CS-5
August 2017 QE

Problem 1.(50pt)

Consider the following 2D system with input $x(m,n)$ and output $y(m,n)$ for $\lambda > 0$.

$$y(m,n) = x(m,n) + \lambda \left(x(m,n) - \frac{1}{9} \sum_{k=-1}^{1} \sum_{l=-1}^{1} x(m-k, n-l) \right) .$$

a) Is this a linear system? Is this a space invariant system?

b) Calculate and sketch the psf, $h(n)$, for $\lambda = 0.5$.

c) Is this a separable system?

d) Calculate the frequency response, $H(e^{j\mu}, e^{j\nu})$. (Express your result in simplified form.)

e) Describe what the filter does and how the output changes as λ increases.

Write in Exam Book Only

Problem 2.(50pt)

Let $x(t) = \mathrm{sinc}^2(t/a)$ for some $a > 0$, and let $y(n) = x(nT)$ where $f_s = 1/T$ is the sampling frequency of the system.

a) Calculate and sketch $X(f)$, the CTFT of $x(t)$.

b) Calculate $Y(e^{j\omega})$, the DTFT of $y(n)$.

c) What is the minimum sampling frequency, f_s, that ensures perfect reconstruction of the signal?

d) Sketch the function $Y(e^{j\omega})$ on the interval $[-2\pi, 2\pi]$ when $T = a/2$.

e) Sketch the function $Y(e^{j\omega})$ on the interval $[-2\pi, 2\pi]$ when $T = a$.