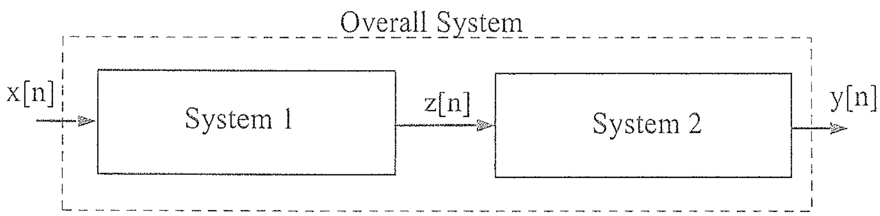
# CS-2
# August 2012 QE



Problem 1. Consider two discrete-time LTI systems in series.




(a) System 1 is described by the following difference equation

$$z[n] = z[n-1] + x[n] - x[n-6] \tag{1}$$

Determine and plot (stem-plot) the impulse response $h_1(n)$ of System 1.

(b) The frequency response $|H_1(\omega)|$ of System 1 is the DTFT of the impulse response $h_1[n]$. Plot the magnitude, $|H_1(\omega)|$, of the frequency response of System 1 over $-\pi < \omega < \pi$.

(i) Explicitly list all frequencies within the range $-\pi < \omega \leq \pi$ for which $H(\omega) = 0$.

(ii) Explicitly state the numerical value of $H(0)$.

(c) The input signal is obtained from sampling a continuous-time signal as

$$x[n] = x_a(nT_s), \qquad x_a(t) = u(t) - u(t-10) \qquad \text{and} \qquad T_s = 4$$

where $u(t)$ is the unit step. Determine and plot (stem-plot) the intermediate output $z(n)$ obtained with this input.

(d) The second system is described by the following difference equation.

$$y[n] = \frac{1}{4}y[n-1] + z[n] - 4z[n-1] \tag{2}$$

Determine and plot the magnitude, $|H_2(\omega)|$, of the frequency response of System 2 over $-\pi < \omega < \pi$.

(e) Determine and plot the magnitude, $|Y(\omega)|$, of the DTFT of the output $y[n]$ obtained with the input $x[n]$ defined in part (c). Clearly indicate the frequencies for which $Y(\omega) = 0$ over $-\pi \leq \omega \leq \pi$ .

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**Problem 2** Consider the signal $x_p(t)$ below, which is the Fourier Series expansion for a periodic sawtooth waveform with period $T = 1$ sec.

$$x_p(t) = \sum_{k=-\infty}^{-1} \frac{j(-1)^k}{k\pi} e^{j2\pi kt} + \sum_{k=1}^{\infty} \frac{j(-1)^k}{k\pi} e^{j2\pi kt}$$

This signal is first low-passed filtered with an analog lowpass filter having the impulse response

$$h_{LP}(t) = \frac{1}{2}\frac{\sin(2\pi 8t)}{\pi t}\frac{\sin(2\pi t)}{\pi t}$$

to form $x(t) = x_p(t) * h_{LP}(t)$ and then $x(t)$ is sampled at a rate of $F_s = 16$ samples/sec to form $x[n]$. $x[n]$ is then the input to a Discrete Time LTI system with impulse response

$$h[n] = 8\,\frac{\sin\left(\frac{\pi}{2}n\right)}{\pi n}\frac{\sin\left(\frac{\pi}{8}n\right)}{\pi n} \qquad (3)$$

Show all work and write your expression for the output $y[n] = x[n] * h[n]$ in the space below.

**Problem 3**

(a) Let $H_0(\omega)$ be the Discrete Time Fourier Transform (DTFT) of the impulse response $h_0[n]$ defined below.

$$h_0[n] = 2\frac{\sin\left(\frac{\pi}{4}n\right)}{\pi n}\sin\left(\frac{\pi}{4}n\right) \qquad (4)$$

Note that $h_0[n]$ is both real-valued and odd-symmetric as a function of time. Thus, $H_0(\omega)$ is purely imaginary-valued and odd-symmetric as a function of frequency. Plot the magnitude $|H_0(\omega)|$ and the phase $\angle H_0(\omega)$ over $-\pi < \omega < \pi$.

(b) Determine and plot the DTFT $X(\omega)$ over $-\pi < \omega < \pi$ of the signal $x[n]$ below:

$$x[n] = \frac{\sin\left(\frac{\pi}{2}n\right)}{\pi n} + \frac{1}{2}\left\{\frac{\sin\left(\frac{\pi}{2}(n-2)\right)}{\pi(n-2)} + \frac{\sin\left(\frac{\pi}{2}(n+2)\right)}{\pi(n+2)}\right\}$$

(c) Determine and plot the Fourier Transform for the signal $y_0[n]$ defined below, where $\hat{x}_0[n] = x[n] * h_0[n]$ with $h_0[n]$ and $x[n]$ defined in parts (a) and (b), respectively. Plot $Y_0(\omega)$ over $-\pi < \omega < \pi$.

$$y_0[n] = x[n] + j\hat{x}_0[n] \quad \text{where:} \quad \hat{x}_0[n] = x[n] * h_0[n]$$

(d) Determine and plot the Fourier Transform for the signal $z_0[n]$ defined below where, as defined previously, $\hat{x}_0[n] = x[n] * h_0[n]$ with $h_0[n]$ and $x[n]$ defined in parts (a) and (b), respectively. Plot $Z_0(\omega)$ over $-\pi < \omega < \pi$.

$$z_0[n] = x[n]\cos\left(\frac{\pi}{2}n\right) - \hat{x}_0[n]\sin\left(\frac{\pi}{2}n\right)$$

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