CS-1
August 2015 QE

1. (25 points) If X and Y are independent Poisson random variables with respective parameters λ_1 and λ_2, calculate the conditional probability mass function of X given that $X + Y = n$.

2. (25 points) Let $Z(t), t \geq 0$, be the random process obtained by switching between the values 0 and 1 according to the event times in a counting process $N(t)$. Let $P(Z(0) = 0) = p$ and

$$P(N(t) = k) = \frac{1}{1 + \lambda t}\left(\frac{\lambda t}{1 + \lambda t}\right)^k$$

for $k = 0, 1, \ldots$. Find the pmf of $Z(t)$.

3. (25 points) Let X and Y be independent identically distributed exponential random variables with mean μ. Find the characteristic function of $X + Y$.

4. (25 points) Consider a sequence of independent and identically distributed random variables $X_1, \ldots, X_n$, where each X_i has mean $\mu = 0$ and variance σ^2. Show that for every $i = 1, \ldots, n$ the random variables S_n and $X_i - S_n$, where $S_n = \sum_{j=1}^{n} X_j$ is the sample mean, are uncorrelated.