Problem 1.[30 pts] An upsampler system is shown in Figure 1. For all parts, consider the ideal case where $h[n] = 3\frac{\sin(\frac{\pi}{3}n)}{\pi n}$ AND the input signal to this system is the sampled signal $x[n] = x_a(nT_s)$ where $T_s = \frac{2\pi}{40}$ and

$$x_a(t) = \left(\frac{2\pi}{40}\right)^2 \frac{\sin(15t)}{\pi t}\frac{\sin(5t)}{\pi t} \tag{1}$$

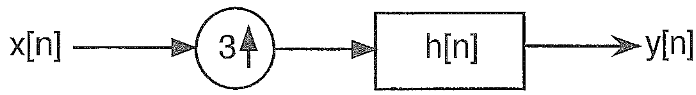

Figure 1.

(a) For the polyphase filter $h_0[n] = h[3n]$, determine an expression for the output, denoted $y_0[n] = x[n] * h_0[n]$.

(b) For the polyphase filter $h_1[n] = h[3n+1]$, determine an expression for the output, denoted $y_1[n] = x[n] * h_1[n]$.

(c) For the polyphase filter $h_2[n] = h[3n+2]$, determine an expression for the output, $y_2[n] = x[n] * h_2[n]$.

(d) Determine an expression for the overall output of this system, denoted $y[n]$.

Problem 2.[40 pts] A second-order digital filter is to be designed from an analog filter having two poles in the s-plane at $p_1 = -1+2j$ and $p_2 = -1-2j$ and two zeros at $z_1 = j$ and $z_2 = -j$,

$$H_a(s) = G\frac{(s-z_1)(s-z_2)}{(s-p_1)(s-p_2)}$$

via the bilinear transformation method characterized by the mapping

$$s = \frac{z-1}{z+1}$$

(a) Is the resulting digital filter (BIBO) stable? Briefly explain why or why not.

(b) Denote the frequency response of the resulting digital filter as $H(\omega)$ (the DTFT of its impulse response). You are given that in the range $0 < \omega < \pi$, there is only one value of ω for which $H(\omega) = 0$. Determine that value of ω.

(c) Draw a pole-zero diagram for the resulting **digital** filter. Give the exact locations of the poles and zeros of the digital filter in the z-plane.

(d) Plot the magnitude of the DTFT of the resulting digital filter, $|H(\omega)|$, over $-\pi < \omega < \pi$. You are given that $H(0) = 0.8$. Be sure to indicate any frequency for which $|H(\omega)| = 0$. Also, specifically note the numerical value of $|H(\omega)|$ for $\omega = \frac{\pi}{2}$ and $\omega = \pi$.

(e) Determine the difference equation for the resulting digital filter.

Problem 3.[30 pts] Consider the length-7 DT signal below where the first value corresponds to $n = 0$.

$$x[n] = \{+1, +1, +1, -1, -1, +1, -1\}$$

One way to compute autocorrelation is $r_{xx}[n] = x[n] * x^*[-n]$. Suppose we want to use DFT-based processing to compute autocorrelation. One issue is that the DFT assumes that signals start at $n = 0$. So, let's time-shift the time-reversed signal to the right by 7-1=6 to form $x[-(n-6)]$ so that it starts at $n = 0$; $h[n] = x[-(n-6)]$ is the causal matched filter. Since $x[n]$ is real-valued,

$$h[n] = x[-(n-6)] \overset{DTFT}{\longleftrightarrow} H(\omega) = X^*(\omega)e^{-j6\omega}$$

(a) A 16-point DFT of $x[n]$ is computed and denoted as $X_{16}(k)$, $k = 0, 1, ...15$. We then form $Y_{16}(k)$ via the point-wise product defined below

$$Y_{16}(k) = X_{16}(k)X_{16}^*(k)e^{-j6\frac{k2\pi}{16}} \quad k = 0, 1, ..., 15$$

Finally, compute $y_{16}[n]$ as the 16-point Inverse DFT of $Y_{16}(k)$. Determine and list all 16 values of $y_{16}[n]$, for $n = 0, 1, ..., 15$. Show all work.

(b) A 13-point DFT of $x[n]$ is computed and denoted as $X_{13}(k)$, $k = 0, 1, ...12$. We then form $Y_{13}(k)$ via the point-wise product defined below

$$Y_{13}(k) = X_{13}(k)X_{13}^*(k)e^{-j6\frac{k2\pi}{13}} \quad k = 0, 1, ..., 12$$

Finally, compute $y_{13}[n]$ as the 13-point Inverse DFT of $Y_{13}(k)$. Determine and list all 13 values of $y_{13}[n]$, for $n = 0, 1, ..., 12$.

(c) An 10-point DFT of $x[n]$ is computed and denoted as $X_{10}(k)$, $k = 0, 1, .., 9$. We then form $Y_{10}(k)$ via the point-wise product defined below

$$Y_{10}(k) = X_{10}(k)X_{10}^*(k)e^{-j6\frac{k2\pi}{10}} \quad k = 0, 1, ..., 9$$

Finally, compute $y_{10}[n]$ as the 10-point Inverse DFT of $Y_{10}(k)$. Determine and list all 10 values of $y_{10}[n]$, for $n = 0, 1, ..., 9$.

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