from ground sensors to a satellite are a few examples [3]. In these applications, sensor nodes are deployed in remote, possibly hazardous places while receiver (base station) is far away from these sensor nodes. Since sensor nodes have limited transmission range due to low transmit power, they have to collaborate to convey information over long distances to the receiver for data analyses. In addition, it is necessary to optimize the battery lifetime of sensor nodes because it is difficult to recharge or replace them. Hence, energy consuming operations should be minimized.

In spite of the promising prospects, collaborative beamforming has several challenging problems such as phase and frequency offset estimation and adjustment, time synchronization, and energy consumption in data sharing process. The phase and frequency offsets resulting from the temperature, humidity, process variations, and aging make the targeted beampattern significantly distorted as time proceeds. Fig. 1 shows the ideal collaborative beamforming and Fig. 2 shows the effect of the phase and frequency offsets on collaborative beamforming. The phase and frequency offsets create strong sidelobes which decrease the energy in the main lobe. Hence, collaborative beamforming losses its advantages. Data synchronization is another major problem. If the data to be transmitted are not aligned, it distorts the signal which causes ISI problem. Therefore, precise timing control and location information are necessary to prevent the distortions. Nevertheless, a recent paper [4] showed that it is possible to reach fine-grained time synchronizations with errors on the order of μs' by using reference broadcast systems (RBS) as opposed to network time protocol (NTP) where synchronization errors are at least eight times greater than those of RBS synchronization algorithm.

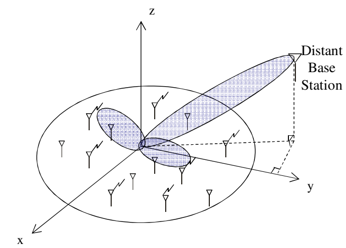

Fig. 1. Ideal collaborative beamforming

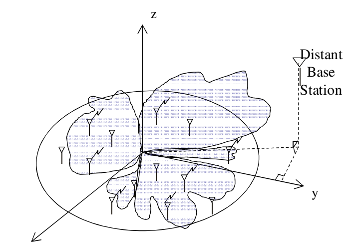

Fig. 2. Effect of nonidealities on collaborative beamforming

Mudumbai et. al. [7] proposed one bit feedback control protocol for the phase synchronization. However, they ignored the frequency offset effect and their proposed protocol requires transmission from local sensor nodes to base station and a feedback control. Hence, this scheme requires a large energy overhead. Another issue with one bit feedback control is that it requires enormous number of iterations that cause latency in the transmission and energy consumption [5]. Master-slave architecture [1] and time-slotted round trip carrier synchroniza-

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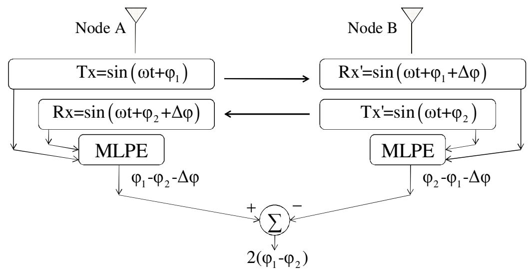

Fig. 3. Two-way phase estimation technique

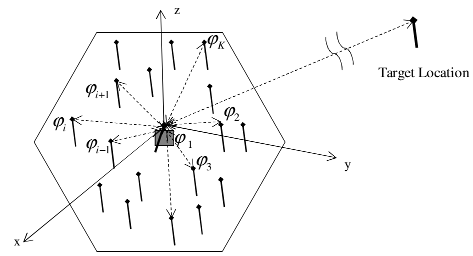

Fig. 4. Phase difference in a collaborative beamforming sensor network

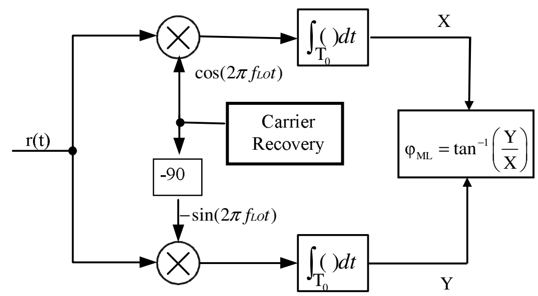

Fig. 5. One-shot ML phase estimate with quadrature carriers

tions [8] are two recent open-loop synchronization schemes. Fast and accurate phase difference (to a reference point) and frequency offset estimations are necessary for the operations in [8] and for reducing the synchronization overhead in both [1] and [8]. The scheme in [8] requires a pair of frequency synthesis PLLs in each node.

Another approach to collaborative beamforming is that phase and frequency offsets are estimated explicitly during pre-beamforming phase and then these estimations with a priori location information are used to steer the beam. The average beampattern for uniformly distributed sensor network in a two dimensional disk was derived in [2] and it was shown that collaborative beamforming increases the signal strength at the receiver. In a two-antenna beamformer case the beamforming efficiency is more than 0.9 even if there exists 30 degree phase offset between the antennas [1]. Chang et al. [6] grouped sensor nodes into phase partitions in a large sensor network. Antennas belonging to the same group is scheduled to transmit together since they have similar arrival phases and the beamforming efficiency is higher. Another property of the phase partition is that in a very large network it is possible to find a group of sensor nodes that have main lobe pointing the desired direction. However, they assumed that phase information is available and ignored the effect of frequency offsets.

In this paper, we propose an effective phase difference and frequency offset estimation technique, which can be adopted to different kinds of network topologies and beamforming methods. We also present the effect of channel noise and multipath fading on the estimation accuracy, providing a guideline for sensor network beamforming optimization including phase and frequency offset compensation.

II. Phase Difference Estimation Technique

Fig. 3 shows the proposed general phase difference estimation technique between two sensor nodes. The two nodes shown have the same angular frequency ω_{LO} but different phases φ_1 and φ_2 for node *A* and node *B*, respectively. Node *A* transmits a signal to node *B* and node *B* receives the phase shifted signal due to the channel and estimates the phase difference, φ_2-φ_1-$\Delta\varphi$ using a modified maximum likelihood phase estimator (MLPE) described in Section III. Then node *B* sends a signal to node *A* and node *A* receives the phase shifted signal and then estimates φ_1-φ_2-$\Delta\varphi$. The subtraction of them executed in the network level after sharing the data gives 2(φ_1-φ_2). Assuming the nodes *A* and *B* have identical RF-front ends, the phase contributions due to the transcievers cancel out in the proposed phase difference estimation.

Suppose that there are *K* numbers of nodes with known locations in a network and one node with phase φ_1 is a coordinator as shown in Fig. 4. A set of $\{\varphi_i\text{-}\varphi_1\}$ where $2 \leq i \leq K$ is calculated by means of the aforementioned technique with complexity *O(K)*. Then using the set $\{\varphi_i\text{-}\varphi_1\}$, and location information, the output phase of each node *i* could be shifted to achieve the required synchronization condition described in [2], or the nodes that should participate in beamforming can be selected according to the technique described in [6]. It can also be used for new beamforming algorithm development.

III. Modified ML Carrier Phase Estimator

For an unmodulated carrier, the ML phase estimate is given in [9] by:

$$\int_{T_0} r(t) sin(2\pi f_c t + \varphi_{ML}) dt = 0 \tag{1}$$

where $r(t)$ is the received signal, f_c is the carrier frequency, and φ_{ML} is the ML carrier phase estimate. An explicit expression for (1) is given in [9] by:

$$\varphi_{ML} = -tan^{-1}\left[\int_{T_0} r(t) sin(2\pi f_c t) dt \Big/ \int_{T_0} r(t) cos(2\pi f_c t) dt\right] \tag{2}$$

Fig. 5 shows an explicit ML phase estimator for an unmodulated carrier. In a collaborative beamforming point of view, the explicit phase information is necessary such that the extraction of the received signal phase information can be used to adjust the phases of the nodes to have a beam pattern pointing the desired target location. For the two-way phase

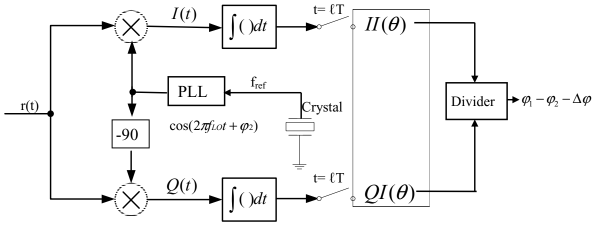

Fig. 6. Modified ML phase estimate for collaborative beamforming

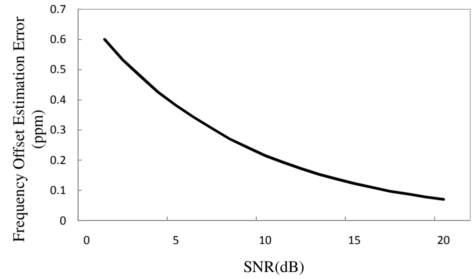

Fig. 7. Frequency offset estimation error v.s. SNR

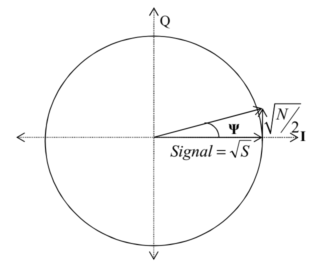

Fig. 8. Representation of signal and noise on a constellation diagram

estimation technique described in Section II, the transmitters and receivers are required to have a constant phase during the data transmission. For this purpose, we modify the ML phase estimator in Fig. 5 to have a constant phase which can be achieved by a crystal based PLL circuit shown in Fig. 6. The modified MLPE consists of I/Q mixers, integrators, a crystal based PLL, and a divider, which are common in radio circuit design. Note that the ratio of the PLL output phase to the crystal phase is fixed in most cases. Hence, phases, φ_1 and φ_2, of the PLL outputs in *A* and *B* in Fig. 3 do not change as long as the crystals remain on. Since the crystal is used to provide a clock signal to other parts of the system, even during hibernation, we can safely assume that it is always on and phases φ_1 and φ_2 are always constant.

Assuming that the input to the modified ML phase estimation receiver is $r(t)=\alpha\cdot\cos(2\pi(f_{LO}+\Delta f)t+\varphi_1+\Delta\varphi)$ where α is the attenuation constant, f_{LO} is the carrier frequency, Δf is the frequency offset, φ_1 is the transmitted signal phase, and $\Delta\varphi$ is the phase shift due to the channel with the receiver carrier of $\cos(2\pi f_{LO}t+\varphi_2)$, then the ML estimated phase difference becomes:

$$tan^{-1}(-QI/II) = \pi\Delta f\ell T + \varphi_1 - \varphi_2 + \Delta\varphi. \quad (3)$$

While deriving (3), it is also assumed that $2f_{LO} \gg \Delta f$ and $sin(\pi\Delta f\ell T)\approx\pi\Delta f\ell T$. Equation (3) is an inverse tangent function and bounded by $\pm 90°$. A whole range (360°) can be covered by checking the polarities of the numerator and denominator in addition to their ratio. Another issue is that $tan^{-1}(-QI/II)$ goes to 90° or 270° as $-QI/II$ approaches $\pm\infty$. This occurs when the value of II is zero. Since a tangent value changes rapidly near 90° and 270°, we mitigate this issue by truncating $-QI/II$ value. For instance, if $|-QI/II|$ is bigger than 30, we assign 89° or 271° to $tan^{-1}(-QI/II)$ depending on the polarity of QI. The numerical error is less than $\pm 1°$.

A. Frequency Offset Estimation

We use Fig. 6 without any alteration for frequency offset estimation. Equation (3) is a linear function with two unknowns; Δf and $\varphi_1 - \varphi_2 + \Delta\varphi$. We can solve this equation by taking samples in series. Assuming that two sampling times are ℓT and $(\ell + m)T$, then the samples provide two equations:

$$y_1 = tan^{-1}(-QI/II)_1 = \pi\Delta f(\ell T) + \varphi_1 - \varphi_2 + \Delta\varphi \quad (4)$$

$$y_2 = tan^{-1}(-QI/II)_2 = \pi\Delta f(\ell + m)T + \varphi_1 - \varphi_2 + \Delta\varphi \quad (5)$$

By subtracting (4) from (5), we obtain:

$$\Delta y = y_2 - y_1 = \pi\Delta f m T. \quad (6)$$

Frequency offset can be estimated from (6). Fig. 7 shows the simulation result for the frequency offset estimation error for $\ell = 800$, $m = 1600$, and $\wp = 100$. The maximum error is 0.2ppm at $SNR = 10dB$. As shown in (3), frequency offset hinders correct phase estimation, and more importantly makes the amplitude of a combined signal fluctuate over time. The estimated frequency offset can be used to adjust or to compensate the offset or to estimate the efficiency of a collaborative beamforming method during its optimization.

B. Effect of Channel Noise

In this subsection, we assume $\Delta f=0$ and an AWGN channel. Hence, (3) becomes:

$$tan^{-1}(-QI/II) = \varphi_1 - \varphi_2 + \Delta\varphi \quad (7)$$

The inphase and quadrature noise vectors are i.i.d zero mean Gaussian random variables with a standard deviation of $\sqrt{N/2}$, where N is the noise power. In [10] the one-way phase accuracy expression is derived for large SNR assuming that signal with power S has only inphase component and ignoring the noise component on the inphase channel, which can also be deduced from Fig. 8, $\sigma_{rad} \approx \sqrt{N/2}/\sqrt{S}$ using $tan(\Psi)\approx\Psi$ for $S \gg N$ where σ_{rad} is the one-way phase estimation error. The error as a function of SNR is given by [10]:

$$\sigma_{rad} = 1/\sqrt{2 \times 10^{(SNR/10)}} \quad (8)$$

Equation (8) is for only one sample. The sample mean theorem states that ℓ number(s) of independent samples reduce the variance by a factor of $\sqrt{\ell}$. Furthermore, $\wp$ times of independent

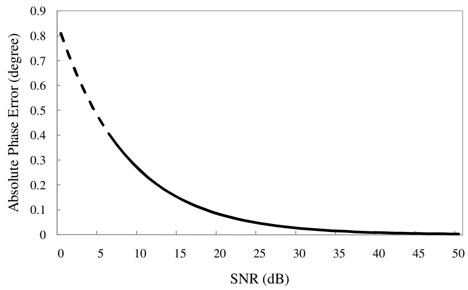

Fig. 9. Two-way phase accuracy v.s. SNR

measurements reduces the variance by a factor of $\sqrt{\wp}$. Hence, the two-way phase accuracy becomes:

$$\sigma_{2,rad} = 2/\sqrt{2 \times \ell \times \wp \times 10^{(SNR/10)}} \quad (9)$$

where $\sigma_{2,rad}$ is the two-way phase estimation error. Equation (9) is plotted in Fig. 9 for $\ell = 1000$ and $\wp = 10$ with dashed part signifying that the accuracy expression is invalid for SNR less than 5dB. The error is less than $0.3°$ at 10dB SNR for the two-way phase estimation.

C. Effect of Multipath on Phase Estimation

Multipath is another important parameter for practical applications. The phase of the signal at the receiver input changes due to scattering, reflection, and diffraction. Amplitude, phase, and frequency of the signal can change at the same time. The collaborative beamforming concept is generally used in outdoor applications and we assume that wireless sensor nodes are stationary or slowly moving devices. Hence, we suppose that the frequency of the signal does not fluctuate (non-selective or flat fading) and the channel is constant. We use the COST 207 typical urban channel model (TU 6-path channel model) [11], which has 5μs delay spread with following coefficients: $\{Power(dB)$, $\tau(\mu$s)$\}$= $\{(-3,0)$, $(0,0.2)$, $(-2,0.5)$, $(-6,1.6)$, $(-8,2.3)$, $(-10,5)\}$. Fig. 10 shows simulation result for three different frequencies ($f_{LO} = 50MHz, 250MHz$ and $1000MHz$). Although the estimation error varies with SNR, its overall variation is less than $0.4°$ with SNR from 30 dB to 0dB, $\ell = 1000$ and $\wp = 10$. Hence, we can assume that multipath introduces an almost constant offset in phase estimation. The constant offset can be estimated from known channel conditions during installation, or by using a location fingerprinting technique [12]. Because of the difficulties and complexities associated with the offset estimation, it becomes the major source of phase estimation error. It is worthwhile to note that the collaborative beamforming technique described in [6] provides reasonable performance even with $60°$ phase error for large numbers of nodes. We also note that in the reciprocal channels multipath does not cause any phase error due to the inherent property of two-way phase estimation technique.

IV. Conclusion

We present an efficient phase difference and frequency offset estimation technique for collaborative beamforming in sensor network and its implementation using standard RF

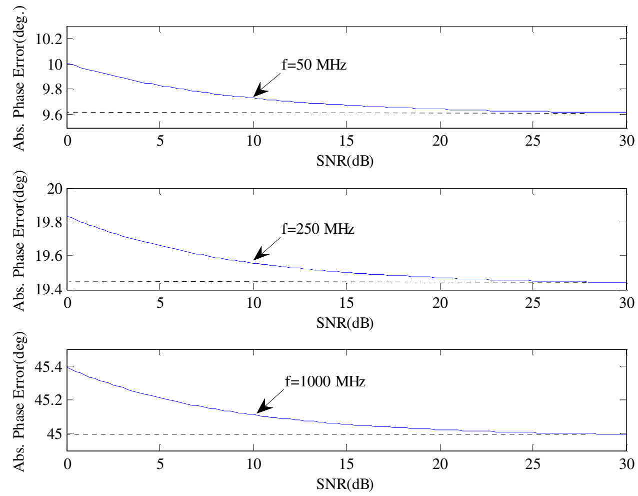

Fig. 10. Multipath effect

blocks. The results show that the errors in phase difference and frequency offset estimations are less than $0.3°$ and 0.2ppm, respectively, for the conditions described and that multipath introduces an almost constant offset in phase estimation. It also shows feasibility for fast and accurate phase and frequency offset estimation required for collaborative beamforming using a simple low-power ML based estimator.

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