

Problem 1. Suppose that one of the baseband signals, $s_0(t) = 0$ or $s_1(t) = (t - T)p_T(t)$, is received in the presence of additive white Gaussian noise (AWGN) $n(t)$ with two-sided power spectral density $N_0/2$. (The function $p_T(t)$ is defined by $p_T(t) = 1$ for $0 \leq t < T$, and $p_T(t) = 0$, elsewhere.) The receiver is shown below.

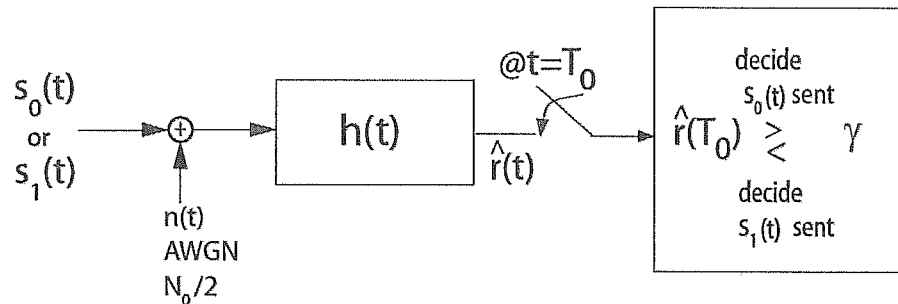


Figure 1: Receiver Used for Problem (1).

The sum of the signal $s_i(t)$, where $i = 0$ or $i = 1$, and noise $n(t)$ passes through a filter with impulse response $h(t)$, and the output of the filter is sampled at time T_0 to produce a decision statistic $\hat{r}(T_0)$. If $\hat{r}(T_0) \geq \gamma$, the receiver decides that $s_0(t)$ was sent, and if $\hat{r}(T_0) < \gamma$, the receiver decides that $s_1(t)$ was sent. Let $P_{e,0}$ denote the probability of error given that $s_0(t)$ is sent and $P_{e,1}$ denote the probability of error given that $s_1(t)$ is sent. (When necessary, express answers in terms of $\Phi(x)$, the cumulative distribution function of a zero-mean, unit-variance Gaussian random variable.)

- (a) (25 pts.) Find the impulse response $h(t)$ of the filter, the threshold γ , and the sample time T_0 that minimize the maximum of $P_{e,0}$ and $P_{e,1}$.
- (b) (10 pts.) Find the values of $P_{e,0}$ and $P_{e,1}$ for the impulse response, threshold, and sample time that you are to find above in Part (a).
- (c) (20 pts.) Now suppose that $h(t) = p_T(t)$, $T_0 = T$, and $\gamma = 0$. Find both $P_{e,0}$ and $P_{e,1}$.

For the remaining parts of this problem, again suppose that $h(t) = p_T(t)$ and $T_0 = T$. You are now interested in setting the threshold γ . You have been informed that the receiver in Figure 1 is located in a control tower at an airport and the received signal represents a reflection off an aircraft that may, or may not, be present in a particular volume of space. If the airplane is present in that particular volume of space $s_1(t)$ is received, but, if the airplane is not present in that volume of space, $s_0(t)$ is received.

- (d) (10 pts.) If the threshold $\gamma = 0$, what is the probability of missed detection of the aircraft, and, also, what is the probability of false alarm (that is, that the receiver decides an aircraft is present when it is not present)?
- (e) (15 pts.) As a function of γ and other necessary parameters, give an expression for the probability of missed detection of the aircraft, and, also, give an expression for the probability of false alarm (that is, that the receiver decides an aircraft is present when it is not present).

Problem 2. Suppose that only additive white Gaussian noise (AWGN) with two-sided power spectral density $N_0/2 = 7 \times 10^{-6} \text{ W/Hz}$ is applied to the input of the filter depicted in the figure shown below:

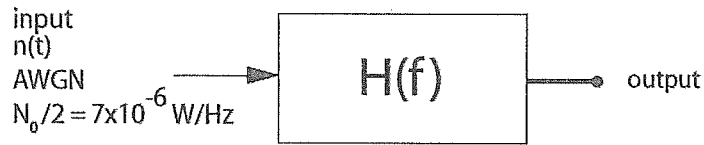


Figure 2: Diagram of System with Noise and Filtering for Problem (2).

- (a) (10 pts.) Consider the operation of a low pass filter and write down an expression for the power at the output of the filter if $H(f) = 3$ for $|f| < 10$, where f is given in Hertz.
- (b) (10 pts.) Now consider the operation of a band pass filter and write down an expression for the power at the output of the filter if $H(f) = 2$ for $10 < |f| < 20$, where f is given in Hertz.

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