CS-5
August 2015 QE

Problem 1.(50pt)
Consider the emissive display device which is accurately modeled by the equation

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} R^\alpha \\ G^\alpha \\ B^\alpha \end{bmatrix}$$

where R, G, and B are the red, green, and blue inputs in the range 0 to 255 that are used to modulate physically realizable color primaries.

a)(10pt) What is the gamma of the device?

b)(10pt) What are the chromaticity components (x_r, y_r), (x_g, y_g), and (x_b, y_b) of the device's three primaries.

c)(10pt) What are the chromaticity components (x_w, y_w) of the device's white point.

d)(10pt) Sketch a chromaticity diagram and plot and label the following on it:

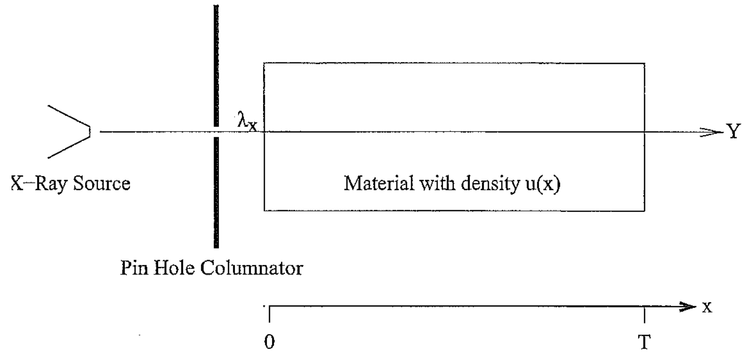
1. $(x, y) = (1, 0)$
2. $(x, y) = (0, 1)$
3. $(x, y) = (0, 0)$
4. $(R, G, B) = (255, 0, 0)$
5. $(R, G, B) = (0, 255, 0)$
6. $(R, G, B) = (0, 0, 255)$

e)(10pt) Imagine that the values of (R, G, B) are quantized to 8 bits, and that you view a smooth gradient from black to white on this device. What artifact are you likely to see, and where in the gradient will you see it?

Write in Exam Book Only

Problem 2.(50pt)

Consider an X-ray imaging system shown in the figure below.



Photons are emitted from an X-ray source and columnated by a pin hole in a lead shield. The columated X-rays then pass in a straight line through an object of length T with density $u(x)$ where x is the depth into the object. The number of photons in the beam at depth x is denoted by the random variable Y_x with Poisson density given by

$$P\{Y_x = k\} = \frac{e^{-\lambda_x}\lambda_x^k}{k!} \; .$$

where x is measured in units of cm and $\mu(x)$ is measured in units of cm^{-1}.

a)(10pt) Calculate the mean of Y_x, i.e., $E\left[Y_x\right]$.

b)(10pt) Calculate the variance of Y_x, i.e., $E\left[(Y_x - E\left[Y_x\right])^2\right]$

c)(10pt) Write a differential equation which describes the behavior of λ_x as a function of x.

d)(10pt) Solve the differential equation to form an expression for λ_x in terms of $u(x)$ and λ_0.

e)(10pt) Calculate an expression for the integral of the density, $\int_0^T u(x)dx$, in terms of λ_0 and λ_T.