Source Shaping for Electromagnetic Optimization via Higher-Order Variational Quantum Algorithms

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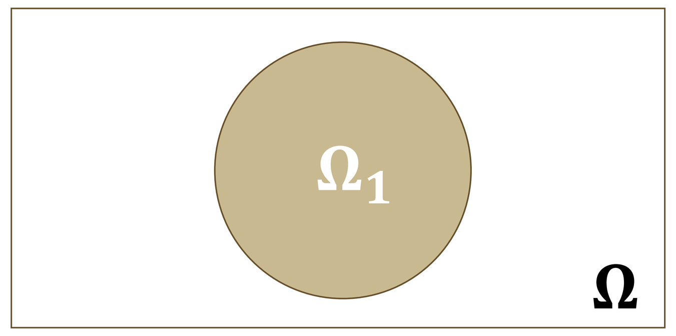
Background

Green's Function: Given some linear differential operator equation $\hat{\boldsymbol{L}}\boldsymbol{E} = \boldsymbol{\eta}$, where $\hat{\boldsymbol{L}}$ is a linear differential operator, $\boldsymbol{E}$ is the unknown electric field, and η is a source function, we can solve for $\boldsymbol{E}$ using the Green's function of $\hat{\boldsymbol{L}}$, i.e.,

$$\boldsymbol{E} = \int_{\Omega} g(\boldsymbol{r},\boldsymbol{r}')\eta(\boldsymbol{r}')d\boldsymbol{r}' . \tag{1}$$

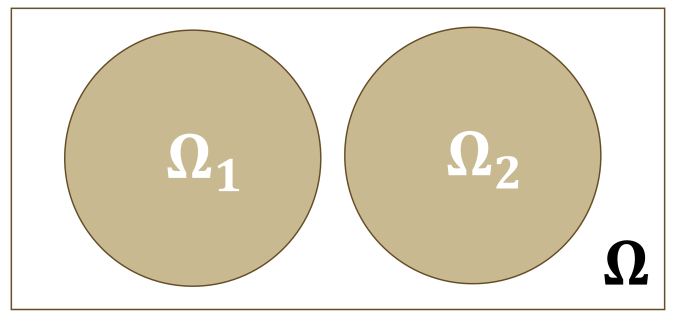
Shape Function: Given some region Ω, we construct n subsets $\Omega = \{\Omega_1, \Omega_2, \dots, \Omega_n\}$, and define a source function η_i over each subset Ω_i such that $\eta_i = 0$ outside of the region, i.e.,



$$\eta_i(\boldsymbol{r}) = \begin{cases} f_i(r) : \forall \boldsymbol{r} \in \Omega_i \\ 0 : Else \end{cases}.$$

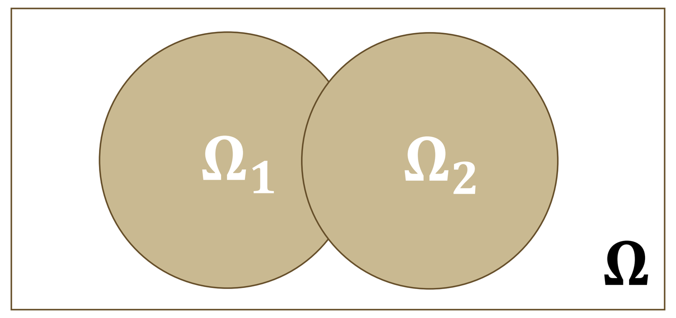
Source Shaping

Consider a subset of non-overlapping shape functions $\{\eta_1, \eta_2\}$ in a region Ω. We can construct an overall shape function $\eta_{\{1,2\}}$ by taking a linear combination,



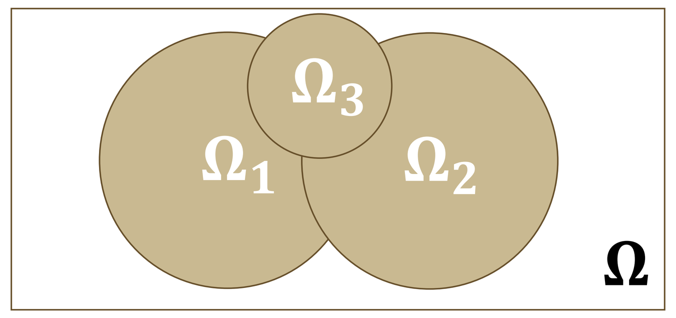
$$\eta_{\{1,2\}} = c_1\eta_1 + c_2\eta_2.$$

However, if the domains overlap, we can introduce an overlap term $c_{1,2}\eta_1\eta_2$ into the overall function $\eta_{\{1,2\}}$.



$$\eta_{\{1,2\}} = c_1\eta_1 + c_2\eta_2 + c_{1,2}\eta_1\eta_2.$$

This carries over into higher-order overlaps as well,



$$\eta_{\{1,2,3\}} = c_1\eta_1 + c_2\eta_2 + c_3\eta_3 + c_{1,2}\eta_1\eta_2 + c_{1,3}\eta_1\eta_3 + c_{2,3}\eta_2\eta_3 + c_{1,2,3}\eta_1\eta_2\eta_3.$$

Problem Definition

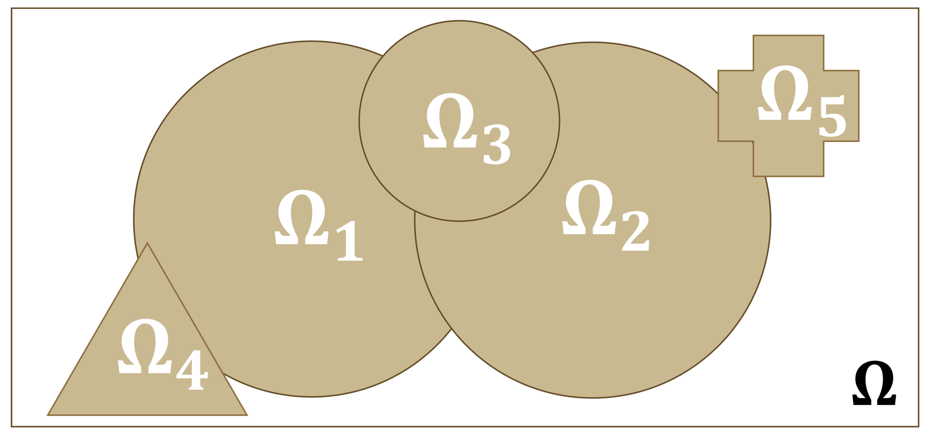
Compute the electrical field $\boldsymbol{E}$ that maximizes the functional of interest $Q: \boldsymbol{E} \to \mathbb{R}$, e.g., power dissipation, beam steering, spectrum overlap, etc.

$$\max_{\boldsymbol{E}} Q[\boldsymbol{E}] \tag{2}$$

Higher-Order Binary Optimization (HUBO)

Consider a set of shape functions $\mathrm{H} = \{\eta_1, \eta_2, \dots, \eta_n\}$. For a given subset $\mathrm{H}_{\boldsymbol{x}} \subseteq \mathrm{H}$, we assign the string $\boldsymbol{x} \in \{0,1\}^n$ such that:

$$\eta_i \in \mathrm{H}_{\boldsymbol{x}} \leftrightarrow x_i = 1.$$



Then, a k-local candidate shape function $\eta_{\boldsymbol{x}}$ parameterized by $\boldsymbol{x}$ is computed by:

$$\eta_x = \sum_{\alpha_1} c_{\alpha_1}x_{\alpha_1}\eta_{\alpha_1} + \sum_{\alpha_1,\alpha_2} c_{\alpha_1\alpha_2}x_{\alpha_1}x_{\alpha_2}\eta_{\alpha_1}\eta_{\alpha_2} + \cdots + \sum_{\alpha_1,\alpha_2,\dots,\alpha_k} c_{\alpha_1\dots\alpha_k}x_{\alpha_1}\dots x_{\alpha_k}\eta_{i_1}\dots\eta_{i_k} . \tag{3}$$

Using the Green's function solution to $\boldsymbol{E}$ in (1) and the binary shape function in (3), we recast the problem in (2) as:

$$\max_{\boldsymbol{x}\in\{0,1\}^n} Q\left[\int_{\Omega} g(\boldsymbol{r},\boldsymbol{r}')\eta_x(\boldsymbol{r}')d\boldsymbol{r}'\right] \tag{4}$$

If Q is a linear functional, then we distribute to each monomial,

$$\max_{\boldsymbol{x}\in\{0,1\}^n} \sum_{\alpha_1} x_{\alpha_1}s_{\alpha_1} + \sum_{\alpha_1,\alpha_2} x_{\alpha_1}x_{\alpha_2}s_{\alpha_1\alpha_2} + \cdots \tag{5}$$

$$s_\alpha = Q\left[\int_{\Omega} g(\boldsymbol{r},\boldsymbol{r}') \prod_{\alpha_i\in\alpha} \eta_{\alpha_i}(\boldsymbol{r}')\, d\boldsymbol{r}'\right]$$

Quantum Approximate Optimization Algorithm

Farhi et al. arXiv:1411.4028 [quant-ph]

1) Convert HUBO to Problem Hamiltonian

$$\max_{\boldsymbol{x}\in\{0,1\}^n} \sum_{\alpha_1} x_{\alpha_1}s_{\alpha_1} + \sum_{\alpha_1,\alpha_2} x_{\alpha_1}x_{\alpha_2}s_{\alpha_1\alpha_2} + \cdots$$

$$\widehat{H}_C = \sum_{\alpha_1} \sigma^z_{\alpha_1}\tilde{s}_{\alpha_1} + \sum_{\alpha_1,\alpha_2} \sigma^z_{\alpha_1}\sigma^z_{\alpha_2}\tilde{s}_{\alpha_1\alpha_2} + \cdots$$

2) Construct Quantum Optimization Problem

$$\max_{\boldsymbol{\gamma},\boldsymbol{\beta}} \langle\boldsymbol{\gamma},\boldsymbol{\beta}|\widehat{H}_C|\boldsymbol{\gamma},\boldsymbol{\beta}\rangle$$

$$|\boldsymbol{\gamma},\boldsymbol{\beta}\rangle = \hat{B}(\beta_p)\hat{C}(\gamma_p)\dots\hat{B}(\beta_1)\hat{C}(\gamma_1)|s\rangle$$

Problem Hamiltonian Evolution

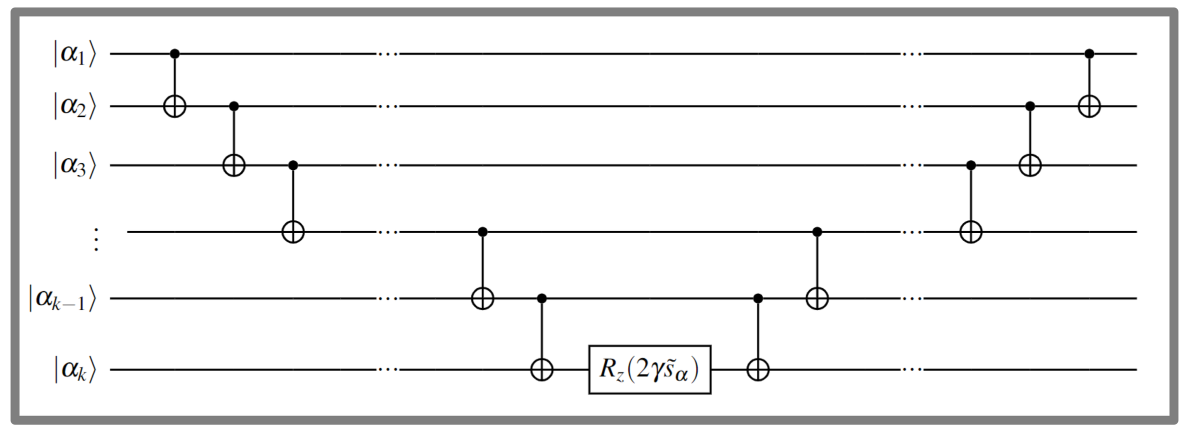
$$\hat{C}(\gamma) = e^{i\gamma\widehat{H}_C} = \prod_{\boldsymbol{\alpha}} e^{i\gamma\widehat{H}_\alpha}$$

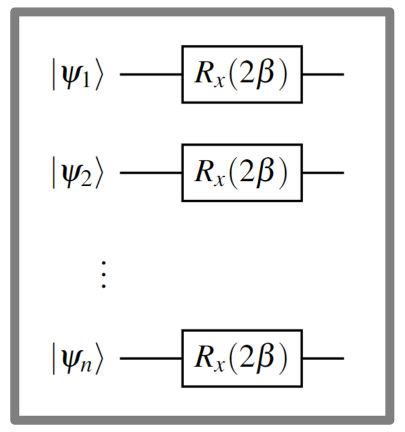
Mixer Hamiltonian Evolution

$$\hat{B}(\beta) = e^{i\beta\widehat{H}_x} = e^{i\beta\sum_i \sigma_i^x}$$

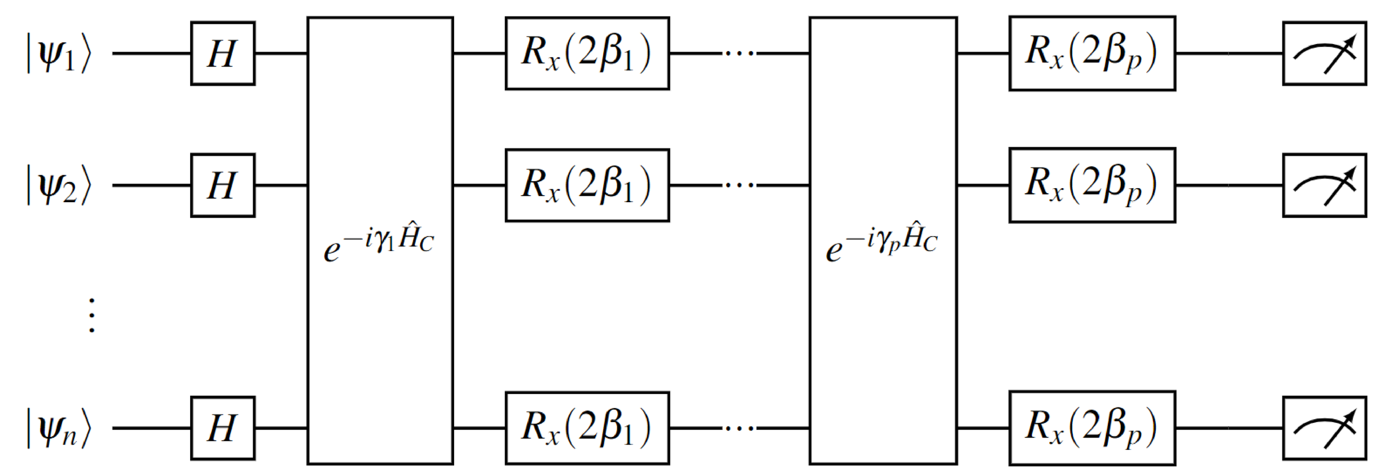
Where the problem Hamiltonian and mixer Hamiltonian are decomposed as:

$$e^{i\gamma\widehat{H}_\alpha} = e^{-i\gamma\tilde{s}_\alpha \prod \sigma^z_{\alpha_i}} \qquad e^{-i\beta\sum_i \sigma_i^x}$$





3) Compute Quantum Circuit to Simulate Time-Evolution



Discussion

Using a functional decomposition of the source function in the Green's function solution to a linear differential operator, we can define a binary optimization problem that maximizes the response of the differential operator with respect to some functional.

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