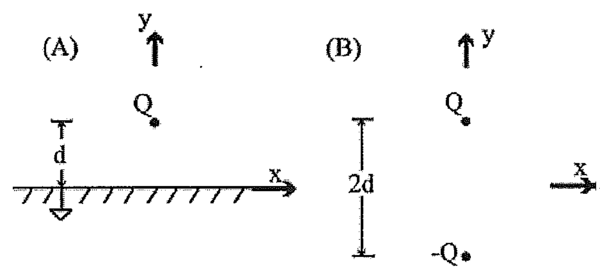
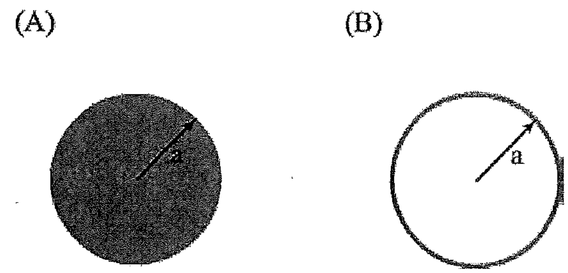
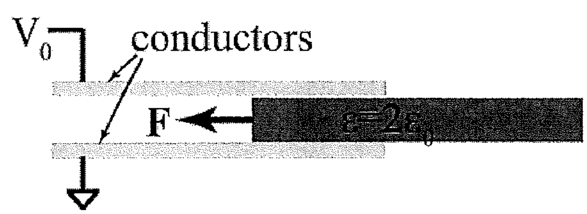
1. (40 pts) Compare the energy of the following configurations. The permittivity ε is ε_0 everywhere.
   a. In (A), a point charge Q is suspended a distance d above a grounded plane. In (B), two point charges, $+Q$ and $-Q$, are separated by a distance $2d$. How does the energy W_A of configuration (A) compare to that of configuration (B), W_B? Specifically, find W_A/W_B. Explain your reasoning.



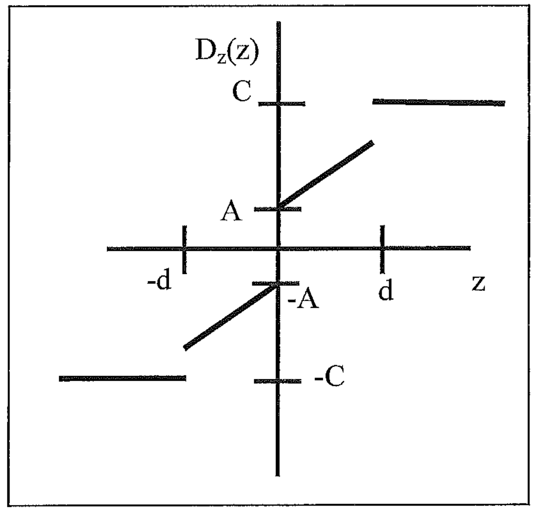
   b. In these configurations, the two spheres are of radius a, and each contains a total charge of Q. In (A), the volume charge density ρ_v is uniform in the region $R < a$, and zero everywhere else. In (B), the only charge is a surface charge, of density ρ_s, uniformly distributed on the surface of the sphere. (i) Compare and contrast the electric field in the regions $R < a$ and $R > a$ for these two charge configurations. [Note: No equations are necessary for your response to this part of the question.] (ii) Explain qualitatively the difference between the energy W_A of configuration (A) and the energy W_B of configuration (B). (iii) Find W_A/W_B.



2. (30 pts) A parallel plate capacitor of spacing d, is charged to a potential V_0. The dimension of the conductors is $a \times b$, where b is the dimension into the page. A dielectric slab, of permittivity $\varepsilon = 2\varepsilon_0$, is partially inserted into the region between the plates, as shown. Neglecting edge effects, approximate the force **F** felt by the dielectric. Is the force pulling the dielectric into the space (as shown), or expelling the dielectric (opposite the direction shown). Explain your answer. [Hint: Recall that the mechanical work done in moving an object against a force **F** is $-\int \mathbf{F} \cdot d\boldsymbol{\ell}$. Except for consideration of signs, the inverse of this relation can be used to find the force **F**.]



3. (30 pts) Consider a charged region of infinite length in the *x* and *y* dimensions. The displacement field **D** has only a D_z component, which is $D_z(z) = A + Bz$ for $0 < z < d$, $D_z(z) = C$ for $z > d$, and $D_z(-z) = -D_z(z)$. *A*, *B*, and *C* are known constants. See the plot to the right.
    a. Determine the volume charge density ρ_V for the four regions $0 < z < d$, $z > d$, $z < -d$, and $-d < z < 0$.
    b. Determine the surface charge density ρ_S at $z = d$.
    c. Determine the total charge *Q* contained within a volume *V* which has surface area ΔS on the faces normal to the *z*-axis, and extends over $-L < z < L$, where $L > d$, in the z-direction.



Write in Exam Book Only

Maxwell's Equations:

$$\nabla\cdot\mathbf{D}=\rho_v \qquad \oint\!\!\oint_S \mathbf{D}\cdot d\mathbf{S}=Q_{enc}$$

$$\nabla\cdot\mathbf{B}=0 \qquad \oint\!\!\oint_S \mathbf{B}\cdot d\mathbf{S}=0$$

$$\nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t} \qquad \oint_c \mathbf{E}\cdot d\mathbf{l}=-\frac{d}{dt}\oint\!\!\oint_S \mathbf{B}\cdot d\mathbf{S}$$

$$\nabla\times\mathbf{H}=\mathbf{J}+\frac{\partial\mathbf{D}}{\partial t} \qquad \oint_c \mathbf{H}\cdot d\mathbf{l}=I_{enc}+\frac{d}{dt}\oint\!\!\oint_S \mathbf{D}\cdot d\mathbf{S}$$

Poynting's Theorem:

$$\nabla\cdot(\mathbf{E}\times\mathbf{H})=-\frac{\partial}{\partial t}\left(\frac{1}{2}\mu\mathbf{H}\cdot\mathbf{H}\right)-\frac{\partial}{\partial t}\left(\frac{1}{2}\varepsilon\mathbf{E}\cdot\mathbf{E}\right)-\mathbf{J}\cdot\mathbf{E}$$

Potentially useful vector algebra

$$\nabla\times\mathbf{A}=\hat{\mathbf{x}}\left(\frac{\partial A_z}{\partial y}-\frac{\partial A_y}{\partial z}\right)+\hat{\mathbf{y}}\left(\frac{\partial A_x}{\partial z}-\frac{\partial A_z}{\partial x}\right)+\hat{\mathbf{z}}\left(\frac{\partial A_y}{\partial x}-\frac{\partial A_x}{\partial y}\right)$$

$$\nabla\cdot\mathbf{A}=\frac{\partial A_x}{\partial x}+\frac{\partial A_y}{\partial y}+\frac{\partial A_z}{\partial z}$$

$$\nabla=\hat{\mathbf{x}}\frac{\partial}{\partial x}+\hat{\mathbf{y}}\frac{\partial}{\partial y}+\hat{\mathbf{z}}\frac{\partial}{\partial z}$$

Potentially useful integral identities

$$\int\frac{dx}{x}=\ln x$$

$$\int\frac{dx}{\sqrt{x^2+a^2}}=\ln\left(x+\sqrt{x^2+a^2}\right) \qquad \int\frac{xdx}{\sqrt{x^2+a^2}}=\sqrt{x^2+a^2}$$

$$\int\frac{dx}{x^2+a^2}=\frac{1}{a}\tan^{-1}\frac{x}{a} \qquad \int\frac{xdx}{x^2+a^2}=\frac{1}{2}\ln\left(x^2+a^2\right)$$

$$\int\frac{dx}{\left(x^2+a^2\right)^{3/2}}=\frac{x}{a^2\sqrt{x^2+a^2}} \qquad \int\frac{xdx}{\left(x^2+a^2\right)^{3/2}}=-\frac{1}{\sqrt{x^2+a^2}}$$

$$\int\sin^2 x dx=\frac{x}{2}-\frac{1}{4}\sin 2x \qquad \int\cos^2 x dx=\frac{x}{2}+\frac{1}{4}\sin 2x$$

$$\int\sin^3 x dx=\frac{\cos^3 x}{3}-\cos x \qquad \int\cos^3 x dx=-\frac{\sin^3 x}{3}+\sin x$$

$$\int\sinh^2 x dx=\frac{1}{2}\left[-x+\sinh x\cosh x\right] \qquad \int\cosh^2 x dx=\frac{1}{2}\left[x+\sinh x\cosh x\right]$$

Write in Exam Book Only