Problem 1. [30 points]

(a) Consider the discrete-time signal below.

$$x[n] = (0.5)^n \, e^{j\frac{\pi}{2}n} \, \{u[n] - u[n-5]\}$$

Determine a closed-form expression for the DTFT, $X(\omega)$, of $x[n]$. Show all work.

(b) Show how your answer to 1(a) changes for the discrete-time signal below.

$$y[n] = (0.5)^n \, e^{j\frac{\pi}{2}n} \, \{u[n-2] - u[n-7]\}$$

Determine a closed-form expression for the DTFT, $Y(\omega)$, of $y[n]$.

(c) The damped sinusoidal signal $x_a(t) = e^{-4ln(2)t} e^{j2\pi t} \{u(t) - u(t-1)\}$ is sampled every $T_s = 0.25$ seconds to form $x[n] = x_a(nT_s)$, where, again, T_s is a quarter of a second. Determine a closed-form expression for the DTFT $X(\omega)$ of the $x[n]$ thus obtained. *Hint:* $ln(2)$ equal to the natural logarithm of 2 was chosen to make the numbers work out nicely; same with the factor of 4 in the exponent. Recall $e^{ln(x)} = x$.

$$x[n] = x_a(nT_s) \quad \text{where: } T_s = 0.25 \text{ secs} \quad \text{and} \quad x_a(t) = e^{-4ln(2)t} e^{j2\pi t} \{u(t) - u(t-1)\}$$

Problem 2. [20 points] Consider a causal FIR filter of length $M = 9$ with impulse response as defined below:

$$h_p[n] = \sum_{\ell=-\infty}^{\infty} \frac{\sin\left[\pi\left(n + \frac{1}{2} + \ell 9\right)\right]}{\pi\left(n + \frac{1}{2} + \ell 9\right)} \{u[n] - u[n-9]\}$$

(a) Determine the 9-pt DFT of $h_p[n]$, denoted $H_9(k)$, for $0 \le k \le 8$. You can EITHER write an expression for $H_9(k)$, OR list the numerical values: $H_9(0) =?$, $H_9(1) =?$, $H_9(2) =?$, $H_9(3) =?$, $H_9(4) =?$, $H_9(5) =?$, $H_9(6) =?$, $H_9(7) =?$, $H_9(8) =?$.

(b) Consider the sequence $x[n]$ of length $L = 9$ below, equal to a sum of 9 finite-length sinewaves.

$$x[n] = \sum_{k=0}^{8} e^{jk\frac{2\pi}{9}n} \{u[n] - u[n-9]\}$$

$y_9[n]$ is formed by computing $X_9(k)$ as an 9-pt DFT of $x[n]$, $H_9(k)$ as a 9-pt DFT of $h[n]$ and, finally, then $y_9[n]$ is computed as the 9-pt inverse DFT of $Y_9(k) = X_9(k)H_9(k)$. Express the result $y_9[n]$ as a weighted sum of finite-length sinewaves similar to how $x[n]$ is written above.

Problem 3. [50 points] Let $x[n]$ and $y[n]$ be DT signals with autocorrelations and cross-correlations defined in terms of convolution as below.

$$r_{xx}[\ell] = x[\ell]*x^*[-\ell] \quad r_{yy}[\ell] = y[\ell]*y^*[-\ell] \quad r_{xy}[\ell] = x[\ell]*y^*[-\ell] \quad r_{yx}[\ell] = y[\ell]*x^*[-\ell] \quad (1)$$

(a) Consider the case where $x[n]$ and $y[n]$ are both causal, finite-length signals of duration N. That is, $x[n]$ and $y[n]$ are both only nonzero for $n = 0, 1, ..., N-1$. A concatenated signal of length $2N$ is formed as below:

$$z[n] = x[n] + y[n-N] \quad (2)$$

Express the autocorrelation, $r_{zz}[\ell]$, for $z[n]$ in terms of $r_{xx}[\ell]$, $r_{yy}[\ell]$, $r_{xy}[\ell]$, and $r_{yx}[\ell]$.

(b) Consider a case where $x[n]$ & $y[n]$ form a complementary pair of +1's and -1's satisfying

$$r_{xx}[\ell] + r_{yy}[\ell] = 2N\delta[\ell] \quad (3)$$

Simplify your answer for $r_{zz}[\ell]$ in part (a) for this special case.

(c) Consider the Barker codes of length 4 below, where the first value corresponds to $n = 0$.

$$x[n] = \{1, 1, 1, -1\} \qquad y[n] = \{1, 1, -1, 1\} \quad (4)$$

such that $z[n] = x[n] + y[n-N]$ is $z[n] = \{1, 1, 1, -1, 1, 1, -1, 1\}$. Determine the autocorrelation $r_{zz}[\ell]$ for $z[n]$ using the results that you derived above (you can compare to a direct calculation of $r_{zz}[\ell]$ to check your answer.)

(d) Repeat the steps above for the case where the concatenated signal of length $2N$ is formed as below with a negative sign on the second term.

$$w[n] = x[n] - y[n-N] \quad (5)$$

Express the autocorrelation, $r_{ww}[\ell]$, for $w[n]$ in terms of $r_{xx}[\ell]$, $r_{yy}[\ell]$, $r_{xy}[\ell]$, and $r_{yx}[\ell]$.

(e) Consider a case where $x[n]$ & $y[n]$ form a complementary pair of +1's and -1's satisfying

$$r_{xx}[\ell] + r_{yy}[\ell] = 2N\delta[\ell] \quad (6)$$

Simplify your answer for $r_{ww}[\ell]$ in part (d) for this special case.

(f) Again, let $x[n] = \{1, 1, 1, -1\}$ and $y[n] = \{1, 1, -1, 1\}$ be Barker codes of length 4, such that

$$w[n] = x[n] - y[n-N] = \{1, 1, 1, -1, -1, -1, 1, -1\} \quad (7)$$

Determine the autocorrelation $r_{ww}[\ell]$ for $z[n]$ using the results derived above.

(g) Sum your answers to parts (b) and (e) to determine $r_{vv}[\ell] = r_{zz}[\ell] + r_{ww}[\ell]$, Do a stem plot of the sum $r_{vv}[\ell]$. Compare against the sum of your answers to (c) and (f).

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