CS-4
August 2015 QE

1. **(40 points)** Consider a random access system with two channels. Two packets can be transmitted simultaneously in the system, as long as they are transmitted on different channels. In this problem, you will be asked to derive the throughput of pure-Aloha for such a two-channel system. Assume that there are N stations. New packets arrive at each station according to a Poisson process with rate λ, independently of other stations. Assume that each packet is of fixed length, and it takes m amount of time to transmit a packet on either channel. However, the two channels have different characteristics: the transmission on channel 1 is always error-free, while the transmission on channel 2 can encounter an error with probability q, independently of collisions or other packet transmissions. When a station has a new packet to send, it will randomly choose channel 1 or channel 2 with probability p and $1-p$, respectively, and immediately transmit the packet on the chosen channel. If there is a collision on the chosen channel, each station involved in the collision will then wait for a random time, again randomly and independently choose channel 1 or channel 2 with probability p and $1-p$, respectively, and retransmit the packet on the chosen channel. Further, if the packet is transmitted on channel 2, even if it does not encounter collisions but instead encounter an error, the station itself will also wait for a random time, again randomly and independently choose channel 1 or channel 2 with probability p and $1-p$, respectively, and retransmit the packet on the chosen channel. Such a procedure continues until the packet is successfully transmitted.

   Let λ' denote the aggregate rate of packet transmissions at each station (including both new and retransmitted packets). **You may assume that the aggregated arrivals of both new and retransmitted packets on each channel are Poisson.**

   (a) (15 points) For a given packet transmission on channel 2 that starts at time t (which can be either a new packet or a retransmitted packet), find the probability that this packet can be transmitted successfully (i.e., with no collision and no error).

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(b) (10 points) Derive an expression that relates λ and λ'. (Note: packet transmissions may happen on either channel 1 or channel 2.)

(c) (15 points) Assume that $p = 1/2$. Determine the maximum possible λ (as a function of q, N and m) that can be supported by this system.

2. (**60 points**) Jobs arrive to a dispatcher according to Poisson process with rate λ. There are L servers who can process the jobs. The dispatcher can dispatch each job to one or more servers. If a job is dispatched to a particular server $i = 1, ..., L$, the time for the server to complete the job is exponentially distributed with mean $1/\mu$, independently across jobs and across servers. In this problem, the dispatcher wishes to reduce the delay in serving the jobs. One idea that the dispatcher can use is to dispatch the same job to more than one servers. Then, the job is considered completed when any one of the servers completes first. However, sending the same job to multiple servers indirectly increase the load in the system. Thus, this mechanism may or may not improve the delay. In this problem, you will use queueing theory to study this tradeoff. You will study two different versions of the system, depending on whether an on-going job at a server can be abandoned or not. For simplicity, we will focus on the case when $L = 2$, i.e., there are only two servers, and we will assume that the dispatcher has infinite waiting space.

(a) (20 points) In the first system, each job is always dispatched to both servers. Further, whenever one of the servers completes the job first, the dispatcher will immediately instruct the other server to abandon the remaining part of the on-going job. Then, the oldest job waiting at the dispatcher will again be dispatched to both servers.

Let P_n be the probability that there are n **distinct** jobs in the system. In other words, if a job is currently being served by two servers, count them as **one** job for the definition of the state n. Draw the state-transition diagram, write down the balance equations, and find P_n.

(b) (15 points) For the system in part (a), let W be the random variable that denotes the delay from the time that a job arrives, to the time that the job is completed. Find $\mathbf{E}[W]$.

(c) (5 points) Compared with a dispatcher that always dispatch a job to **one** server, does the above system lead to smaller delay? (You only need to answer Yes or No.)

(d) (10 points) In the second system, each job may be dispatched to either one or two servers. Specifically, if both servers are idle when a new job arrives to the dispatcher, then the dispatcher will dispatch the job to both servers. However, unlike the first system in part (a), even when one of the servers completes first, the dispatcher has NO way to instruct the other server to abandon the remaining part of the on-going job. Rather, the other server still must continue with the ongoing job until it finishes. On the other hand, whenever one server is available, the oldest job waiting at the dispatcher is then dispatched (only) to this server. This dispatch-to-only-one-server procedure continues until both servers become idle (because of no jobs waiting), then the entire process repeats itself.

Let P_n be the probability that there are n jobs in the system. Here, if a job is currently being served by two servers, count it as **two** jobs for definition of the state n. Draw the state-transition diagram that can be used to find P_n, and write down the balance equations that can be used to find P_n. (*Hint:* You do NOT need to solve P_n).

(e) (10 points) For the second system in part (d), assume that you can solve for P_n. Let W be the random variable that denotes the delay from the time that a job arrives, to the time that the job is **(first)** completed. Write down an expression that you can use to find the expected delay $\mathbf{E}[W]$. (*Hint:* You may not be able to apply Little's Law directly.)

$$\sum_{n=0}^{K-1} p^n = \frac{1-p^K}{1-p}$$

$$\sum_{n=0}^{K-1} np^n = \frac{p[1+(K-1)p^K - Kp^{K-1}]}{(1-p)^2}$$

Table 1: Formulas for some frequently-used summations