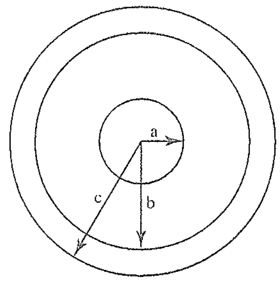
The coaxial cable shown has two concentric conductors indicated by the gray shaded regions and is translationally invariant in the direction normal to the page. You can assume a coordinate system with the z-axis normal to the page and also that the material between the two conductors (in the radial distance range $a < \rho < b$) has a relative dielectric constant ϵ_r and a conductivity σ.



1. (30 points) Find the static electric field everywhere, assuming a uniform surface charge density at the surface $\rho = a$ of ρ_{sa} C/m^2.
2. (20 points) Find an expression for the capacitance per unit length for this coaxial cable.
3. (20 points) Find an expression for the magnetic field in the region $a < \rho < b$, under the assumption of a uniform current I within the inner conductor ($\rho < a$).
4. (15 points) Find an expression for the external inductance per unit length, that is, the inductance per unit length due to flux linkage over $a < \rho < b$, under the assumption that the material in this space in non-magnetic.
5. (15 points) Find an expression for the conductance per unit length due the dielectric leakage current.

Write in Exam Book Only

Potentially Useful Information

$$\epsilon_0 \approx \frac{1}{36\pi \times 10^9} \text{ F/m}$$
$$c \approx 3 \times 10^8 \text{ m/s}$$
$$e \approx 1.6 \times 10^{-19} \text{ C}$$

$$\nabla \cdot \bar{D} = \rho \qquad \oint_S \bar{D} \cdot d\bar{S} = Q_{enc}$$
$$\nabla \cdot \bar{B} = 0 \qquad \oint_S \bar{B} \cdot d\bar{S} = 0$$
$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} \qquad \oint_c \bar{E} \cdot d\bar{l} = -\frac{d}{dt} \int_S \bar{B} \cdot d\bar{S}$$
$$\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t} \qquad \oint_c \bar{H} \cdot d\bar{l} = I + \frac{d}{dt} \int_S \bar{D} \cdot d\bar{S}$$

$\bar{D} = \varepsilon_0 \bar{E} + \bar{P}$ and if $\bar{P} = \varepsilon_0 \chi_E \bar{E}$ then $\bar{D} = \varepsilon_0 \varepsilon_r \bar{E} = \varepsilon \bar{E}$

$\bar{B} = \mu_0 (\bar{H} + \bar{M})$ and if $\bar{M} = \chi_H \bar{H}$ then $\bar{B} = \mu_0 \mu_r \bar{H} = \mu \bar{H}$

$$W_E = \frac{1}{2} \int_V \bar{D} \cdot \bar{E} \, dv$$
$$W_H = \frac{1}{2} \int_V \bar{B} \cdot \bar{H} \, dv$$

$$\int \nabla \cdot \bar{A} dv = \oint \bar{A} \cdot d\bar{S}$$

$$\int \nabla \times \bar{A} \cdot d\bar{S} = \oint_s \bar{A} \cdot d\bar{l}$$

Write in Exam Book Only