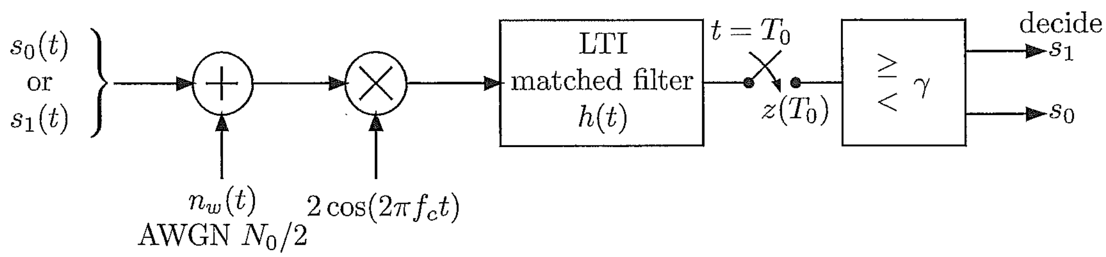
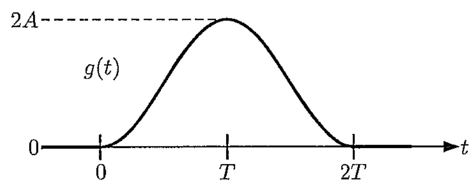
CS-3
August 2015 QE



Problem 1. [45 pts. total] The purpose of this problem is step through the development of the optimal receiver for ASK. The signals $s_0(t) = 0$, $s_1(t) = +g(t)\cos(2\pi f_c t)$, where $g(t)$ is the time-domain raised cosine shaped pulse:

$$g(t) = \begin{cases} A(1 + \cos(\pi(t - T)/T). & 0 \leq t \leq 2T \\ 0 & \text{else} \end{cases} .$$



Assume that $f_c \gg 1/T$ (which will suggest a certain approximation simplifying the results below).

(a) [4 pts.] For the receiver shown above and the assumed signals $s_0(t)$ and $s_1(t)$ choose the impulse response $h(t)$ of the matched filter and specify the sampling time T_0 to minimize the probability of error. Note that the downconversion via multiplication by $2\cos(2\pi f_c t)$ occurs before the matched filter in this architecure.

(b) [15 pts.] Assuming that the transmitted signal is actually $s_1(t)$ find:

(b1) The message-related part of $z(T_0)$.

(b2) The noise-related part of $z(T_0)$: Specify its distribution and its mean and variance.

(b3) What is the distribution of the random variable $z(T_0)$ conditioned on $s_1(t)$ being transmitted?

(c) [10 pts.] Repeat part (b) assuming that $s_0(t)$ is transmitted. You can do this by inspection given your derivation from (b) if you give the proper justification.

(d) [6 pts.] Assuming that the prior probabilities of $s_0(t)$ and $s_1(t)$ are 1/2 choose the threshold γ for minimum average probability of error.

(e) [10 pts.] Find the average probability of error.

Problem 2. [25 pts. total] In binary frequency shift keying (BFSK) a single binary bit is transmitted in a signaling interval of length T by sending one of two possible frequencies as short bursts of a carrier wave. At the receiver the signals are modeled as:

$$s_0(t) = \begin{cases} \sqrt{2}A\cos(2\pi f_0 t + \phi_0) & 0 \le t \le T \\ 0 & t < 0 \text{ or } t > T \end{cases}$$

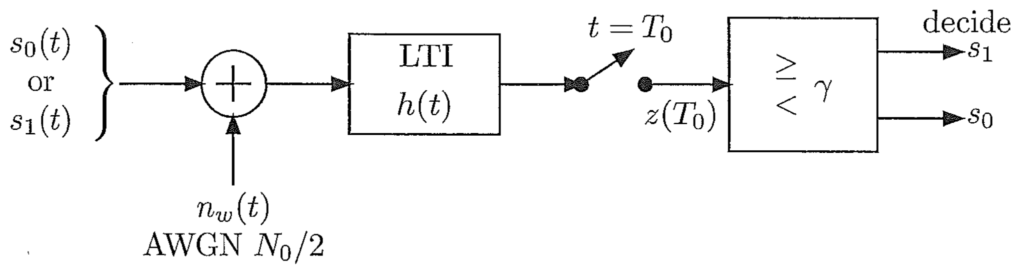
$$s_1(t) = \begin{cases} \sqrt{2}A\cos(2\pi f_1 t + \phi_1) & 0 \le t \le T \\ 0 & t < 0 \text{ or } t > T \end{cases}.$$

Usually the bit length and the frequencies are picked to satisfy a condition such that $f_0 T$, $f_1 T$ are distinct integers, so we assume this. For a non-coherent receiver it is assumed that the signal phases ϕ_0 and ϕ_1 are unknown and that no attempt is made to estimate them.

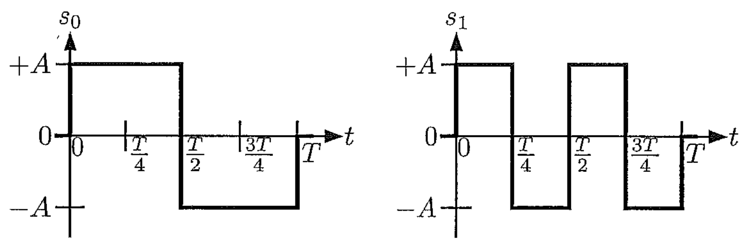
Assuming that the two signals are equally likely to have been sent and that they are received in additive white Gaussian noise draw the block diagram of the optimal non-coherent receiver for this problem. You may use local oscillators, mixers (i.e., multipliers), integrators, memoryless non-linearities, adders, and comparators. You may assume perfect knowledge of the signaling interval, $0 \le t \le T$, and the frequencies f_0 and f_1.

Give an engineering common sense explanation for how the non-coherent receiver is able to work in the absence of phase information.

Write in Exam Book Only



Problem 3. [30 pts. total] Consider the receiver shown above and the two signals $s_0(t)$ and $s_1(t)$ shown below. The channel is an additive white Gaussian noise channel with psd height $N_0/2$. Assume that the two signals are equally likely to be selected for transmission.



(a) [15 pts.] Find and plot the impulse response of a filter $h(t)$ which achieves the minimum average error probability in the optimal receiver for this problem. Simplify as much as possible and explain your work.

(b) [15 pts.] Find the optimum sampling time and optimum threshold for the receiver which uses the filter of part (a). Explain your work.