

CS-1
August 2017 QE

1. (25 points) A moving average random process X_n is produced as follows:

$$X_n = W_n + c_1 W_{n-1} + \dots + c_p W_{n-p}$$

where W_n is a zero-mean white noise process with $R_W(k) = E[W_n W_{n+k}] = \sigma^2 \delta(k)$, and c_1, \dots, c_p are real-valued constants. Find $R_X(k) = E[X_n X_{n+k}]$.

2. (25 points) Consider the system

$$Y(t) = e^{-t} \int_{-\infty}^t e^r X(r) dr$$

Let $X(t)$ be wide-sense stationary zero-mean white noise with power spectral density $S_X(f) = N_0/2$.

- (a) (10 points) Find $R_{XY}(t_1, t_2)$.
- (b) (10 points) Find $R_{YY}(t_1, t_2)$.
- (c) (5 points) Are $X(t)$ and $Y(t)$ jointly WSS? You must justify your answer.
3. (25 points) Let $X(t)$ be a continuous-time random process.
- (a) If $X(t_1), \dots, X(t_n)$ are iid random variables for any positive integer n and any t_1, \dots, t_n , where $t_i \neq t_j$ if $i \neq j$, is $X(t)$ strict-sense stationary? You must justify your answer.
- (b) If $X(t_1), \dots, X(t_n)$ are independent but not necessarily identically distributed for any positive integer n and any t_1, \dots, t_n , where $t_i \neq t_j$ if $i \neq j$, is $X(t)$ strict-sense stationary? You must justify your answer.
4. (25 points) A total of m items are to be sequentially distributed among n cells, with each item independently being put into cell j with probability $p_j, j = 1, \dots, n$. Find the expected number of collisions that occur, where a collision occurs whenever an item is put into a nonempty cell.

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