CS-1
August 2014 QE

1. A coin is bent such that the probability of tail coming up is 3/4. The coin is tossed until a tail occurs. (a) Deermine the probability that the coin was tossed less than 10 times, (b) Determine the probability that an even number of tosses were made before a tail comes up.

(Hint: $\sum_{n=1}^{N} r^n = \frac{1-r^N}{1-r}$ $if\ 0<r<1$)

2. A random process is given by

$$X(t) = a\sin(2\pi ft + \theta)$$

where θ is uniformly distributed in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. Determine (a) the autocorrelation function of $X(t)$, (b) the power spectral density of $X(t)$.

3. Consider random sequence $X[k]$ of IID standard Gaussian random variables defined on $\Gamma = Z^+$ (positive integer time). Let

$$Y[k] = X[k] + \alpha X[k-1]$$

where $k \in \Gamma$, $\alpha \in R$. (a) Find an expression for $R_{YY}[m]$, (b) Determine if $Y[k]$ is an independent increment sequence.

4. (a) Prove that the covariance function $C\left(t_i, t_j\right)$ of a random process $X(t)$ is positive semidefinite (hint: a function $z\left(t_i, t_j\right)$ is positive semidefinite if $\sum_i \sum_j a_i a_j^* z\left(t_i, t_j\right) \geq 0$)

(b) A and B are IID, normally distributed RVs with mean zero and variance σ^2. A random process is defined by

$$X(t) = A\cos(wt) + B\cos(wt), -\infty < t < \infty$$

Show that $X(t)$ is a Gaussian process.

Each Question is 25 Points

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