Problem 1. [40 points] Consider a two-pole surface-mounted permanent magnet ac machine. The magnet poles have an angular span of α electrical radians ($0 < \alpha < \pi$), and they create a radial field in the air gap given by

$$B_m(\phi_r) = \begin{cases} B_0 & \text{for } -\frac{\pi}{2} - \frac{\alpha}{2} \le \phi_r \le -\frac{\pi}{2} + \frac{\alpha}{2} \\ -B_0 & \text{for } \frac{\pi}{2} - \frac{\alpha}{2} \le \phi_r \le \frac{\pi}{2} + \frac{\alpha}{2} \\ 0 & \text{otherwise} \end{cases}$$

where $B_0 > 0$ is constant, and ϕ_r (such that $-\pi \le \phi_r < \pi$) denotes electrical angle relative to the rotor (see Fig. 1 below). The B-field is positive when directed outwardly from the center. The a-phase in the stator has winding function

$$w_{as}(\phi_s) = W_1 \cos(\phi_s)$$

where W_1 is constant, and ϕ_s denotes electrical angle relative to the stator.
Derive an expression for the flux linkage $\lambda_{asm}(\theta_r)$ due to the permanent magnets as a function of rotor position θ_r. The expression should contain machine design parameters such as the ones listed above, or others that have not been explicitly defined so far.

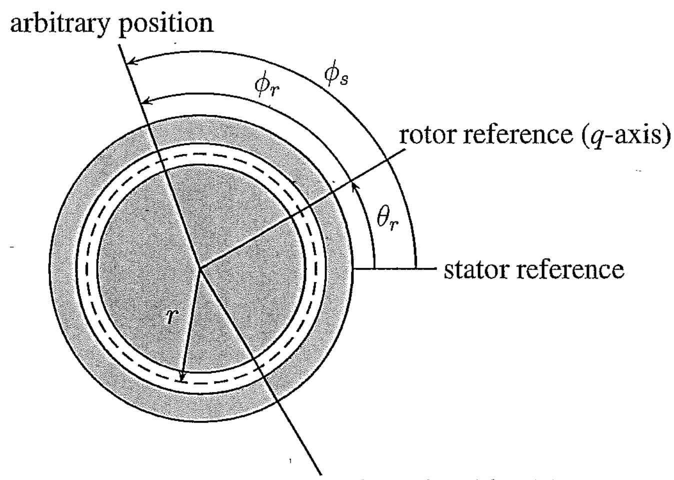

Figure 1: Angle definitions for Problem 1.

Potentially useful formulas (see page 4 of this QE question for trigonometric formulas):

$$\lambda_{xm} = \int_0^{2\pi} B(\phi_m) w_x(\phi_m) lr \, d\phi_m$$

$$L_{xym} = \mu_0 r l \int_0^{2\pi} \frac{w_x(\phi_m) w_y(\phi_m)}{g(\phi_m)} d\phi_m$$

Problem 2. [40 points] Consider a Δ-connected three-phase circuit, as shown in Fig. 2. (This is *not* a rotating electric machine, so the circuit can be assumed to be stationary.) Each side of the Δ is a series *RL* branch, and all branches are identical with resistance $R = 1\ \Omega$, self-inductance $L_s = 6$ mH, and mutual inductance between branches $M = -2.5$ mH. At time $t = 0$, a transformation of the Δ branch currents (i_{AB}, i_{BC}, i_{BC}) to a reference frame with $\theta(0) = 0°$ yields (in amperes): $i_{q\Delta}(0) = 4$, $i_{d\Delta}(0) = -2$, and $i_{0\Delta}(0) = 3$. (Do not worry about how these currents were established in the first place; just accept the fact that they are initially present in the circuit.)

(a) Calculate $i_B(0)$.

(b) Find a numerical expression for $i_{0\Delta}(t)$, $t \geq 0$.

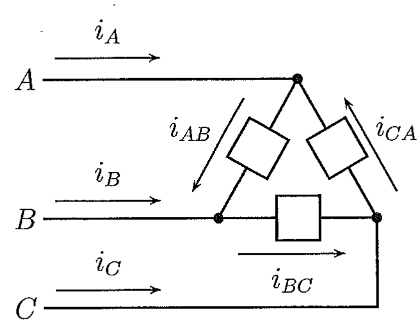

Figure 2: Circuit for Problem 2.

Recall the transformation of stationary circuit variables to the arbitrary reference frame: $\mathbf{f}_{qd0} = \mathbf{K}_s \mathbf{f}_{abc}$, where

$$\mathbf{K}_s = \frac{2}{3}\begin{bmatrix} \cos\theta & \cos\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta + \frac{2\pi}{3}\right) \\ \sin\theta & \sin\left(\theta - \frac{2\pi}{3}\right) & \sin\left(\theta + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$(\mathbf{K}_s)^{-1} = \begin{bmatrix} \cos\theta & \sin\theta & 1 \\ \cos\left(\theta - \frac{2\pi}{3}\right) & \sin\left(\theta - \frac{2\pi}{3}\right) & 1 \\ \cos\left(\theta + \frac{2\pi}{3}\right) & \sin\left(\theta + \frac{2\pi}{3}\right) & 1 \end{bmatrix}$$

$$\theta(t) = \int_0^t \omega(\tau)d\tau + \theta(0)$$

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Problem 3. [20 points] An online forum for electrical engineers contains the following post (which is copied here more or less verbatim, including grammar mistakes etc.):

1 Dear all,
2
3 I have come to a question regarding what has been extensively studied in
4 electromagnetic energy conversion principles.
5
6 Assume a single coil in a linear magnetic circuit. If we consider the instantaneous
7 energy flow we write:
8 $v = Ri + p\lambda$
9 $\lambda = Li$
10 Thus:
11 $v = Ri + Lpi + ipL = Ri + Lpi + i(dL/d\theta)\omega$
12 Then the input power will be:
13 $P_{\text{in}} = Ri^2 + Lip(i) + i^2\omega(dL/d\theta)$
14 The right hand side has 3 terms; the first is the copper loss, the second is
15 the rate of change of the stored magnetic energy and finally, the last one is
16 the output mechanical power (P_{mech}).
17 Therefore: $P_{\text{mech}} = T\omega = i^2\omega(dL/d\theta)$
18 Therefore: $T = i^2(dL/d\theta)$
19 This is in contrast with the coenergy method which gives:
20 $T = dW_c/d\theta = d(\frac{1}{2}Li^2)/d\theta = \frac{1}{2}i^2(dL/d\theta)$
21
22 Why is the result of one method double the other one? I would appreciate
23 if any one gives me their valuable points of view.

Provide an answer to this apparent paradox. Identify the line number(s) where the error is made, and explain the error in the reasoning. Justify your answer using an energy balance argument (using W_E, W_e, W_m, etc.), in order to convince the person who posted the question.

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Potentially useful trigonometric identities:

$$\sin(\tfrac{\pi}{2} - x) = \cos x$$
$$\sin(x + \pi) = -\sin x$$
$$\cos(x + \pi) = -\cos x$$
$$\cos\theta - \cos\phi = -2\sin\left(\frac{\theta+\phi}{2}\right)\sin\left(\frac{\theta-\phi}{2}\right)$$
$$\sin\theta - \sin\phi = 2\cos\left(\frac{\theta+\phi}{2}\right)\sin\left(\frac{\theta-\phi}{2}\right)$$

A.2. THREE-PHASE TRIGONOMETRIC RELATIONS

$$\cos x + \cos\left(x - \frac{2\pi}{3}\right) + \cos\left(x + \frac{2\pi}{3}\right) = 0$$

$$\sin x + \sin\left(x - \frac{2\pi}{3}\right) + \sin\left(x + \frac{2\pi}{3}\right) = 0$$

$$\cos^2 x + \cos^2\left(x - \frac{2\pi}{3}\right) + \cos^2\left(x + \frac{2\pi}{3}\right) = \frac{3}{2}$$

$$\sin^2 x + \sin^2\left(x - \frac{2\pi}{3}\right) + \sin^2\left(x + \frac{2\pi}{3}\right) = \frac{3}{2}$$

$$\sin x \cos x + \sin\left(x - \frac{2\pi}{3}\right)\cos\left(x - \frac{2\pi}{3}\right) + \sin\left(x + \frac{2\pi}{3}\right)\cos\left(x + \frac{2\pi}{3}\right) = 0$$

$$\sin x \cos y + \sin\left(x - \frac{2\pi}{3}\right)\cos\left(y - \frac{2\pi}{3}\right) + \sin\left(x + \frac{2\pi}{3}\right)\cos\left(y + \frac{2\pi}{3}\right) = \frac{3}{2}\sin(x - y)$$

$$\sin x \sin y + \sin\left(x - \frac{2\pi}{3}\right)\sin\left(y - \frac{2\pi}{3}\right) + \sin\left(x + \frac{2\pi}{3}\right)\sin\left(y + \frac{2\pi}{3}\right) = \frac{3}{2}\cos(x - y)$$

$$\cos x \cos y + \cos\left(x - \frac{2\pi}{3}\right)\cos\left(y - \frac{2\pi}{3}\right) + \cos\left(x + \frac{2\pi}{3}\right)\cos\left(y + \frac{2\pi}{3}\right) = \frac{3}{2}\cos(x - y)$$

$$\sin x \cos y + \sin\left(x + \frac{2\pi}{3}\right)\cos\left(y - \frac{2\pi}{3}\right) + \sin\left(x - \frac{2\pi}{3}\right)\cos\left(y + \frac{2\pi}{3}\right) = \frac{3}{2}\sin(x + y)$$

$$\sin x \sin y + \sin\left(x + \frac{2\pi}{3}\right)\sin\left(y - \frac{2\pi}{3}\right) + \sin\left(x - \frac{2\pi}{3}\right)\sin\left(y + \frac{2\pi}{3}\right) = -\frac{3}{2}\cos(x + y)$$

$$\cos x \cos y + \cos\left(x + \frac{2\pi}{3}\right)\cos\left(y - \frac{2\pi}{3}\right) + \cos\left(x - \frac{2\pi}{3}\right)\cos\left(y + \frac{2\pi}{3}\right) = \frac{3}{2}\cos(x + y)$$