CE-3
August 2012 QE

Problem A (30%)

Part 1 (10%)
Define the notion of a *constraint satisfaction problem*.

Part 2 (10%)
Define what it means for a constraint satisfaction problem to be *arc consistent*.

Part 3 (10%)
Give an algorithm for making a constraint satisfaction problem arc consistent.

Problem B (70%)
You are given a video that has T frames. We use $t = 1, \ldots, T$ to denote the indices of the frames. A person detector is run on each frame to yield an ordered set of detections for each frame. There are D_t detections for frame t. We denote the d-th detection, $d = 1, \ldots, D_t$, for frame t as b_{td}. Each detection b is an axis-aligned rectangle (box) specified by four numbers: the x coordinates of the left and right edges, left(b) and right(b), and the y coordinates of the top and bottom edges, top(b) and bottom(b). We assume a Cartesian coordinate frame with x increasing upward and y increasing rightward.

We wish to construct an *interpretation* I of the video. An interpretation has R *tracks*. We use $r = 1, \ldots, R$ to denote the indices of the tracks. Each track contains exactly one detection from each frame. We use I_{rt} to denote the index d of the detection for track r in frame t.

A box b overlaps a box b' iff $\mathit{overlaps}(b, b')$, which is defined as follows:

$$\mathit{overlaps}(b,b') \triangleq \left(\begin{array}{l} \text{left}(b) < \text{right}(b')\wedge \\ \text{left}(b') < \text{right}(b)\wedge \\ \text{bottom}(b) < \text{top}(b')\wedge \\ \text{bottom}(b') < \text{top}(b) \end{array}\right)$$

The center of a box b, denoted $\mathbf{c}(b)$, is $\left(\frac{\text{right}(b)-\text{left}(b)}{2}, \frac{\text{top}(b)-\text{bottom}(b)}{2}\right)$. If a track r contains a box b in frame t and a box b' in frame $t+1$ then we take its velocity $\mathbf{v}_{rt}$ in frame t to be $\mathbf{c}(b') - \mathbf{c}(b)$. Note that boxes don't have velocity; only frames of tracks in the context of an interpretation do and tracks do not have velocity in the last frame.

If track r has velocities $\mathbf{v}_{rt}$ and $\mathbf{v}_{r,t+1}$ in frames t and $t+1$ respectively, we say that it *changes direction* in frame t iff the orientations of $\mathbf{v}_{rt}$ and $\mathbf{v}_{r,t+1}$ differ by more than 90° (i.e., $\mathbf{v}_{rt} \cdot \mathbf{v}_{r,t+1} < 0$). We say that a track has a *velocity spike* in frame t iff it changes direction in frames t and $t+1$. Let b, b', b'', and b''' denote the boxes for track r in frames t, $t+1$, $t+2$, and $t+3$ respectively. Track r has a velocity spike in frame t iff $\mathit{spike}(b, b', b'', b''')$, which is defined as follows:

$$\mathit{spike}(b,b',b'',b''') \triangleq \left(\begin{array}{l} (\mathbf{c}(b') - \mathbf{c}(b)) \cdot (\mathbf{c}(b'') - \mathbf{c}(b')) < 0\wedge \\ (\mathbf{c}(b'') - \mathbf{c}(b')) \cdot (\mathbf{c}(b''') - \mathbf{c}(b'')) < 0 \end{array}\right)$$

Let a and p denote the indices of two tracks, the *agent* and the *patient*. We say that track a *approaches* track p in frame t iff track p has zero velocity in frame t and the distance between the centers of the boxes from tracks a and p decreases from frame t to frame $t+1$. We say that track a *chases* track p in frame t iff the orientations of the velocities of tracks a and p in frame t differ by less than 90° (i.e., $\mathbf{v}_{at} \cdot \mathbf{v}_{pt} > 0$) and track a is moving faster than track p in frame t. Let b and b' denote the boxes for track a in frames t

Write in Exam Book Only

and $t+1$ respectively and let b'' and b''' denote the boxes for track p in frames t and $t+1$ respectively. Track a approaches track p in frame t if $\mathit{approaches}(b, b', b'', b''')$, which is defined as follows:

$$\mathit{approaches}(b, b', b'', b''') \stackrel{\triangle}{\leftrightarrow} \left(\begin{array}{l} \mathrm{c}(b'') = \mathrm{c}(b''')\wedge \\ \|\mathrm{c}(b') - \mathrm{c}(b''')\| < \|\mathrm{c}(b) - \mathrm{c}(b'')\| \end{array} \right)$$

Track a chases track p in frame t iff $\mathit{chases}(b, b', b'', b''')$, which is defined as follows:

$$\mathit{chases}(b, b', b'', b''') \stackrel{\triangle}{\leftrightarrow} \left(\begin{array}{l} ((\mathrm{c}(b') - \mathrm{c}(b)) \cdot (\mathrm{c}(b''') - \mathrm{c}(b''))) > 0\wedge \\ \|(\mathrm{c}(b') - \mathrm{c}(b))\| > \|(\mathrm{c}(b''') - \mathrm{c}(b''))\| \end{array} \right)$$

We say that an interpretation depicts *approaching* if track 1 approaches track 2 in every frame. We say that an interpretation depicts *chasing* if track 1 chases track 2 in every frame.

We impose the following consistency conditions on interpretations:

1. No two tracks can contain the same box in any frame.
2. No two tracks can have overlapping boxes in any frame.
3. No track can have a velocity spike in any frame.
4. An interpretation must depict either approaching or chasing.

The above four consistency conditions are pre-theoretic, and specified in a somewhat informal fashion. In parts 1 and 2 below you will need to formally specify these four consistency conditions more precisely using mathematical notation.

Part 1 (50%)

Describe a procedure that take an arbitrary video as input, in the form of the output of a person detector, and reduces the problem of finding consistent interpretations to a constraint satisfaction problem. You should support videos with an arbitrary number of frames T, arbitrary numbers of detections D_t in each frame t, and an arbitrary number of tracks R specified as input to the reduction. The videos will be specified as the boxes b_{td} with $d = 1, \ldots, D_t$ and $t = 1, \ldots, T$. Describe your answer in English and mathematical notation using the notation defined above.

Part 2 (20%)

Give an example in English of a single arc-consistency inference that might conceivably occur when processing the constraint-satisfaction problem generated by the above reduction for a video to ensure arc consistency. If you wish, you can assume that multiple constraints asserted between the same set of variables are coalesced into a single constraint. Also, if you wish, you can consider generalized forward checking to be a special case of arc consistency and report a single such inference.

Write in Exam Book Only