P-Bit and FPGA Acceleration of Sampling for Modeling Log-Normal Colored Noise in Non-Linear Oscillator

Nathaniel James Nauman

Supervisors: Jan Kaiser, Rishi Jaiswal, Professor Supriyo Datta

Elmore Family School of Electrical and Computer Engineering, Purdue University, West Lafayette, IN 47907

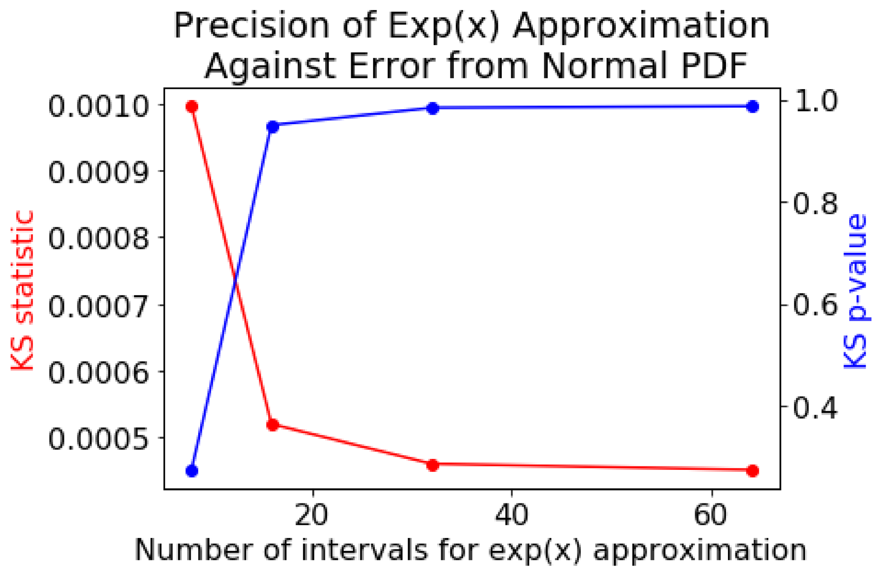
Introduction

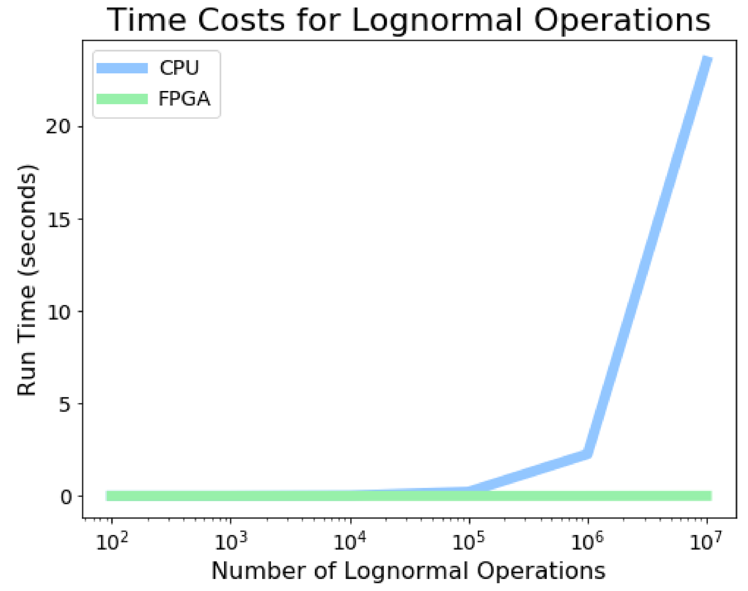
Probabilistic computing is emerging as an exciting bridge between deterministic digital computing and quantum computing by utilizing p-bits as hardware accelerators for stochastic algorithms and processes. The p-bit is composed of a magnetic tunnel junction with a low-barrier magnet in place of the free layer. This study uses FPGA hardware to transform uniformly generated random numbers into samples from normal and log-normal distributions. This process can be augmented by replacing the linear-feedback shift registers with p-bits.

The normal and log-normal samples are white and colored noise, respectively, in the stochastic differential equation of a non-linear oscillator. By tuning the noise intensity, the oscillator can jump into another state. A state's stability and the distribution of the mean first-exit time not only depends upon the noise intensity, but the type of noise in the system as well.

Log-Normal Transformation

This study introduces a simple SystemVerilog range reduction technique for the exponential function on the output of the Box-Muller transformation. The power can be separated into integer and fractional components, and then the exponential of each is calculated by indexing two lookup tables. The number of entries in the fractional table must maintain balance between memory and precision. The Kolmogorov-Smirnov test quantified the quality of the distribution after 1 million samples, and the amount of consumed BRAM space was measured for 8, 16, 32, and 64 entries. The KS statistic should be near 0 and the p-value should exceed 0.05 and ideally approach 1. The hardware with 16 intervals had a sufficiently high p-value and consumed the same number of memory tiles as the hardware with 8 intervals.



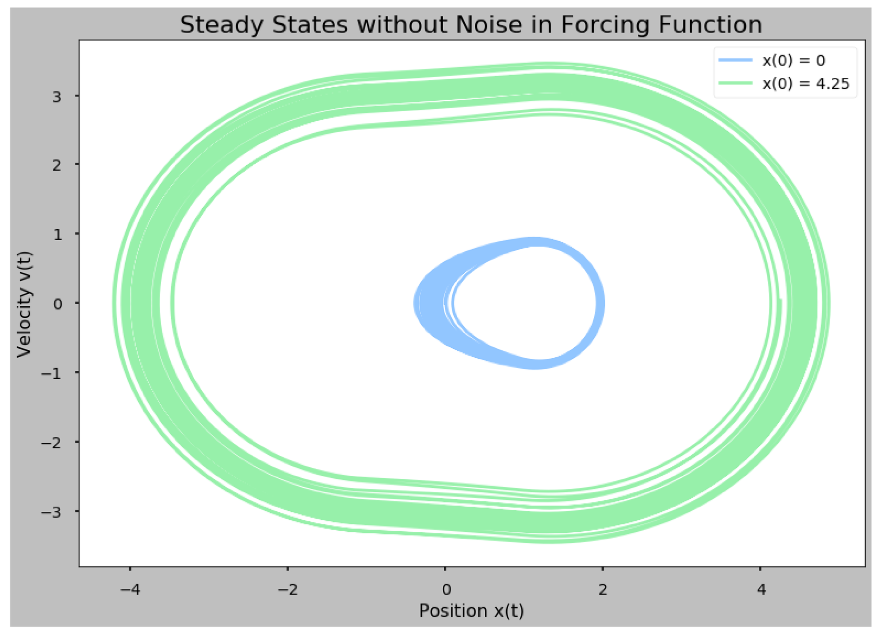


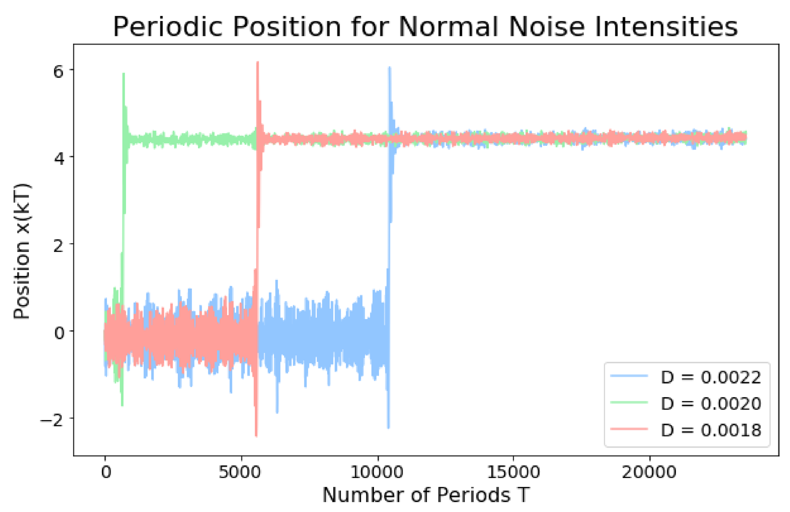
Steady States and Transitions

This study's non-linear stochastic differential equation is given by:

$$d^2x/dt^2 + 0.02*dx/dt + f(x) = 0.25 + 0.25*\cos(0.80*t) + u$$

where *u* is the noise term and *f(x)* is the piecewise restoring force. The two steady states are shown below alongside transitions from one state to another, induced by white noise at different intensities.

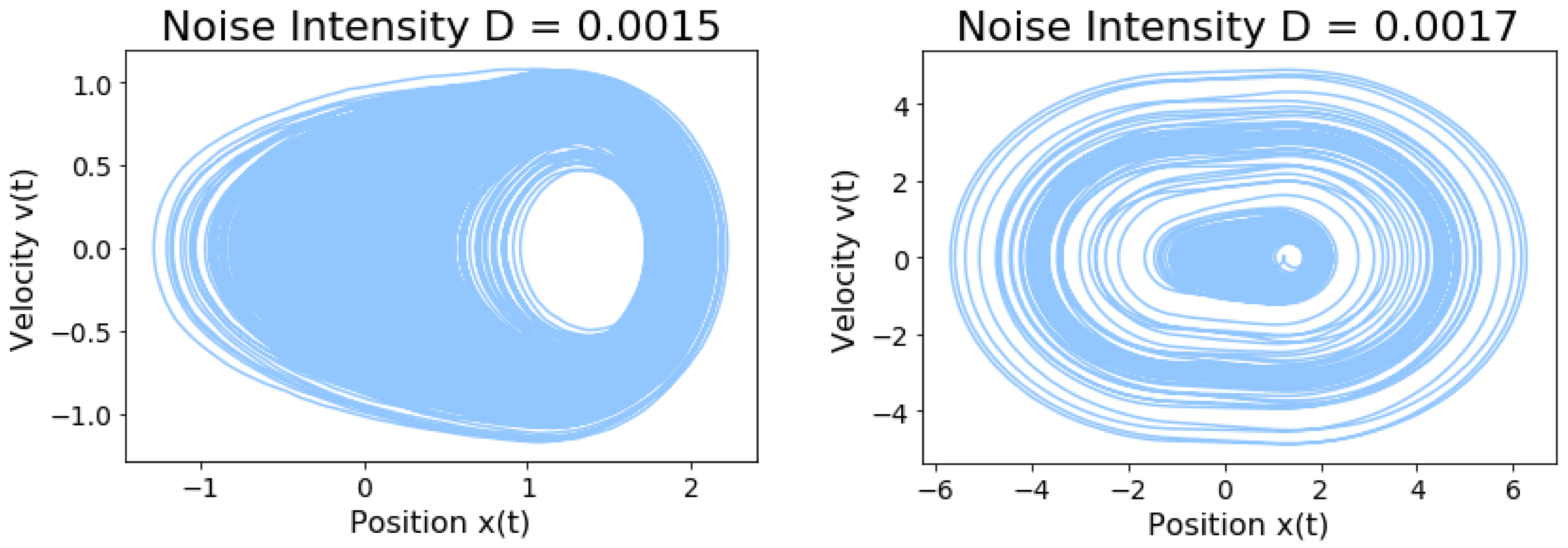




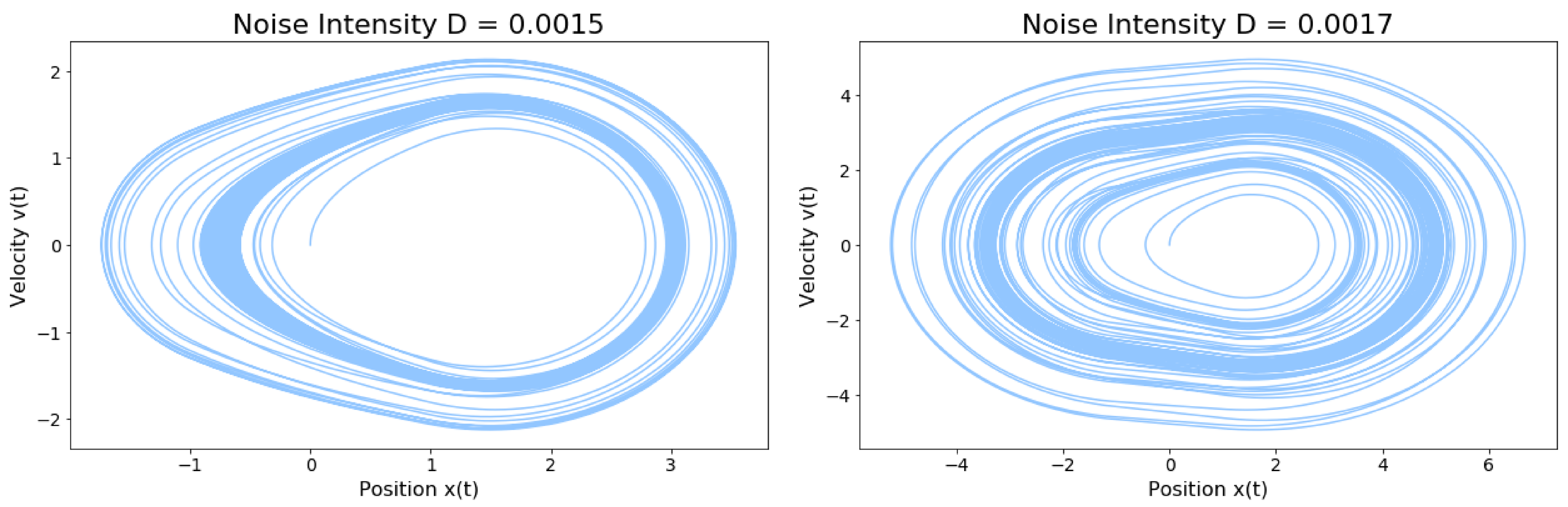
Normal and Log-Normal Noise Effects

The effects of normal noise are highly dependent on intensity such that a slight variation consistently implies whether a state transition will occur, although the exit time is still highly variable. However, log-normal noise has a much higher consistency in exit time.

State Stability and Transition from Normal Noise



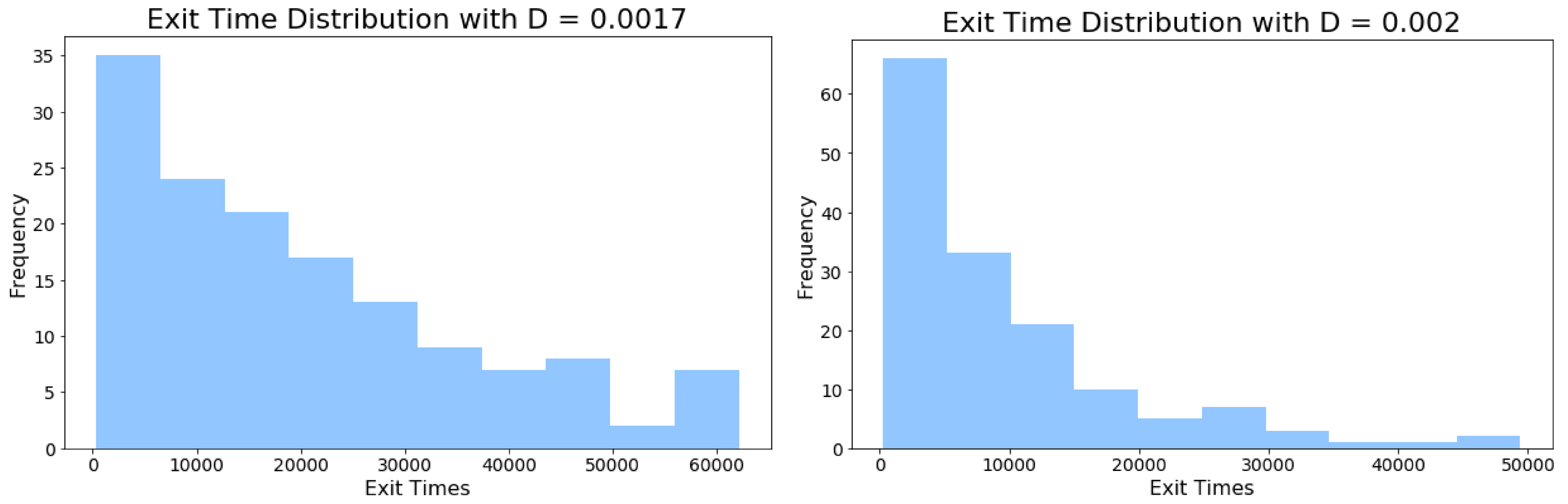
State Stability and Transition for Log-Normal Noise



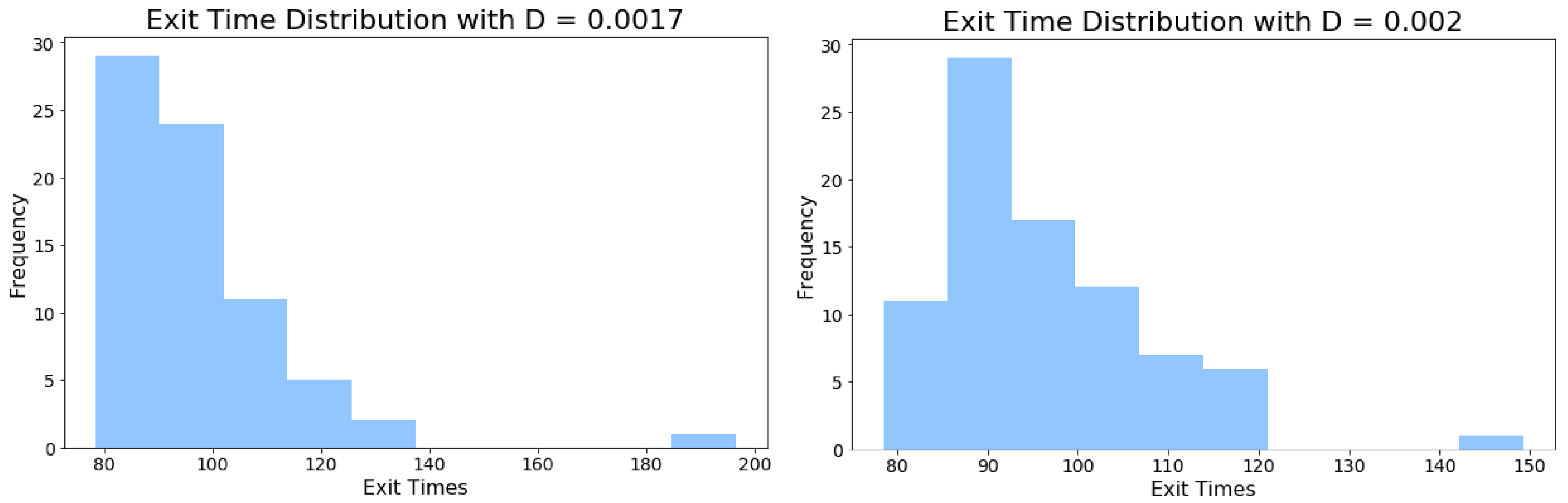
First-Exit Time Results

The means of the exit time distributions are significantly different.

Distributions of Exit Times from Normal Noise



Distributions of Exit Times from Log-Normal Noise



Discussion and Future Work

The inclusion of log-normal noise to the non-linear oscillator results in greater stability within both steady states, despite adding non-negative and often larger samples to the harmonic forcing function. Due to log-normal noise, the transition to the outer state occurs less frequently and far sooner than from the effects of white noise.

One promising future application of p-bit-accelerated sampling is in extrinsic noise-induced transitions to bimodal dynamics in both chemical and biological systems. Much like the mechanical system presented in this study, white noise is often assumed for the sake of simplicity in repressed gene models.

References

Gerardo Aquino, Andrea Rocco. Bimodality in gene expression without feedback: from Gaussian white noise to log-normal coloured noise[J]. Mathematical Biosciences and Engineering, 2020, 17(6): 6993-7017. doi: 10.3934/mbe.2020361

Kerem Y. Camsari, Brian M. Sutton, Supriyo Datta. P-bits for probabilistic spin logic. Applied Physics Reviews, 2019, 6. doi: 10.1063/1.5055860

R. V. Roy, E. Nauman. Noise-induced effects on a non-linear oscillator. Journal of Sound and Vibration, 1995, 183(2): 269-295.