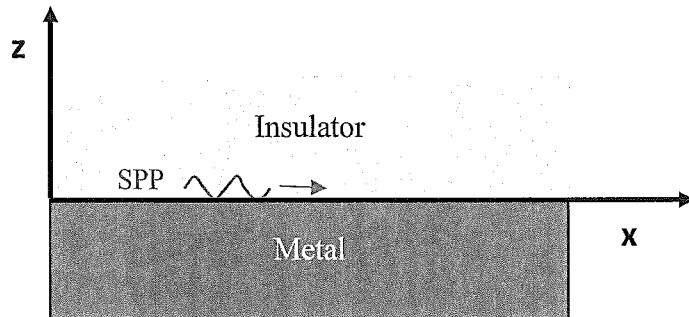


**Question 1. (70 points)**

A surface plasmon polariton (SPP) is a collective oscillation of electrons propagating along the interface of a metal and an insulator. In this problem we will derive the existence of such waves and solve for their dispersion directly from Maxwell's equations.

Assume we have a system as shown below. The SPP propagates along the interface in the x-direction and decays into both insulator and metal in the z direction.



- a. Starting with Maxwell's equations, and assuming all fields are time varying, i.e.  $\mathbf{E}(x,y,z,t) = \mathbf{E}(x,y,z) \exp(-i\omega t)$ , find explicit expressions for all field components of  $\mathbf{E}$  and  $\mathbf{H}$ . Note that the field components may be related to the spatial derivatives of other field components.
- b. Since the SPP propagates in x-direction and decays in the z-direction, we can say the  $\mathbf{E}(x,y,z) = \mathbf{E}(z) \exp(i\beta x)$ , where  $\beta = k_x$  is called the propagation constant of the travelling wave and corresponds to the component of the wave vector in the direction of propagation. Using this and the fact that the system is invariant in y-direction, simplify the 6 expressions from part a.
- c. Now, we will specifically look at only transverse-magnetic (TM) waves. Isolate the TM field components from part b and derive a wave equation for the TM modes.
- d. Define the general form of TM fields for the insulator and metal regions that satisfy the equations from part c. Let the metal have permittivity  $\epsilon_1$ , wave vector  $k_1$  and amplitude  $A_1$  and insulator having  $\epsilon_2, k_2$  and  $A_2$ . Using standard boundary conditions for Maxwell's equations, solve for any coefficients and derive the following relation

$$\frac{k_2}{k_1} = -\frac{\epsilon_2}{\epsilon_1}$$

- e. Since  $\epsilon_2 > 0$  for insulators, what limitation is there for  $\epsilon_1$ ?
- f. Now solve for  $k_1$  and  $k_2$  as a function of  $\beta$  and  $k_0$ .

- g. Finally, solve for  $\beta$  using the answer from Question 6 and the relation in Question 4.  $\beta$  should only depend on the constants and material permittivities.

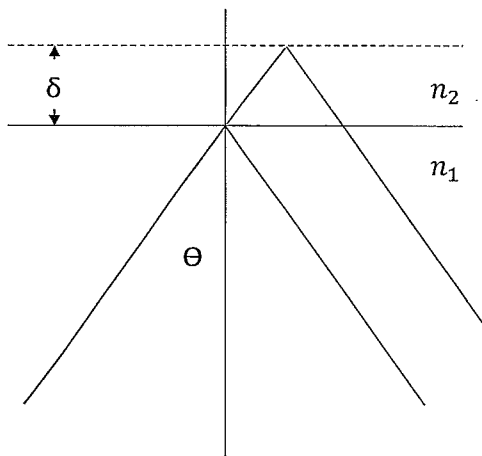
**Question 2.** (30 points)

We have an electromagnetic wave incident on a dielectric layer with lower refractive index than the host ( $n_2 < n_1$ ), as shown in the figure below. Beyond a critical angle of incidence, the wave can experience total internal reflection, where the refracted wave is propagated only parallel to the surface and is attenuated exponentially beyond the surface. Given that the Poynting vector has the form

$$S \cdot \hat{n} = \frac{1}{2} \text{Re}[\hat{n} \cdot (\mathbf{E} \times \mathbf{H}^*)],$$

prove that even though the fields exist beyond the interface, there is no energy flow through the surface. (Hint: you should use somehow introduce the wave vector in Maxwell's 4<sup>th</sup> equation

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}).$$



Write in Exam Book Only