AC-2
August 2017 QE

LTI and LT Systems – State-Space Approach
August 2017

Unless otherwise stated, you need to justify your answers to get the full credit.

Problem 1. (20 points) Consider the matrix $A \in \mathbb{R}^{3\times 3}$ given by $A = \begin{bmatrix} 3 \\ -2 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & 1 & -1 \end{bmatrix}$. Note that A has rank one.

(a) **(6 pts)** Find the three eigenvalues of matrix A and their corresponding right eigenvectors.

(b) **(7 pts)** Consider the discrete-time LTI system $x[k+1] = Ax[k]$, $k = 0, 1, \ldots$. Is the system asymptotically stable? Find *all the possible* initial conditions $x[0]$ starting from which the solution $x[k]$ as $k \to \infty$ will (i) converges to zero; (ii) not converge to zero, respectively.

(c) **(7 pts)** Consider the continuous-time LTI system $\dot{x}(t) = Ax(t)$, $t \geq 0$. Is the system asymptotically stable? Find *all the possible* initial conditions $x(0)$ starting from which the solution $x(t)$ as $t \to \infty$ will (i) converges to zero; (ii) not converge to zero, respectively.

Problem 2. (30 points) Consider the following LTI system:

$$x[k+1] = \begin{bmatrix} 1 & 0 & 1 \\ -1 & -1 & 1 \\ 0 & 0 & 2 \end{bmatrix} x[k] + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u[k]$$
$$y[k] = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x[k].$$

(a) **(5 pts)** Is the system controllable? What is its reachable subspace?

(b) **(10 pts)** Assume $x[0] = 0$. Find the minimum time $T \geq 0$ and a sequence of controls $u[0], \ldots, u[T-1]$ that can drive the system state to $x[T] = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}^T$.

(c) **(5 pts)** Is the system observable? What is its unobservable subspace?

(c) **(10 pts)** Suppose $u[k] = 0$ for all $k = 0, 1, \ldots,$ and it is observed that

$$y[0] = -1, \quad y[1] = 1, \quad y[2] = 5.$$

Can $x[0]$ be uniquely determined? If so, find $x[0]$; otherwise, describe all possible values of $x[0]$.

Problem 3. (25 pts) The following LTI system is given

$$\dot{x} = \underbrace{\begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix}}_{A} x + \underbrace{\begin{bmatrix} -1 \\ 1 \end{bmatrix}}_{B} u.$$

Can you design a state feedback controller $u = Kx$ for some gain matrix $K \in \mathbb{R}^{1\times 2}$ so that the eigenvalues of the closed-loop system matrix $A_{\text{cl}} = A + BK$ are placed at -1 and -2, respectively? If so, find such a K; otherwise, state your reason.

Problem 4. (25 points) Consider the LTV system

$$\dot{x}(t) = \begin{bmatrix} -1 & t \\ 0 & -2 \end{bmatrix} x(t).$$

Find the fundamental matrix $\Phi(t)$ of the system for $t \geq 0$.

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