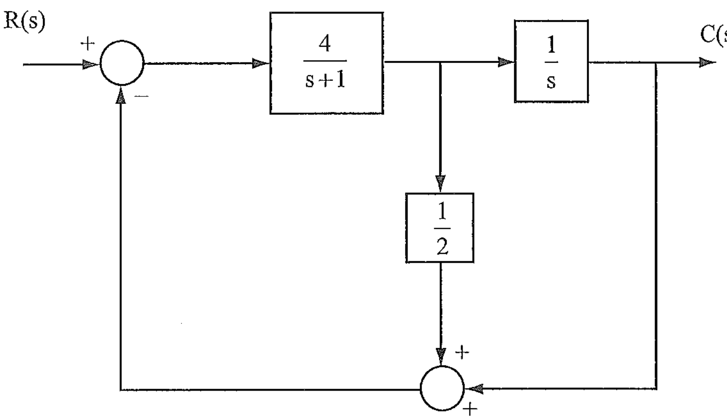
AC-1
August 2017 QE

General Instructions:

1. AC-1 has **four** problems and a total of **5 pages**.
2. Write all your answers in your answer book.
3. Use the Math Table at the end of the exam for some of your calculations.

(I) Consider the feedback control system shown below.
Without any block diagram reduction, provide concise answer to the following questions:



[20 points total; 2 points each]

(A) The feedforward transfer function is ______

(B) The open-loop transfer function is ______

(C) The closed-loop transfer function is ______

(D) The characteristic equation of the system is ______

(E) The time constant of the system is ______

(F) The damped natural frequency of the system is ______

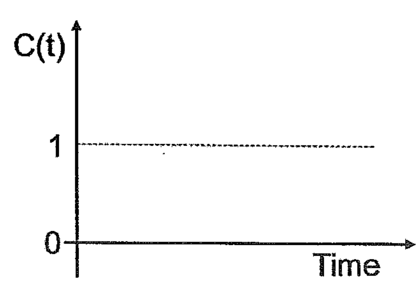
(G) The undamped natural frequency of the system is ______

(H) The type number of the system is ______

(I) The differential equation that describes the system is ______

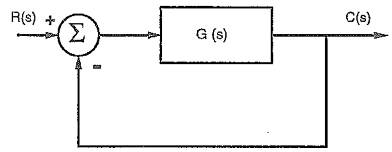
(J) Graph the output of the system due to a unit step input.

Write in Exam Book Only



(II) Given a negative unity feedback control system with **[20 points]**

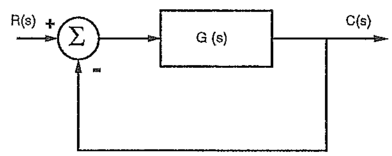
$$G(s) = \frac{K}{s(s+3)(s+2)}.$$



If the input to the system is $2+2t$, determine the value of K such that the system can achieve a steady-state error of 0.2 .

(III) The characteristic equation of a negative unity feedback control system is given as: **[30 points total]**

$$(1+K)s^2+(2-2K)s+2K=0.$$



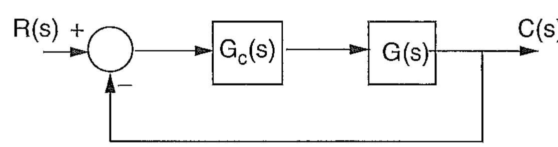
(A) Sketch the root locus for $K > 0$. **[10 points.]**

(B) Determine the angle of departure/arrival, if any. (If none, state none!) Some trigonometric function values are given at the end of the exam question. **[10 points.]**

(C) Determine the value of K and the frequency at which the loci cross the $j\omega$-axis, if any. (If none, state none!) **[10 points.]**

(IV) Given a negative unity feedback control system with a compensator $G_c(s)$ and a plant $G(s)$ as shown below. **[30 points total, 10 points each]**

$$G(s) = \frac{2}{s(s+1)}.$$



It is desirable to place the dominant closed-loop poles of the compensated system at $s_1^*, s_2^* = -2 \pm j2\sqrt{3}$ with a desirable $K_v = 20$ sec^{-1}.

A student decides to design a PID compensator, $G_c^{PID}(s) = \frac{K_D(s+z_1)(s+z_2)}{s}$, for the above system.

(A) Determine the angle of deficiency ϕ to the dominant closed-loop pole at $s_1^* = -2 + j2\sqrt{3}$. **[10 points.]**

(B) The student decides to use the 10% rule to design the PI portion of the PID compensator. What is the resulting transfer function of the PI portion of the PID compensator? **[10 points.]**

(C) Assuming the angle contribution from the PI portion of the PID compensator is $-3°$, where should the student place the zero of the PD portion of the PID compensator? What is the resulting transfer function of the PID compensator? You do not need to determine the open-loop gain of $G_c^{PID}(s)$. **[10 points.]**

Write in Exam Book Only

Values of Some Trigonometric Functions and Other Functions (I)

$\sqrt{2} = 1.4142$	$\sqrt{3} = 1.7321$	$\sqrt{5} = 2.2361$
$\sqrt{6} = 2.4495$	$\sqrt{7} = 2.6458$	$\sqrt{8} = 2.8284$
$\sqrt{10} = 3.1623$	$\sqrt{11} = 3.3166$	$\sqrt{12} = 3.4641$
$\tan^{-1}(\frac{1}{4}) = 14.04°$	$\tan^{-1}(\frac{1}{3}) = 18.43°$	$\tan^{-1}(\frac{1}{2}) = 26.57°$
$\tan^{-1}(\frac{2}{3}) = 33.69°$	$\tan^{-1}(\frac{3}{4}) = 36.87°$	$\tan^{-1}(\frac{4}{5}) = 38.66°$
$\tan^{-1}(\frac{\sqrt{3}}{4}) = 23.41°$	$\tan^{-1}(\frac{\sqrt{3}}{3}) = 30.00°$	$\tan^{-1}(\frac{\sqrt{3}}{2}) = 40.89°$
$\tan^{-1}(\sqrt{3}) = 60.00°$	$\tan^{-1}(2\sqrt{3}) = 73.90°$	$\tan^{-1}(3\sqrt{3}) = 79.11°$
$\tan^{-1}(1) = 45°$	$\tan^{-1}(2) = 63.43°$	$\tan^{-1}(3) = 71.57°$
$\tan^{-1}(4) = 75.96°$	$\tan^{-1}(5) = 78.69°$	$\tan^{-1}(6) = 80.54°$
$\tan^{-1}(7) = 81.87°$	$\tan^{-1}(8) = 82.88°$	$\tan^{-1}(9) = 83.66°$
$\tan^{-1}(10) = 84.29°$	$\tan^{-1}(11) = 84.81°$	$\tan^{-1}(12) = 85.24°$
$\tan(5°) = 0.0875$	$\tan(10°) = 0.1763$	$\tan(15°) = 0.2679$
$\tan(20°) = 0.3640$	$\tan(25°) = 0.4663$	$\tan(30°) = 0.5774$
$\tan(35°) = 0.7002$	$\tan(40°) = 0.8391$	$\tan(45°) = 1.0000$
$\tan(43°) = 0.9325$	$\tan(43.1°) = 0.9358$	$\tan(43.2°) = 0.9391$
$\tan(44°) = 0.9657$	$\tan(44.1°) = 0.9691$	$\tan(44.2°) = 0.9725$
$\tan(45°) = 1.0000$	$\tan(45.1°) = 1.0035$	$\tan(45.2°) = 1.0070$
$\tan(46°) = 1.0355$	$\tan(46.1°) = 1.0392$	$\tan(46.2°) = 1.0428$
$\tan(47°) = 1.0724$	$\tan(47.1°) = 1.0761$	$\tan(47.2°) = 1.0799$
$\tan(48°) = 1.1106$	$\tan(48.1°) = 1.1145$	$\tan(48.2°) = 1.1184$
$\tan(49°) = 1.1504$	$\tan(49.1°) = 1.1544$	$\tan(49.2°) = 1.1585$
$\tan(50°) = 1.1918$	$\tan(55°) = 1.4281$	$\tan(60°) = 1.7321$
$\tan(65°) = 2.1445$	$\tan(70°) = 2.7475$	$\tan(75°) = 3.7321$
$\tan(80°) = 5.6713$	$\tan(85°) = 11.4301$	

Values of Some Trigonometric Functions and Other Functions (II)

$\sin(43^\circ) = 0.6820$	$\sin(43.1^\circ) = 0.6833$	$\sin(43.2^\circ) = 0.6845$
$\sin(44^\circ) = 0.6947$	$\sin(44.1^\circ) = 0.6959$	$\sin(44.2^\circ) = 0.6972$
$\sin(45^\circ) = 0.7071$	$\sin(45.1^\circ) = 0.7083$	$\sin(45.2^\circ) = 0.7096$
$\sin(46^\circ) = 0.7193$	$\sin(46.1^\circ) = 0.7206$	$\sin(46.2^\circ) = 0.7218$
$\sin(47^\circ) = 0.7314$	$\sin(47.1^\circ) = 0.7325$	$\sin(47.2^\circ) = 0.7337$
$\sin(48^\circ) = 0.7431$	$\sin(48.1^\circ) = 0.7443$	$\sin(48.2^\circ) = 0.7455$
$\sin(49^\circ) = 0.7547$	$\sin(49.1^\circ) = 0.7559$	$\sin(49.2^\circ) = 0.7570$
$\sin(50^\circ) = 0.7660$	$\sin(55^\circ) = 0.8192$	$\sin(60^\circ) = 0.8660$
$\sin(65^\circ) = 0.9063$	$\sin(70^\circ) = 0.9397$	$\sin(75^\circ) = 0.9659$
$\sin(80^\circ) = 0.9848$	$\sin(85^\circ) = 0.9962$	$\sin(90^\circ) = 1.0000$
$\cos(43^\circ) = 0.7314$	$\cos(43.1^\circ) = 0.7302$	$\cos(43.2^\circ) = 0.7290$
$\cos(44^\circ) = 0.7193$	$\cos(44.1^\circ) = 0.7181$	$\cos(44.2^\circ) = 0.7169$
$\cos(45^\circ) = 0.7071$	$\cos(45.1^\circ) = 0.7059$	$\cos(45.2^\circ) = 0.7046$
$\cos(46^\circ) = 0.6947$	$\cos(46.1^\circ) = 0.6934$	$\cos(46.2^\circ) = 0.6921$
$\cos(47^\circ) = 0.6820$	$\cos(47.1^\circ) = 0.6807$	$\cos(47.2^\circ) = 0.6794$
$\cos(48^\circ) = 0.6691$	$\cos(48.1^\circ) = 0.6678$	$\cos(48.2^\circ) = 0.6665$
$\cos(49^\circ) = 0.6561$	$\cos(49.1^\circ) = 0.6547$	$\cos(49.2^\circ) = 0.6534$
$\cos(50^\circ) = 0.6428$	$\cos(55^\circ) = 0.5736$	$\cos(60^\circ) = 0.500$
$\cos(65^\circ) = 0.4226$	$\cos(70^\circ) = 0.3420$	$\cos(75^\circ) = 0.2588$
$\cos(80^\circ) = 0.1736$	$\cos(85^\circ) = 0.0872$	$\cos(90^\circ) = 0.00$
$\log_{10}(2) = 0.3010$	$\log_{10}(3) = 0.4771$	$\log_{10}(4) = 0.6021$
$\log_{10}(5) = 0.6990$	$\log_{10}(6) = 0.7782$	$\log_{10}(7) = 0.8451$
$\log_{10}(8) = 0.9031$	$\log_{10}(9) = 0.9542$	