

You must show work for credit

- Problem 1. 33 pts. The torque of a magnetically linear single-phase electromechanical device may be expressed

$$T_e = 8 \cos(4\theta_{rm}) i^2$$

When the rotor position is zero, the inductance is 5 H. What is the flux linkage equation for the device?

- Problem 2. 33 pts. The  $a$ -phase conductor density of a machine is given by

$$n_{as} = -6 \cos(4\phi_{sm})$$

where  $\phi_{sm}$  is position measured relative to the stator. The radial flux density at the stator inner radius is given by

$$B(\phi_{rm}) = \frac{2}{3} \cos(4\phi_{rm})$$

where  $\phi_{rm}$  is position measured relative to the rotor. The machine is 0.1 m long. With the stated flux density, and neglecting winding resistance and leakage inductance, it is desired that the  $a$ -phase voltage is a 100 V zero-to-peak sinewave when the mechanical rotor speed is  $1000/\pi$  rad/s. What should be the radius (stator inner radius) of the machine?

Handy Fact:  $2 \sin A \cos B = \sin(A+B) + \sin(A-B)$

- Problem 3. 34 pts. Transform the following flux linkage equation to the rotor reference frame:

$$\lambda_{abc} = \begin{bmatrix} 4 & -1 & -1 \\ -1 & 4 & -1 \\ -1 & -1 & 4 \end{bmatrix} i_{abc} + 2 \begin{bmatrix} \cos(\theta_r) \\ \cos(\theta_r - 2\pi/3) \\ \cos(\theta_r + 2\pi/3) \end{bmatrix}$$

Handy facts:

$$\cos(x) + \cos(x - 2\pi/3) + \cos(x + 2\pi/3) = 0$$

$$\sin(x) + \sin(x - 2\pi/3) + \sin(x + 2\pi/3) = 0$$

$$\cos(x)\cos(y) + \cos(x - 2\pi/3)\cos(y - 2\pi/3) + \cos(x + 2\pi/3)\cos(y + 2\pi/3) = \frac{3}{2} \cos(x - y)$$

$$\sin(x)\sin(y) + \sin(x - 2\pi/3)\sin(y - 2\pi/3) + \sin(x + 2\pi/3)\sin(y + 2\pi/3) = \frac{3}{2} \cos(x - y)$$

$$\sin(x)\cos(y) + \sin(x - 2\pi/3)\cos(y - 2\pi/3) + \sin(x + 2\pi/3)\cos(y + 2\pi/3) = \frac{3}{2} \sin(x - y)$$