Example Problems for Lecture on Quantitative Tools for Reliability Analysis

ECE 60872 – Fault-Tolerant Computer System Design
School of Electrical and Computer Engineering
Purdue University
Spring 2024

1. Failure rate and Reliability

Consider that the failure rate of a component is $h(t) = \lambda_0 t$ where $\lambda_0 > 0$ is a constant.
Which phase of the component's life does this failure rate characterize?
What is the reliability of the component? What is the expected life of the component (i.e., its MTTF)?

2. Probability bounds

The CPU time per request is known to be exponentially distributed with a mean of 4.39 seconds for a compute server. We classify a request as a trivial request if it takes less than 1 second to complete, a moderate request if it takes between 1 and 5 seconds, and a number crunching request otherwise.

Obtain a bound on the probability that a given request is a number-crunching request. *Hint*: Get familiar with Chebyshev's inequality.

3. Reliability of a parallel system

Consider a parallel system of n independent components. Let X denote the lifetime of the overall system. Assume that the lifetime of each component, X_i, is exponentially distributed with parameter λ (all components have the same parameter). What is the reliability and MTTF of the parallel system?

4. Reliability of a series system

Consider a system made out of n components in series. Component i has lifetime $X_i \sim \text{WEI}(\lambda_i, \alpha)$. What can you say about the reliability of the overall system?

5. Error distribution

Consider that the measurement error is given by the variable X. We are interested in the distribution of the square of the error, i.e., of the variable $Y = X^2$. Write down the cdf of Y, i.e., formulate $F_Y(y)$.
If $X \sim \text{EXP}(\lambda)$, what form does $F_Y(y)$ take?