

August 2014 QE

Consider the following game. Two players, A and B, alternate adding stones to a pile. The pile starts out empty. The stones are not differentiated by player. All that matters is the number of stones in the pile. Player A goes first. At each turn, a player flips an unbiased coin. If it comes up heads, the player must add either one or two stones to the pile in that turn. If it comes up tails, the player must add either two or three stones to the pile in that turn. The game ends when the pile has seven or more stones at the end of a turn. If the pile has exactly seven stones at the end of a turn, the player whose turn just ended wins. If the pile has more than seven stones at the end of a turn, the player whose turn just ended loses.

Describe in English how you construct the full game tree for this game and determine the expected outcome distribution at each game state (what is the probability that player A will win and the expected probability that player B will win) given optimal play (30% credit). There is no depth bound and no static evaluator. Draw this game tree (20% credit). Represent the game tree as a directed acyclic graph that shares equivalent nodes reachable through different move sequences. At each game state indicate the expected outcome distribution (30% credit). Indicate the optimal move in each game state (20% credit).