Summary Sheet for Continuous Distributions

ECE 695/CS 590 –Fault-Tolerant Computer System Design
School of Electrical and Computer Engineering
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Notation	Interpretation	Relations
$F_X(x)$	Pr(Random variable X has a value $\leq x$) Called Cumulative Distribution Function, or simply, Distribution Function (cdf)	
$f_X(x)$	Probability density function (pdf) $\int_{-\infty}^{\infty} f(x) = 1$ $f(x) \geq 0$, for all x	$F_X(x) = \int_{-\infty}^{x} f_X(t)dt$ $Pr(a < X \leq b) = \int_a^b f_X(t)dt$ $= F_X(b) - F_X(a)$
$R(t)$	Reliability Pr(No failure occurs in time (0, t))	$R_X(t) = 1 - F_X(t)$ Typically R(0) = 1 and $\lim_{t\to 0} R(t) = 0$
$h(t)$	Instantaneous failure rate $\frac{1}{R(t)} \lim_{x\to 0} \frac{R(t+x)-R(t)}{x}$ Pr(Failure in ($t0$, $t1$)|System has survived till $t0$) $\neq \int_{t0}^{t1} h(x)dx$ It is $= 1 - \frac{R(t1)}{R(t0)}$	$h(t) = \frac{f(t)}{R(t)}$ $R(t) = \exp[-\int_0^t h(x)dx]$ If $h(t)\Uparrow$ as $t\Uparrow$, this represents an Increasing Failure Rate (IFR). If $h(t)\Downarrow$ as $t\Uparrow$, this represents a Decreasing Failure Rate (DFR). If $h(t)$ stays constant as $t\Uparrow$, this represents a Constant Failure Rate (CFR).

Distribution	Characteristic Measures	Interpretation
Exponential distribution	$F(t) = 1 - e^{-\lambda t}$ $h(t) = \lambda$	If failures arrive with a constant rate λ, then the time to failure follows the Exponential distribution. Exponential distribution represents the CFR part of a system's lifetime.
Hypoexponential distribution	For two stage HYPO with parameters λ_1 and λ_2 ($\lambda_1 \neq \lambda_2$) $F(t) = 1 - (\lambda_2/(\lambda_2 - \lambda_1))e^{-\lambda_1 t} + (\lambda_1/(\lambda_2 - \lambda_1))e^{-\lambda_2 t}$	If a process passes through several sequential phases, and time spent in each is independent and exponentially distributed. It represents the IFR part of a systems lifetime, going from 0 to

		min{λ_1, λ_2, 1}.
Erlang distribution	$F(t) = 1 - \sum_{k=0}^{r-1} e^{-\lambda t} \frac{(\lambda t)^k}{k!}$ Parameters: r, λ	Process passes through r sequential phases, each of which has an identical exponential distribution. Exponential is a special case of Erlang, with $r = 1$ If a system can survive up to r-1 shocks and fails upon the arrival of the r-th shock, and the shocks arrive with a Poisson distribution, then time to failure follows Erlang.
Gamma distribution	$f(t) = \frac{\lambda^{\alpha} t^{\alpha-1} e^{-\lambda t}}{\Gamma(\alpha)}$, $\alpha>0$ Γ is the gamma function, defined as $\Gamma(\alpha) = \int_0^{\infty} x^{\alpha-1} e^{-x} dx$ Parameters: λ, α	If r in the Erlang distribution can take non integer values, it gives the Gamma distribution. For $0 < \alpha < 1$, Gamma distribution is DFR; for $\alpha > 1$, it is IFR; for $\alpha = 1$, it is CFR.
Hyperexponential distribution	$F(t) = \sum_{i=1}^{k} \alpha_i (1 - e^{-\lambda i t})$ where $\sum_{i=1}^{k} \alpha_i = 1$ Parameters: λ_i, α_i, k	If a process faces k parallel phases and it will go through only one of these, then the time follows a Hyperexponential distribution. It is a DFR from $\sum_{i=1}^{k} \alpha_i \lambda_i$ to min{λ_1, λ_2, …}.
Weibull distribution	$F(t) = 1 - e^{-\lambda t^{\alpha}}$ $h(t) = \lambda \alpha t^{\alpha-1}$ Parameters: α (shape parameter), λ (scale parameter)	Most widely used parametric family of distributions. $\alpha = 1 \Rightarrow$ CFR $\alpha < 1 \Rightarrow$ DFR $\alpha > 1 \Rightarrow$ IFR