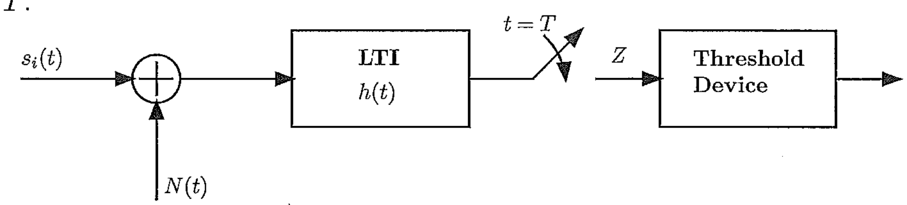
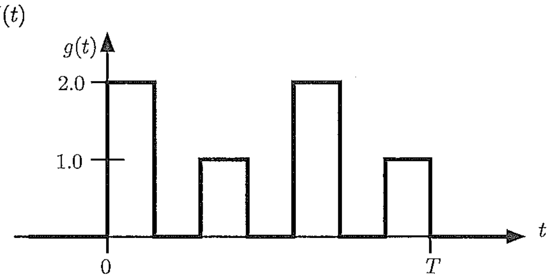
Problem 1. [50 pts. total] Consider the binary baseband communication and basic pulse $g(t)$, which are shown in the figures below. The noise $N(t)$ is AWGN with power spectral density height equal to $N_0/2$. The signals under the two hypotheses are $s_0(t) = \alpha_0 g(t)$ and $s_1(t) = \alpha_1 g(t)$ where $\alpha_0 > \alpha_1$. In this problem the performance criterion is average probability of error (i.e., it is a Bayes problem) where the priors on the two signals are $\pi_0 = \pi_1 = 1/2$.

Let hypothesis H_i correspond to the transmission of signal s_i for $i = 0, 1$. Assume that the LTI filter $h(t)$ is chosen to be the matched filter for pulse shape $g(t)$ and sampling time $t = T$.





(a) Find the probability density functions $f_i(z)$, $i = 0, 1$, for the decision statistic Z under the two hypotheses including writing the parameters of the pdfs in terms of α_0, α_1, N_0, and T.

(b) Starting from the likelihood ratio test for this Bayesian hypothesis test show that an equivalent test is of the form

$$Z \begin{array}{c} > \\ \leq \end{array} \gamma \quad \begin{array}{l} \text{decide } H_0 \\ \text{decide } H_1 \end{array}$$

and find the threshold γ for minimum average probability of error.

(c) Find the average probability of error in terms of α_0, α_1, N_0, and T.

(d) Characterize the loss in performance if the filter

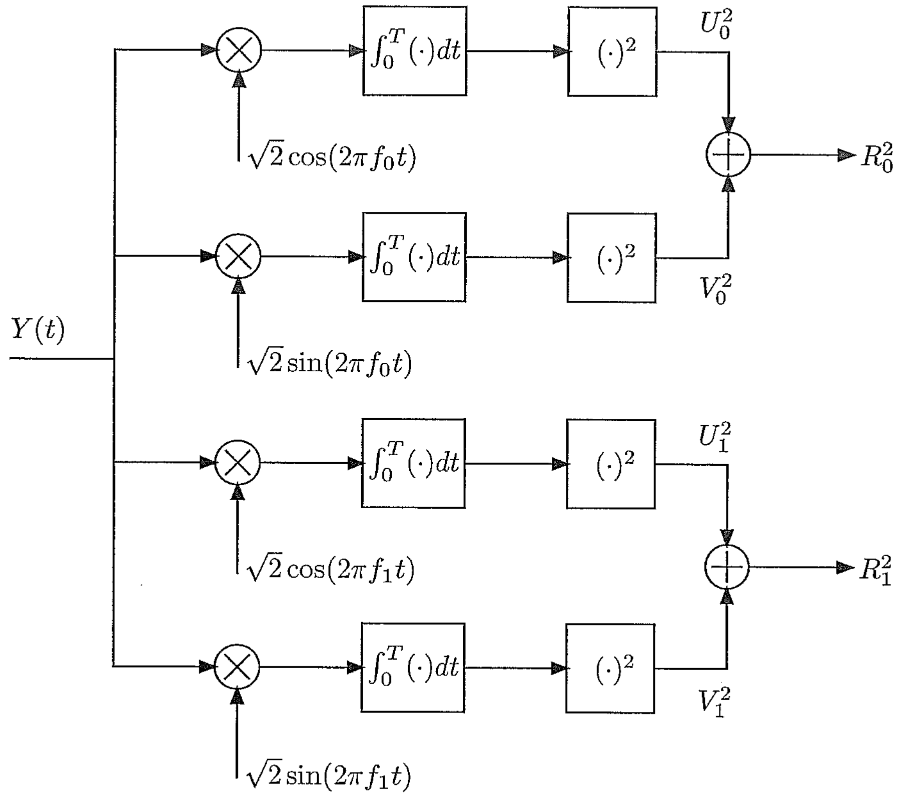
$$h(t) = \begin{cases} 1 & \text{for} \quad 0 \leq t \leq T \\ 0 & \text{for} \quad t < 0 \text{ or } t > T \end{cases}$$

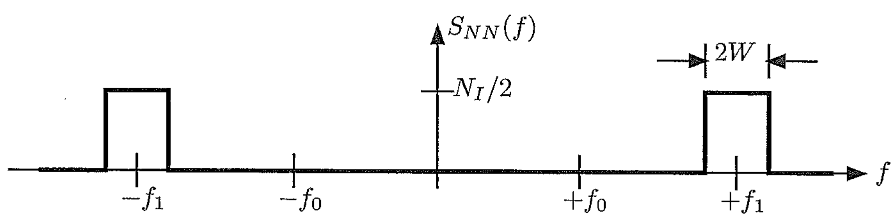
is used in place of the matched filter.

Problem 2. [50 pts. total] For the non-coherent BFSK receiver shown below suppose that

$$Y(t) = \sqrt{2}A\cos(2\pi f_0 t + \phi) + N(t)$$

for $0 \leq t \leq T$ where ϕ is a realization of a random variable uniform on $[0, 2\pi)$, $N(t)$ is a zero-mean Gaussian random noise with a flat-topped power spectral density centered at f_1 as illustrated, and phase angle and noise are statistically independent.





(a) Under certain assumptions the deterministic part of $Y(t)$ produces a nonzero response only in the f_0 path of the receiver. State and **justify** an adequate assumption for this to be true.

(b) Under certain assumptions the random noise part of $Y(t)$ produces a nonzero response only in the f_1 path of the receiver and this non-zero response looks just like a white noise is present at the input to the f_1 path. State and **justify** adequate assumptions for this to be true.

(c) Assuming that the implications mentioned in parts (a) and (b) hold, find the probabilty that $R_1 > R_0$. Show your work and express your answer in terms of the parameters A, T, and N_I.