FO-3
August 2012 QE

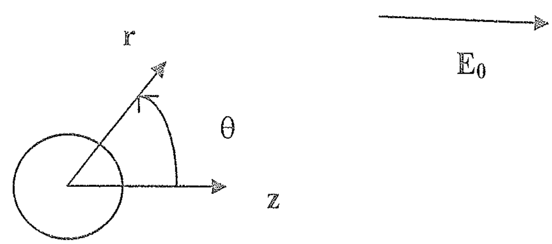
Problem 1. (50 points)

An insulating sphere with radius a is immersed in a conducting medium with conductivity σ and relative dielectric constant ε_r. There is a uniform background electric in the conductor and far from the sphere the electric field is $\mathbf{E}=E_0\hat{\mathbf{z}}$.

Let the origin of the sphere coordinate system be at the center of the sphere. Assume that the electric field is DC - i.e. it is steady in time.

(a) (45 points) Derive an expression for the current density in the region outside the sphere.

(b) (5 points). What is the current density inside the sphere? Explain your answer.



Possibly useful information.

With ϕ symmetry, Laplace's equation in spherical coordinates is

$$\frac{\partial}{\partial r}\left(r^2\frac{\partial\Phi}{\partial r}\right)+\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Phi}{\partial\theta}\right)=0 \qquad (1)$$

The solution to Eq. 1 is

$$\Phi(r,\theta)=\sum_{n=0}^{\infty}\left(A_n r^n+\frac{B_n}{r^{n+1}}\right)P_n(\cos\theta) \qquad (2)$$

P_n are Legendre polynomials. The first few of which are

$P_0=1$ $\qquad P_1=\cos\theta$ $\qquad P_2=\frac{1}{2}\left(3\cos^2\theta-1\right)$

$$\nabla\Phi=\frac{\partial\Phi}{\partial r}\hat{\mathbf{r}}+\frac{1}{r}\frac{\partial\Phi}{\partial\theta}\hat{\boldsymbol{\theta}} \qquad (3)$$

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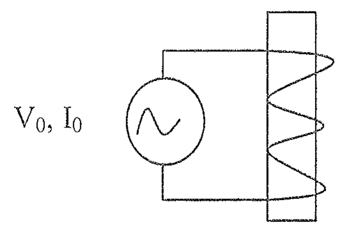
Problem 2. (50 points)

A solenoidal coil is filled with water with NaCl added to yield a conductivity σ (units S/m). The height of the solenoid is h, radius is a, and there are N turns. Assume h>>a. The winding is connected to an continuous wave electric power source with angular frequency ω, voltage amplitude V_0 and current amplitude I_0.

Assume that the conductivity and frequency are sufficiently low so that the magnetic field produced by the eddy currents in the salt water is small compared to the magnetic field produced by the coil.



(a) (20 points). Derive an expression for the magnetic field inside the solenoid.

(b) (20 points). Derive an expression for the average power dissipation in the salt water inside the solenoid.

(c) (10 points). What is the impedance $Z=V_o/I_o$ of the solenoid?

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