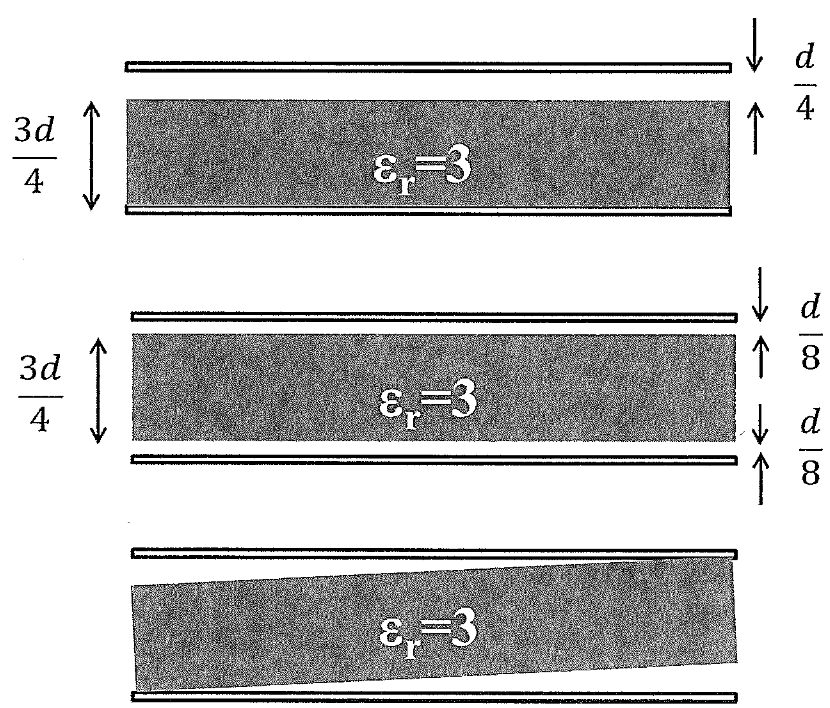
FO-1
August 2014 QE

Problem 1 (20 points):

You are asked to determine the total charge of a conductor that has an irregular shape. You have a magic instrument that allows you to measure the electric potential without perturbing the original field, but you can only use it once, *i.e.* you can only measure the electric potential at one specific location. The potential at infinity is set to 0. However, you have unlimited quantity of metal sheets that you can mold into any shape with very high geometrical accuracy. You can also measure the distance and sizes of an object very accurately. Other than those, you don't have any more toys. Please describe the method you plan to use to determine the total charge of the conductor. Use appropriate formulas to strengthen your argument if needed.

Problem 2 (30 points):

Two parallel conducting plates with an area of A are separated by a distance of d which satisfies $A \gg d$. A dielectric slab of thickness 3d/4 and relative permittivity of $\varepsilon_r = 3$ is placed either next to the bottom plate or at the middle of the two plates. Calculate the capacitance for both scenarios. What would be the capacitance if the dielectric slab is tilted in-between the two parallel plates? Notice that the tilt angle is very small.



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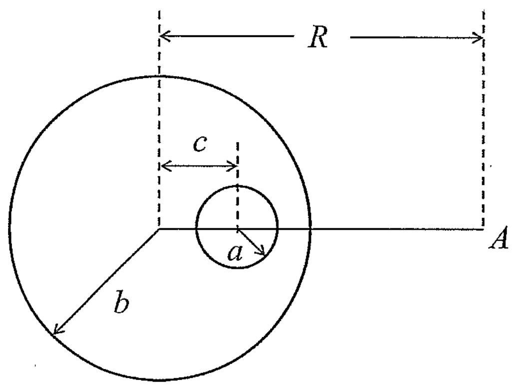
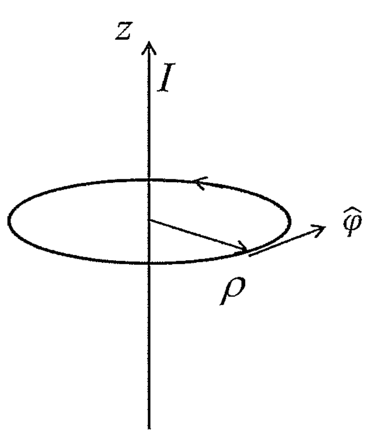
Problem 3 (30 points):

An infinitely long tubular conductor has outer radius b and inner radius a offset by a distance c from the axis of the outer cylinder, as shown below. The inner eccentric tubular conductor carries a uniform current density $\boldsymbol{J}$ going into the paper, and the outer cylinder carries a uniform current density $\boldsymbol{J'}$ that is going out of the paper. The net current is zero which means that the total currents going into and out of the paper have the identical value of I. Find the $\boldsymbol{H}$ field at point A shown below.



Problem 4 (20 points):

The magnetic field due to a steady current I of an infinitely long wire is $\boldsymbol{H} = \hat{\varphi}\frac{I}{2\pi\rho}$, where $\hat{\varphi}$ is one of the unit vectors of the cylindrical coordinate system $(\hat{\rho}, \hat{\varphi}, \hat{z})$, ρ is the distance from the wire, and I is the current. The current is flowing along the $+z$ direction. Bob is a math student and believes Maxwell's equations much more than any experimental results in physics. Can you convince him rigorously (remember Bob is a math major) from $\nabla \times \boldsymbol{H} = \boldsymbol{J}$ and $\nabla \cdot \mathbf{B} = 0$ that the magnetic field of an infinitely long wire current has components $H_z = 0$ and $H_\rho = 0$ at all locations? You can assume the space is non-magnetic thus $\mathbf{B} = \mu_0\boldsymbol{H}$.



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