FO-3
August 2014 QE

1. (30 points)

   Consider a capacitor with vacuum between the plates and operated in the low-field condition so that there is no breakdown, with electrostatic capacitance $C \equiv Q/V$, with Q the charge in the positive plate and V the potential difference between the plates. The capacitor has electrical connections with lossless wires that carry some time-varying current $i(t)$. Starting with Ampere's law and making use of Gauss's law for the electric field, derive an expression for the relationship between the current in the wire, $i(t)$, and the voltage across the plates of the capacitor, $v(t)$.

2. (30 points)

   Consider an inductor with inductance $L = \Phi/i$, where Φ is the total flux linked and $i(t)$ is the time-varying current in the lossless wire from which the inductor is formed. Starting with Faraday's law, derive an expression for the relationship between the current in the wire, $i(t)$, and the voltage across the terminals of the inductor.

3. (40 points)

   Consider time harmonic fields, where the sinusoidal steady state circular frequency is ω and the time convention is $\exp(j\omega t)$. A plane wave (with peak field E_0) is normally incident (in the z-direction) in Region 1 onto an infinite planar surface with the semi-infinite Region 2.

   (a) For the case where Region 2 is a perfect electric conductor (PEC), find an expression for the Region 1 electric field, $\mathbf{E}_1(z,t)$.

   (b) Now consider Region 2 as a good conductor, defined as $\sigma >> \omega\epsilon$, where σ is the conductivity and ϵ is the permittivity. Find an expression for the time-average power flow into the conductor in terms of the electric field reflection coefficient at the surface, $\Gamma(0)$. Also find an expression for the skin depth (δ), the depth into the conductor at which the field has decayed to e^{-1} of its value at the surface of the metal ($z = 0$).

Write in Exam Book Only

Potentially Useful Information

$$\begin{aligned}\epsilon_0 &\approx \frac{1}{36\pi \times 10^9} \text{ F/m} \\ c &\approx 3 \times 10^8 \text{ m/s} \\ e &\approx 1.6 \times 10^{-19} \text{ C}\end{aligned}$$

$$\nabla \cdot \bar{D} = \rho \qquad \oint_S \bar{D} \cdot d\bar{S} = Q_{enc}$$

$$\nabla \cdot \bar{B} = 0 \qquad \oint_S \bar{B} \cdot d\bar{S} = 0$$

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} \qquad \oint_c \bar{E} \cdot d\bar{l} = -\frac{d}{dt}\int_S \bar{B} \cdot d\bar{S}$$

$$\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t} \qquad \oint_c \bar{H} \cdot d\bar{l} = I + \frac{d}{dt}\int_S \bar{D} \cdot d\bar{S}$$

$$\bar{D} = \varepsilon_0 \bar{E} + \bar{P} \quad \text{and if} \quad \bar{P} = \varepsilon_0 \chi_E \bar{E} \quad \text{then} \quad \bar{D} = \varepsilon_0 \varepsilon_r \bar{E} = \varepsilon \bar{E}$$

$$\bar{B} = \mu_0(\bar{H} + \bar{M}) \quad \text{and if} \quad \bar{M} = \chi_H \bar{H} \quad \text{then} \quad \bar{B} = \mu_0 \mu_r \bar{H} = \mu \bar{H}$$

$$W_E = \frac{1}{2}\int_V \bar{D} \cdot \bar{E}\, dv$$
$$W_H = \frac{1}{2}\int_V \bar{B} \cdot \bar{H}\, dv$$

$$\int \nabla \cdot \bar{A} dv = \oint \bar{A} \cdot d\bar{S}$$

$$\int \nabla \times \bar{A} \cdot d\bar{S} = \oint_s \bar{A} \cdot d\bar{l}$$

For $\exp(j\omega t)$ time convention, the complex form of Maxwell's equations in an isotropic material can be written

$$\begin{aligned}\nabla \cdot \bar{D} &= \rho \\ \nabla \cdot \bar{B} &= 0 \\ \nabla \times \bar{E} &= -j\omega\mu\bar{H} \\ \nabla \times \bar{H} &= (\sigma + j\omega\epsilon)\bar{E},\end{aligned}$$

where ϵ is the permittivity and σ the conductivity.

For spatial variation $\exp(-jkz) = \exp(-\gamma z)$,

$$\begin{aligned}\gamma &= \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} \\ &= jk\end{aligned}$$